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REPORTS, PAPERS, DISCUSSIONS, AND MEMOIRS

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CONTENTS

Papers:	PAGE
Hydraulic Studies and Operating Results on the Miami Flood Control System. By FRANCIS LEE STUART, M. AM. SOC. C. E.	1391
Modernizing the Suburban Transit of the Metropolitan District. By FRANCIS LEE STUART, M. AM. SOC. C. E.	1391
The Design of Tall Building Frames to Resist Wind. By ALBERT W. ROSS, JR., JUN. AM. SOC. C. E., and CLYDE T. MORRIS, M. AM. SOC. C. E.	1395
Proposed Formula for Columns. By T. F. HICKERSON, M. AM. SOC. C. E.	1435
Effect of Earth Shocks on Structures. By M. DE BUSSY, ESQ.	1449
The Stiffness of Suspension Bridges. By S. TIMOSHENKO, ESQ.	1464
Discussions:	
Notes on Arched Gravity Dams. By B. F. JAKOBSEN, M. AM. SOC. C. E.	1479
Precise Weir Measurements. By MESSRS. R. E. BALLESTER and GARDNER S. WILLIAMS.	1493
Historic Review of the Development of Sanitary Engineering in the United States During the Past One Hundred and Fifty Years: A Symposium. By MESSRS. JAMES E. BROOKS, W. G. STROMQUIST, S. H. MCCRORY, and J. A. LE PRINCE.	1501
Problems in Concrete Dam Construction on the Pacific Coast. By Fred A. NOETZLI, M. AM. SOC. C. E.	1509
New Theory for the Centrifugal Pump. By MESSRS. R. T. LIDDICOAT, LLOYD H. DONNELL, and S. TIMOSHENKO.	1511
Water Power Appraisals. By MESSRS. J. H. LANCE and WILLIAM H. CUSHMAN.	1521
Side Spillways for Regulating Diversion Canals. By MESSRS. A. P. FLOCKART and GEORGE M. BACON.	1527
A Graphic Method for Determining the Stresses in Circular Arches Under Normal Loads by the Cain Formulas. By FRED A. NOETZLI, M. AM. SOC. C. E.	1531
Surveying and Mapping in the United States. By C. H. BIRDSEYE, M. AM. SOC. C. E.	1535
The Delaware River from Philadelphia to the Sea. By MESSRS. WILLIAM GERIG and F. C. BOGGS.	1537
Traffic Control by Electric Signal Lights. By M. O. ELDRIDGE, ASSOC. M. AM. SOC. C. E.	1541
Boundary Surveys. By MESSRS. ALEXANDER BLAIR and C. T. JOHNSTON.	1545
Baffle-Pier Experiments on Models of Pit River Dams. By MESSRS. RALPH L. PARSHALL, H. W. DENNIS, and I. A. WINTER.	1549
The Science of Foundations—Its Present and Future. By T. L. CONDRON, M. AM. SOC. C. E.	1561
Flood Control with Special Reference to the Mississippi River: A Symposium. By W. J. SCHLICK, M. AM. SOC. C. E.	1565
Emergency Dams on Inner Navigation Canal at New Orleans, Louisiana. By T. B. MÖNNICHE, M. AM. SOC. C. E.	1569
Unusual Engineering Features of an Immense Theatre Building. By MESSRS. THEODORE BELZNER and FRED A. NOETZLI.	1579
The Virginian Railway Electrification. By JOHN P. HOGAN, M. AM. SOC. C. E.	1581
Analysis of Arch Dams by the Trial Load Method. By MESSRS. C. A. P. TURNER, F. H. CONSTANT, and F. W. HANNA.	1585
Stream Flow in General Terms. By MESSRS. FRANK S. BAILEY and CECIL E. PEARCE.	1605
Design and Construction of Concrete Pavements. By MESSRS. G. E. HAWTHORN and H. O. ROOT.	1617

CONTENTS (*Continued*)

Discussions (*Continued*): PAGE

General Contract System Versus Segregated Contracts.
By MESSRS. C. C. MUHS, W. J. BARNEY, LAZARUS WHITE, L. S. STILES, GUSTAVE R. TUSKA, T. KENNARD THOMSON, and H. M. DOUGHERTY..... 1623

The O'Shaughnessy Dam and Reservoir.
By W. H. DITTOE and G. GALE DIXON, MEMBERS, AM. SOC. C. E..... 1635

Crown Stresses in a Skew Arch.
By MESSRS. B. F. JAKOBSEN and EDWARD GODFREY..... 1639

Inlets on Sandy Coasts.
By W. M. BLACK, M. AM. SOC. C. E..... 1645

Silting of the Lake at Austin, Texas.
By MESSRS. CHARLES SCHULTZ and JOHN B. HAWLEY..... 1649

Upward Pressure Under Dams: Experiments by the United States Reclamation Service.
By B. F. JAKOBSEN, M. AM. SOC. C. E..... 1653

Continuous Beams Over Three Spans.
By FRANK S. BAILEY, ASSOC. M. AM. SOC. C. E..... 1655

Progress Report of the Special Committee on Flood-Protection Data.
By MESSRS. GERARD H. MATTHES, F. G. SWITZER, HARRISON P. EDDY, C. E. GRUNSKY, ARTHUR E. MORGAN, A. N. TALBOT, A. J. HAMMOND, WILLIAM BOWIE, and JOHN R. FREEMAN..... 1657

Memoirs:

WARREN MARTIN ARCHIBALD, M. AM. SOC. C. E..... 1671
GEORGE ANSEL CARPENTER, M. AM. SOC. C. E..... 1672
ROBERT L'HOMMEDIEU TATE, M. AM. SOC. C. E..... 1674
PHILIP AYLETT, ASSOC. M. AM. SOC. C. E..... 1675
HUNTER IMBODEN SNYDER, ASSOC. M. AM. SOC. C. E..... 1677
FRANK WAGNER, ASSOC. M. AM. SOC. C. E..... 1678

For Index to all Papers, the discussion of which is current in *Proceedings*, see the second page of the cover.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

PAPERS AND DISCUSSIONS

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HYDRAULIC STUDIES AND OPERATING RESULTS ON THE MIAMI FLOOD CONTROL SYSTEM

BY C. H. EIFFERT,* M. AM. SOC. C. E.

SYNOPSIS

The flood control system of the Miami Conservancy District, a description of which has been published,† consists of a combination of retarding basins and channel improvements. The dams and retarding basins are five in number while channel improvements were carried out in eight cities and towns. The purpose of this paper is to state as fully as possible the results obtained thus far in the operation of the works of the District. The observations and measurements made to check the design are recorded.

The complete system has been in operation for about six years. Sufficient time therefore has not elapsed to make definite conclusions possible in all respects; but it is thought best to report at the present time and make known such information as is now available.

In general, the assumptions on which the designs of dams, conduits, and channels were based, have been confirmed. The discharge of the conduits under given heads has been somewhat larger than the quantities used in the calculations, but this was expected while the conduits were new. The outlet structures have fully met the requirements for dissipating the energy of the water as it leaves the conduits.

The improved channels at Dayton and Hamilton, Ohio, have maintained themselves better than was anticipated. The construction of cut-off channels has been justified. The dams at Dayton and Hamilton together with the gravel plants have greatly reduced the cost of channel maintenance.

Valuable information is being secured from the rainfall and run-off records. Additional information could be secured by the use of a number of recording rain gauges and recording back-water gauges at the dams. The installation of such gauges is being considered.

NOTE.—Written discussion on this paper will be closed in September, 1928.

* Chf. Engr. and Gen. Mgr., The Miami Conservancy Dist., Dayton, Ohio.

† *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1503.

Design

The flood-control works of the Miami Conservancy District are designed for a maximum flood 40% greater than that of 1913, although the studies which were made indicated that this maximum was larger than could occur in this locality. Should such a flood occur, however, it would just reach the spillways of the dams and in the improved channels it would reach maximum heights about 3 ft. below the tops of the levees. The system is so balanced that the maximum discharge through the conduits in the dams will not be greater than can be carried safely by the improved channels below. An important feature of the design is the fact that the flow through the conduits is not controlled by gates. These conduits are smooth open tunnels, the discharge being entirely automatic.

Some of the assumptions used in the design of the conduits and of the improved channels will be restated as it was to check these that measurements, observations, and investigations have been carried on.

Conduits.—The value of the roughness factor, n, in Kutter's formula used in the design of the conduits was 0.013. It was well known, of course, that the value for smooth concrete conduits was often lower than this, even as low as 0.010. It was thought possible, however, that the concrete surfaces would deteriorate somewhat in time, therefore 0.013 was considered a proper mean between conditions in the newly built conduits and those that might exist in the future.

In calculating the discharge through the conduits it was assumed that the only loss of head necessary to be considered was that due to friction. Every effort was made to design entrance and outlet portions of the structure to eliminate losses at these points.

In making the original discharge calculations the head was calculated from the top of the outlet end of the conduits. S. M. Woodward, M. Am. Soc. C. E., has reached the conclusion* that this head should be computed from a point somewhere between the extreme top and the average top of the outlet, although the exact point cannot be determined. In recalculating the discharge curves an elevation half way between the top and the average top has been used.

River Channels.—In designing the channel improvements, values of n from 0.0225 to 0.025 were adopted after a careful study of available data. The cross-section of the improved channels, in the more extensive channel improvements at Hamilton and Dayton, is a modified trapezoid (Fig. 1). This shape concentrates the low-water flow and is also satisfactory for high water. On the Miami River channel the width, *B-E*, varies from 530 to 650 ft.; the width, *C-D*, is 150 ft. in nearly all cases. The depth of maximum flow between Lines *A-F* and *C-D* is from 20 to 34 ft. Points *B* and *E* are, as a rule, 8 ft. above Line *C-D*. The low bottom Line *C-D*, varies in its location relative to the sides. Where the channel is straight, it is in the center; on curves, it is nearer the concave bank.

* Technical Repts., The Miami Conservancy Dist., Pt. VII, p. 144.

Stream Flow Measurements

A most important factor in the hydraulic computations are the stream flow measurements. On this work the latest improved methods, as developed by the U. S. Geological Survey, are being used throughout. Measurements are made from cableways at the dams and from bridges in the cities. Price current meters with 50-lb. weights of the new low-resistance shape are used. Meter and weight are suspended by a single $\frac{1}{8}$-in. airplane cable, which greatly reduces the resistance due to the former double insulated cable. As the weight and meter cannot be raised and lowered conveniently by hand with a $\frac{1}{8}$-in. wire, a small windlass is attached to each of the cable cars and a light, portable, folding derrick with a windlass is used on the bridges.

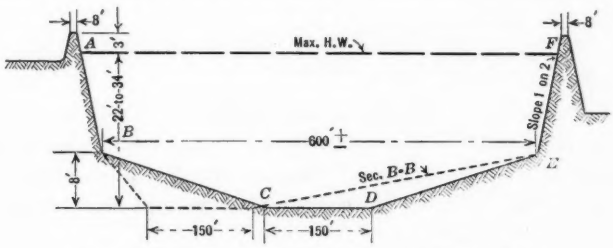

Fig. 1.—Typical Section of Improved Channel, The Miami Conservancy District.

Two men are required to take flood measurements. In most cases the use of the heavy weight and small cable eliminates the necessity of making corrections for the vertical angle; such corrections, however, are made when necessary. Current-meter readings are taken at 0.2 and 0.8 of the depth at 8 or 10-ft. intervals. Every possible precaution has been taken to eliminate error and the results appear to be very consistent.

In co-operation with the U. S. Geological Survey automatic recording gauges have been installed below the five dams and at the City of Hamilton. An automatic long-distance recorder for the river stage at Dayton is located in the District Office. Ordinary staff gauges, which are read daily, are maintained at a number of other locations by the District, the U. S. Geological Survey, and the U. S. Weather Bureau. The locations of all rain and river gauges are shown on Fig. 12.

The automatic gauges at the dams and at Hamilton are placed in concrete houses. The cost of these houses with their pipe intakes varies from $400 to $800, according to the nature of the excavation, length of intake, and height of house. The instruments themselves, known as the Au type of water-stage recorder, cost $200 each. They are very reliable, requiring attention not more than once a month. The pencil making the record is moved by a float in the well under the gauge house and the motion of the paper on which the record is made, is controlled by a weight-driven clock.

The first two installations were made at the expense of the District. Later ones were made by the District while the instruments were furnished by the U. S. Geological Survey. All records obtained are used jointly by the

District and the U. S. Geological Survey, the same being true of records obtained by the Geological Survey in other parts of the valley. Thus, more complete records are obtained than would be possible with either organization working alone.

The automatic recorder in the Dayton Office is a Stevens long-distance water-stage recorder. It consists of a sending apparatus located on a pier of the Main Street Bridge, with a recording apparatus located in the office of the District, all operated by 110-volt current. This instrument, while it is fairly satisfactory, requires more attention than the others.

Recently, some special staff gauges have been installed in Dayton and Hamilton for the purpose of obtaining surface slopes during high water. The results will be used for making further calculations of the coefficient of roughness, n.

Results

Conduits and Outlets

The measurements of outflow through conduits, herein given, are confined to the Englewood and Germantown Dams. At the other dams high stages are less frequent, and sufficient information has not yet been obtained to make definite conclusions possible.

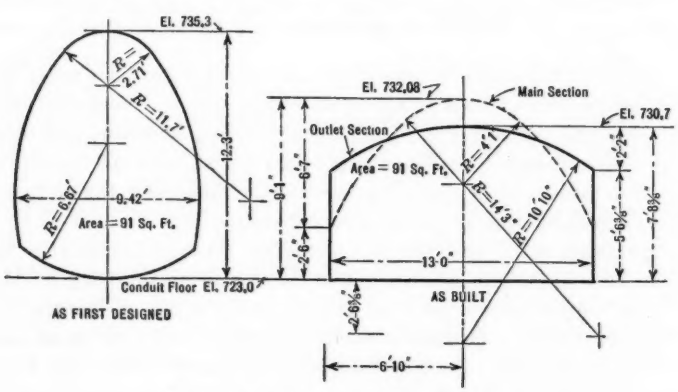

Fig. 2.—Germantown Conduits.

In Figs. 2 and 3 are shown comparisons of the shapes and sizes of the conduits used in the calculations for the Official Plan of the District, with those that were finally adopted and built. The areas are practically the same but, in the conduits as built at the Englewood and Germantown Dams, the sections change near the outlets (to help spread the jet) so that the top of the end of the conduit is 4.6 ft. lower at Germantown, and 4.44 ft. lower at Englewood, than in the original computations. This change, together with the fact that the point of zero head is being taken a little below the top of the outlet portal rather than at the top, as was done originally, gives a greater head and, consequently, greater velocity and discharge for the same head-water elevations. The difference is most apparent for the lower heads. It

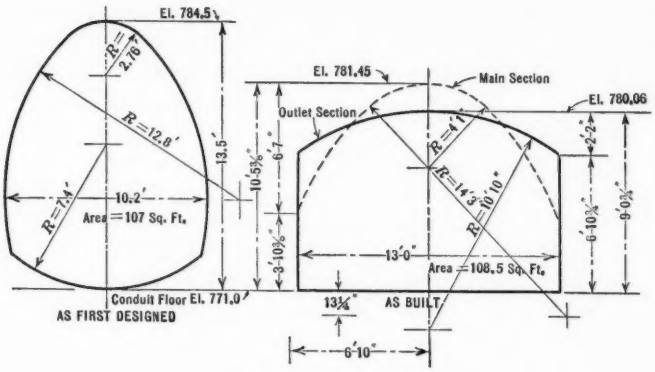

FIG. 3.—ENGLEWOOD CONDUITS.

becomes less in percentage as the head increases, for the reason that the theoretical velocity increases as the square root of the head.

In Figs. 4 and 5 are shown discharge curves for these conduits, and Table 1 gives the corresponding hydraulic values. The original curves used in the design with $n = 0.013$ are shown as well as new curves, the formulas for which were developed from the data contained in Table 2. The results of actual measurements are also platted.

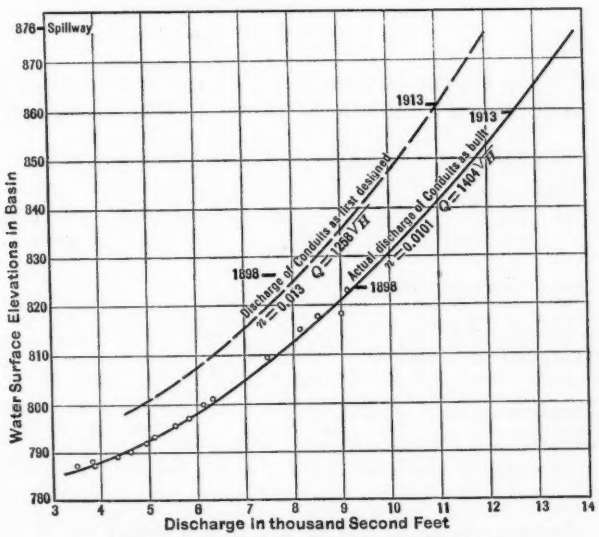

FIG. 4.—ENGLEWOOD DISCHARGE CURVES.

The discharge coefficient, c, is slightly lower for the Germantown than for the Englewood conduits (Table 1). This fact is difficult to explain, as the interiors of the conduits at the two dams appear to be equally smooth. The entrance to the conduits at Germantown, however, is at one side of the

valley and is curved; while that at Englewood is near the middle of the dam and is more nearly straight. There may be a slight loss of head at Germantown for this reason, but this cannot be stated with certainty.

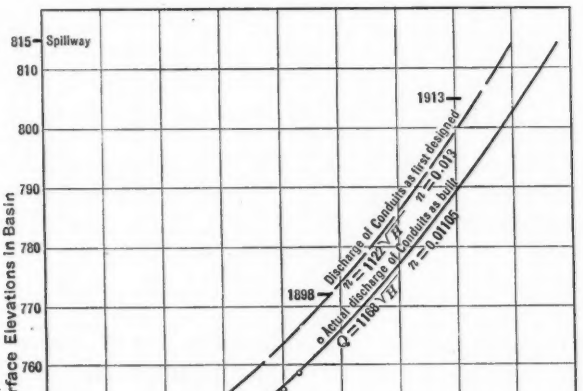

FIG. 5.—GERMANTOWN DISCHARGE CURVES.

In Table 2 are shown the data for all measurements taken at the Germantown and Englewood conduits to March 1, 1927. The formula used for

TABLE 1.—HYDRAULIC ELEMENTS OF CONDUITS.

Item.	Germantown Dam.	Englewood Dam.
Given Data:		
Area, A, two conduits, in square feet	182	217
Length, L, in feet	546	712
Hydraulic radius, r, in feet	2.43	2.71
Zero point for calculating head, in feet	730.35	779.7
Values Derived from Measurements:		
Discharge, Q	$1\ 168\sqrt{h}$	$1\ 404\sqrt{h}$
Velocity, V	$6.42\sqrt{h}$	$6.47\sqrt{h}$
Discharge coefficient, c	0.797	0.804
Kutter coefficient, n	0.01105	0.0101
Chezy coefficient, C	159	176

the conduit discharge is $Q = c\,A\sqrt{2\,g\,h}$, in which, $c = \sqrt{\dfrac{1}{1 + \dfrac{2\,g\,L}{C^2\,r}}}$; A is the cross-sectional area of the conduits, in square feet; h is the head; L, the length of the conduits, in feet; C, the Chezy coefficient; g, the acceleration due to gravity; and r, the hydraulic radius. The values of c in Table 2 were obtained from the other factors, all of which were known. The values of n

were then calculated. The final discharge formulas were developed by using the average value of c for each dam. These may be changed slightly in the future as more discharge measurements are obtained and as the interior surfaces of the conduits change.

With the exception of three measurements which were rejected, the maximum variation between the measured discharge and the discharge given by the formula for either Germantown or Englewood is less than 3 per cent.

As computed from the data shown in Table 2, the value of n is 0.0105 at Englewood and 0.01105 at Germantown. This is based on the assumption that there is no appreciable loss of head except by friction. It is quite probable that a slight entrance loss occurs, but its practical determination would be very difficult. Any such loss is now included with the friction loss and its separation therefrom would make the value of n still lower. A much lower value is improbable and, therefore, it is fair to assume that the entrance loss is insignificant.

The dissipation of the energy of the water as it leaves the conduits at a high velocity has been satisfactorily accomplished by means of the hydraulic jump and the outlet structure. For instance, at the Germantown dam, in the flood of September 2, 1926, the velocity in the conduits at the time the discharge measurement was taken, was 34.6 ft. per sec. At the end of the outlet structure the average velocity was only 7.9 ft. per sec. In Fig. 6 are shown the elevations of the tail-water for various heads at the Germantown Dam.

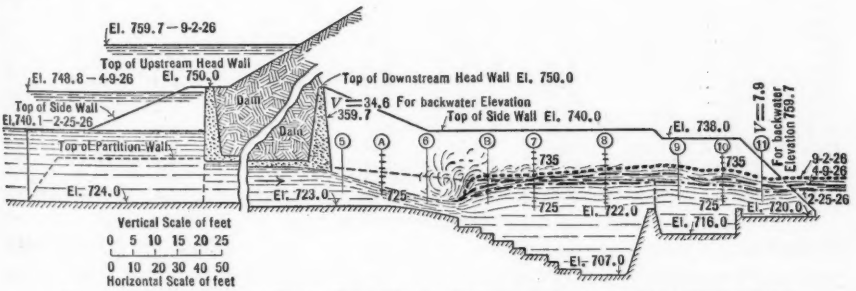

FIG. 6.—GERMANTOWN DAM, THE MIAMI CONSERVANCY DISTRICT, HIGH-WATER PROFILE THROUGH OUTLET WORKS, SHOWING HYDRAULIC JUMP.

A careful inspection of the interiors of the conduits shows the finishers' brush marks still visible on the sides and top. On the bottom, the surface finish has been worn off. Accurate levels, however, show that the wear is scarcely measurable at this time. Very accurate cross-sections have been taken of the interior of the conduits at several points. These will be repeated annually in order to determine the amount of wear.

Re-Routing Floods.—In making the original studies for the retarding basins a number of past floods were routed through these basins.* The maximum elevations reached and the length of time required for the basin to empty were calculated for each flood.

* A complete description of the routing of floods is given in "Hydraulics of the Miami Flood Control Project", by Prof. Sherman M. Woodward, Technical Repts., The Miami Conservancy Dist., Pt. VII.

TABLE 2.—DISCHARGE DATA.

Date.	Head-water elevation, in feet.	Total head, h, in feet.	Discharge, in cubic feet per second.	Theoretical V, $V = \sqrt{2gh}$, in feet per second.	Actual V in conduits, in feet per second.	V at end of outlet works, in feet per second.	Value of c.	Value of n.
	GERMANTOWN DAM.*							
Jan. 11, 1924....	749.5	19.15	5 098	35.1	28.0	....	0.80	0.0109
Jan. 16, 1924....	740.15	9.80	3 470	25.4	19.1	....	‡0.75	.....
Mar. 29, 1924....	764.2	33.85	6 640	46.7	36.5	....	0.78	0.0117
June 6, 1924....	749.8	19.45	5 038	35.4	27.7	....	0.78	0.0117
June 27, 1924....	752.0	21.65	5 365	37.4	29.4	....	0.79	0.0113
Sept. 19, 1925....	743.5	13.15	4 253	29.1	23.4	....	0.81	0.0106
Oct. 2, 1925....	748.9	18.55	4 936	34.5	27.2	....	0.79	0.0113
Nov. 13, 1925....	759.0	28.65	6 294	42.9	34.5	....	0.805	0.0107
Jan. 18, 1926....	740.25	9.90	3 747	25.3	20.6	7.0	0.81	0.0106
Feb. 25, 1926....	740.5	10.15	3 653	25.6	20.2	6.9	0.79	0.0113
Apr. 8, 1926....	746.75	16.40	4 617	32.5	25.4	....	0.78	0.0117
Sept. 2, 1926....	756.1	25.75	6 019	40.8	33.0	7.8	0.81	0.0106
Oct. 29, 1926....	742.5	12.15	4 196	28.0	23.0	6.8	0.82	0.0102
Average.....	.....	.....	.....	....	....	....	0.797	0.01105
	ENGLEWOOD DAM.†							
May 16, 1923....	791.5	11.80	4 920	27.6	22.7	....	0.82	0.0095
Dec. 14, 1923....	790.0	10.30	4 630	25.8	21.4	....	0.83	0.0092
Mar. 26, 1924....	787.3	7.60	3 880	22.2	17.9	....	0.81	0.0099
Mar. 31, 1924....	809.5	29.80	7 497	43.8	34.5	....	0.785	0.0108
Apr. 1, 1924....	802.6	22.90	6 102	38.4	28.2	....	‡0.735	.....
June 9, 1924....	817.8	38.10	8 543	49.5	39.4	....	0.80	0.0103
June 10, 1924....	822.9	43.20	9 140	52.7	42.2	....	0.80	0.0103
Apr. 23, 1925....	795.2	15.50	5 560	31.6	25.6	....	0.81	0.0099
Nov. 9, 1925....	787.5	7.80	3 829	22.4	17.6	3.4	0.785	0.0108
Nov. 13, 1925....	796.7	17.00	5 866	33.1	27.0	....	0.82	0.0095
Nov. 14, 1925....	799.8	20.15	6 155	35.9	28.3	4.3	0.79	0.0106
Nov. 16, 1925....	793.0	13.30	5 100	29.3	23.5	3.8	0.80	0.0103
Feb. 26, 1926....	788.9	9.20	4 323	24.4	19.9	3.6	0.82	0.0095
Apr. 9, 1926....	801.05	21.35	6 345	37.1	29.3	4.7	0.79	0.0106
Oct. 6, 1926....	786.9	7.20	3 501	21.5	16.1	3.4	‡6.75	.....
Oct. 30, 1926....	793.0	13.30	5 084	29.3	23.4	4.1	0.80	0.0103
Average..... ...	.....	.....	.....	....	....	....	0.804	0.0101

*Discharge formula from these data, $Q = 1168\sqrt{h}$.

†Discharge formula from these data, $Q = 1404\sqrt{h}$.

‡Rejected.

From the information now available it is evident that, due to the greater capacity of the conduits, these floods would not rise as high in the basins as calculated and that the time required to empty the reservoirs would be reduced. The 1913 and the 1898 floods, therefore, were re-routed through the Englewood Basin. The calculations show that the maximum elevation for the 1913 flood would be reduced 1.9 ft. and for the 1898 flood, 2.7 ft.

Drift.—When the plans for the dams were first developed, it was thought that some kind of drift barriers would have to be provided above the conduits, but, later, it was decided that these would be unnecessary. Experience thus far indicates that they are not needed. Drift reaches the conduits mostly

during the early stages of a flood. During the intermediate and high stages the velocity in the basin is very low and the drift is carried about by the wind. That reaching the conduits during the first stages is usually light material and passes through easily. By the time the water is deep enough to carry large trees, the velocity has been reduced so that very few of these reach the dam.

There has never been any tendency to clog up the conduits. A number of times, as a flood went down, drift was left on the center wall. In all cases, however, this material would easily have passed through the conduits under a considerable head. As an extra precaution against the accumulation of drift, the caretakers go over their respective basins during the fall and winter and cut all "down timber" into short lengths that will pass through without difficulty.

Channel Improvements

The most extensive channel improvements were made at Dayton and Hamilton. The waterways were straightened, widened, and deepened and made more uniform in cross-section and regular in alignment. The condition of the channels at present is excellent, although there have been some slight changes. The stability of these channels has been greatly increased by the low concrete dams at Island Park and Wolf Creek, in Dayton, and at Twomile Creek, in Hamilton; and by the operation of gravel plants in both cities. These plants, above the limits of the improvements, dig large quantities of gravel from the river bed. The excavations thus made serve to catch most of the gravel carried down stream during floods and prevent its deposit in the improved channel. Besides saving maintenance expense, the District obtains some revenue from the sale of the gravel.

Some of the changes in cross-section of the improved channels are illustrated in Figs. 7 and 8. In Dayton there has been considerable deposit above the Island Park Dam. Fig. 7 shows a section 100 ft. above the dam (taken in 1916 before the dam was built), another in 1923, and a third in 1926. The progressive filling indicates that it will not be long until material will begin to pass over this dam and enter the channel below. However, the material deposited here seems to be rather fine and possibly may be carried on through the improved channel. Small bars have formed at other points in Dayton but, taken as a whole, the sections show the remainder of the channel to be practically as built.

At Hamilton a bar has formed below the dam, probably due partly to the gravel carried over. This material is now being removed by the Hamilton Gravel Company as part of its regular operations. Below the railroad bridge the channel widens causing a considerable deposit along the west side (Fig. 8). The alignment here was made to fit the original bank for economy in construction. Such a deposit would not occur in a channel of uniform width and more regular alignment. A flood somewhat larger than those which have occurred, may remove it.

A flexible revetment is used as protection against erosion. This consists of concrete blocks, 5 by 12 by 24 in., each reinforced and cast with two

transverse holes. The blocks were manufactured at a central plant. They are held together by ½-in. galvanized steel cables, the ends of which are embedded in the toe-wall of the monolithic slope revetment on the land side, and in the larger blocks, cast in place, on the river side. A detail drawing of this revetment has been published.* As scouring occurs, this revetment settles to a depth where further scour is prevented.

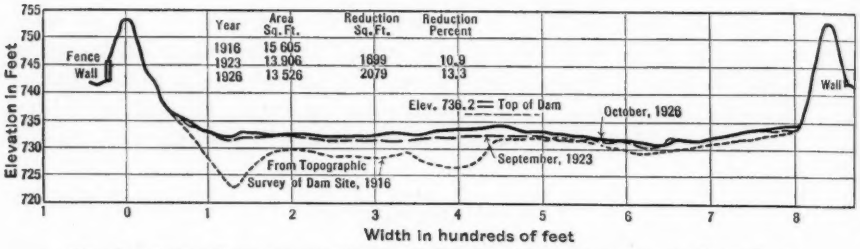

FIG. 7.—CROSS-SECTIONS OF MIAMI RIVER 100 FEET ABOVE ISLAND PARK DAM.

Value of n for Channels.—The discharge measurements made thus far in Dayton and Hamilton show the coefficient of friction, n, to vary from approximately 0.015 to 0.0285. The larger value was obtained at Dayton with a gauge height of 9 ft. At this stage the water just covered the entire bottom from toe to toe of the levees, the depth varying from 0 at the sides to 8 or 10 ft. in the center, over a width of about 600 ft. The lowest value of n obtained was 0.015; this was above the Main Street Bridge, at Hamilton, at a 12.4-ft. stage. The channel here is straight, has a uniform width of 520 ft., and a smooth bottom practically free from grass or weeds, with concrete revetment on the slopes.

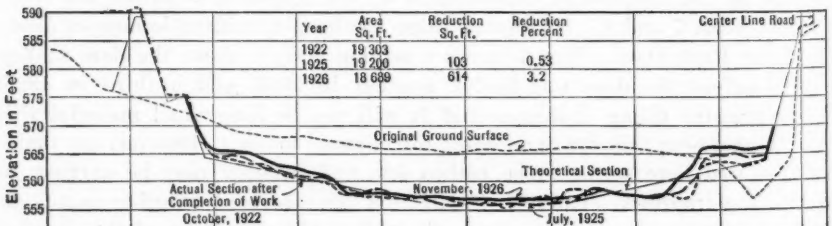

FIG. 8.—CROSS-SECTIONS OF MIAMI RIVER AT STATION 72 + 00, HAMILTON, OHIO.

The detailed procedure in making two determinations of the value of n was as follows: In the storm of March 19 to 21, 1927, the stage of the river at Dayton stayed close to the 12-ft. mark for about two days. The discharge was measured at the Main Street Bridge while the average gauge height was 12 ft. This discharge, 38 300 sec-ft., was used in the computations. Permanent gauges reading to sea-level elevations had previously been installed above and below the Main Street Bridge as shown in Fig. 9(*a*). These gauges were

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), Fig. 35, p. 1508.

read before, during, and after the discharge measurement, the entire set being read five times; three times by the man who took the discharge measurements, and once each by two other men independently.

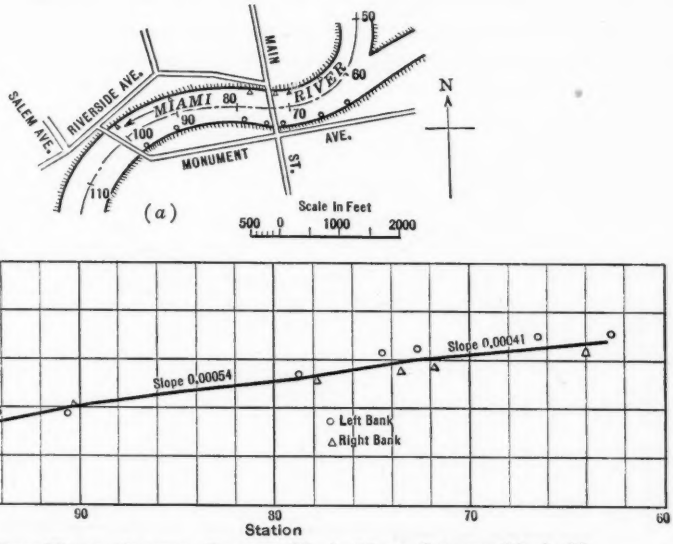

FIG. 9.—HIGH-WATER PROFILE, STATION 63 + 00 TO STATION 98 + 00, THE MIAMI CONSERVANCY DISTRICT, MARCH 21, 1927.

It was recognized that the chief difficulty lay in obtaining accurate slope measurements, and considerable care was taken in obtaining the readings. As each set of slope gauge readings was made, the Main Street river gauge was also read. As there was only a small variation in the gauge height during this time, the surface slopes were practically parallel. Therefore, all slope gauge readings were corrected by addition or subtraction to reduce them to a 12-ft. stage. The five readings for each gauge were then averaged to obtain the final figure. The slope profile was then platted for both sides of the channel, and the slope used in the determination of n was taken as a mean between the surface elevations on the two sides.

This surface slope was used in determining the cross-sectional areas. Accurate cross-sections at 100-ft. intervals were available. Only part of these were used, however, as the areas are quite uniform and the changes gradual. Fig. 9(*b*) shows the final water surface profile. To determine n, three reaches were used, as shown. The slope, hydraulic radius, and velocity having been determined, the value of n was taken from a diagram for the solution of Kutter's formula. For this part of the river it averaged 0.027.

Perhaps the most satisfactory determination was made on a reach of about 3 000 ft. below Stewart Street, at Dayton. Here, the channel is of uniform section throughout and of uniform curvature, with a radius of 4 093 ft. on the center line. The profile in this case was determined by driving stakes flush with the water surface every 200 or 300 ft. on both sides of the channel. While these stakes were being driven the gauge height at Main Street was

11.9 ft. Levels were read on the stakes and the elevations corrected for the 12-ft. stage at which the discharge measurement was taken. The resulting profile is shown in Fig. 10(*b*). The uniformity of the slope and the number of elevations taken indicate the accuracy of this method. The value of n determined from this set of measurements was 0.023, which was an exact check on two previous measurements for the same reach.

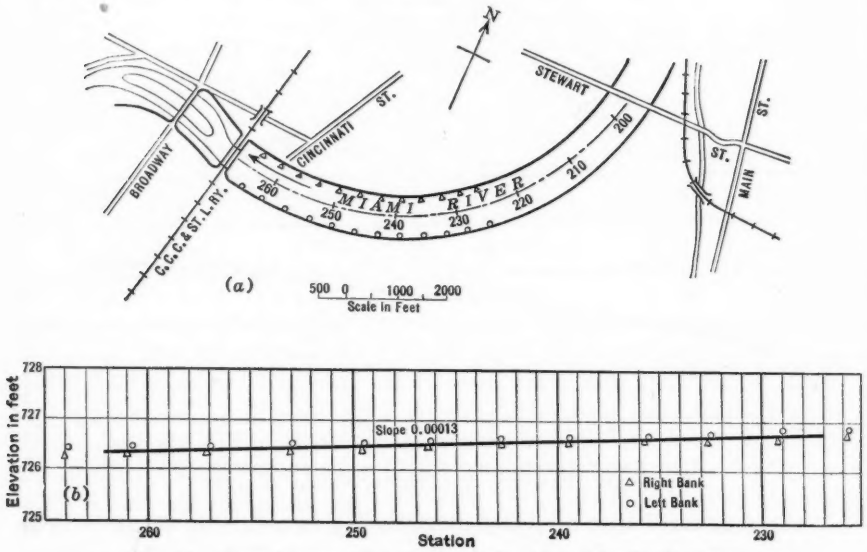

FIG. 10.—HIGH-WATER PROFILE, DAYTON, OHIO, MARCH 21, 1927.

Taking the channels at Dayton and Hamilton as a whole, the average value of n appears to be below that used in the design, and therefore the channel capacity will be somewhat greater than provided by the plan. It is reasonable to assume that at higher stages the values of n will be less than those obtained from the measurements made thus far.

Cut-Off Channels.—At a number of points where more extensive channel improvements were not warranted, cut-off channels were required to facilitate the flow. These were built of small cross-section on the assumption that they would be enlarged by scouring. Each of them reduces the length of the channel, thereby increasing the grade and the velocity. Such channels are located at Troy, Dayton, Poasttown, Middletown, and Hamilton.

The Troy cut-off channel (Fig. 11) is 4 000 ft. long. The original bottom width was 40 ft. which has now increased to about 65 ft. Depth and alignment have been well maintained. It assists greatly in passing the water through the City of Troy and its capacity, of course, will continue to increase. The protection levee for the city is located close to this channel for some distance and as the channel is widening about equally on both sides some bank protection may be necessary along the levee.

In constructing this channel the bank nearest the levee was cut on a 2 to 1 slope while the other bank was made vertical. It was thought that cutting

would occur mostly on the vertical side and that the 2 to 1 slope would remain intact. The cutting has actually been about equal on the two sides.

The cut-off channel at the Broadway Bridge, just below Dayton, has not enlarged, due to the fact that it is rather close, and nearly parallel, to the old channel. It has been effective, however, in increasing the channel capacity. In a large flood no doubt considerable enlargement will take place.

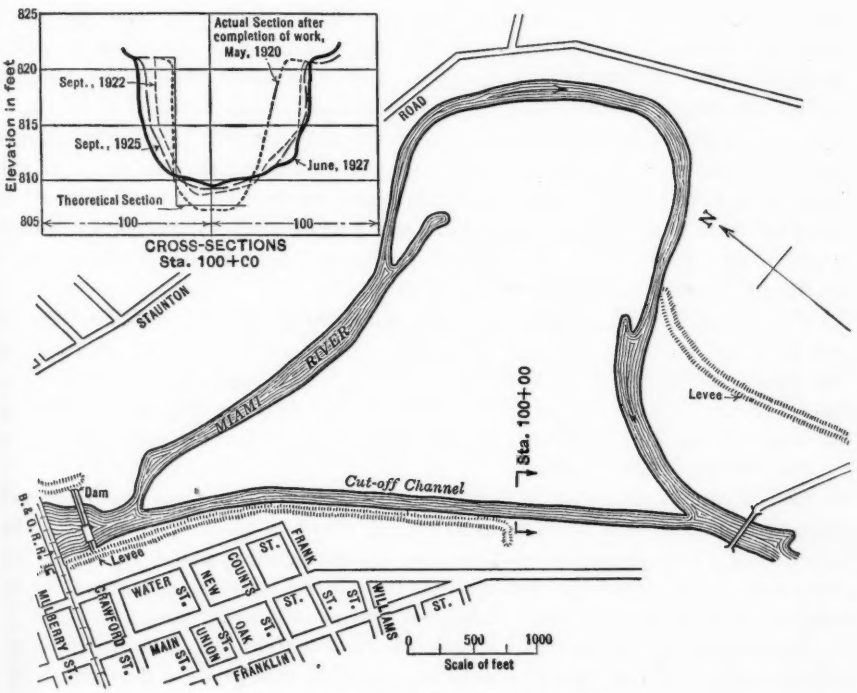

FIG. 11.—CUT-OFF CHANNEL, TROY, OHIO.

A rather remarkable result was obtained north of Middletown on a bend in the river just above the Poasttown Bridge. Here, no excavating was done. The timber was cleared from a strip where a cut-off channel would naturally be located and the river has done the rest. This channel gives the river a more direct approach to the bridge and protects the road on the east, which otherwise would be endangered by scouring.

Opposite Middletown, the Miami River is in the form of a big bend. The overflow area is large, extending, in high water, to the railroad on the west. A concrete bridge about 1 800 ft. long crosses almost the entire bottom. The alignment of the old channel greatly retarded the flow and considerable scour was taking place along the east bank. A cut-off channel with a bottom width of 40 ft. was constructed. At first, only a small part of the flow passed through this channel, but it has been enlarged by floods to such an extent that the cross-sectional area has increased 110% and it now carries the entire low-

water flow and a large proportion of the flood flow. It has also eliminated the scouring along the east bank of the old channel. In this case the results have probably exceeded the expectations of the designers.

Prior to the execution of the District's plan a bad situation existed north of Hamilton, near the mouth of Fourmile Creek, which has a drainage area of 178 sq. miles. The direction of flow of this creek was almost directly opposite that of the river. The retarding action resulting raised the water level in the river at this point and increased the danger of flooding in the City of Hamilton from this direction. To remedy this condition a cut-off channel with a bottom width of 100 ft. was excavated. The excavated material was placed on the east side of the channel in the form of an embankment to prevent the water from taking the old channel. The new channel has been widened to such an extent that practically all the embankment has been washed away, and it has not maintained its depth. Present indications are that some form of bank protection will be necessary to make this channel a permanent success. A considerable further increase in width might result in the creek returning to its old channel.

Dams and Gravel Plants.—As previously stated, important factors in the maintenance of the improved channels through Hamilton and Dayton are the dams and gravel plants at the upper ends of the improvements. The Island Park Dam at Dayton was built by the City of Dayton in 1916 to provide boating and swimming at the park, but no doubt it has also served to stabilize the channel below.

The Twomile Dam, at Hamilton, built solely to protect the improved channel from gravel deposit, is at the upper limit of the improvement. The plant of the Hamilton Gravel Company is immediately above it. Since its construction this plant has removed 156 000 cu. yd. of sand and gravel. The material is replaced by the river about as fast as it is excavated.

The bottom grade of Wolf Creek where it enters the Miami River, at Dayton, is 5 ft. above that of the river. To hold this grade a dam was built at the mouth of the creek. A gravel plant on the creek above the improvement helped to maintain the channel. This plant was abandoned after about 42 000 cu. yd. had been removed. Gravel was not carried down fast enough to supply the plant, for the reason that material was being taken out in large quantities about ½ mile farther up stream.

Gravel plants were located also on Mad River, below Webster Street and below Findlay Street in Dayton. The former has been abandoned, at least temporarily, because the supply was exhausted. It removed a total of about 57 000 cu. yd. Two plants are operating below Findlay Street. They have moved approximately 222 000 cu. yd.

The plants operating on District property have removed from the river a total of 477 000 cu. yd. since the early part of 1922. There can be no doubt that much of this gravel would have passed down stream had it not been removed. A large part of it would certainly have stopped in the improved channels and its removal would cost probably not less than 75 cents per cu. yd.

Rainfall and Run-Off

The District maintains rain gauges at all the dams and at a number of other points. The U. S. Weather Bureau also has gauges in various cities in the Miami Valley. The reports from all these stations are used co-operatively by the District and the Weather Bureau. All rainfalls of more than 1 in. in 24 hours are reported by telephone or telegraph. Other rainfall reports are sent in weekly or monthly by mail. The map, Fig. 12, shows the location of all the gauging stations in the water-shed.

No storms have occurred, since the works of the District were completed, approaching in magnitude the one which produced the 1913 flood. A study of some of the smaller storms, however, is of value. The difference in the percentage of run-off in these storms is of special interest, because it illustrates the difficulty encountered in predicting or estimating river stages from rainfall data. In other words, it shows that it is impossible to determine the run-off by a fixed rule or formula. Predictions of this kind may be made by using the records of the past, but in each case the existing conditions of the season, soil, and vegetation, must be taken into consideration and a coefficient of common sense and judgment must be applied liberally to the result.

Table 3 shows the rainfall and run-off above Dayton, from an area of 2 525 sq. miles, for a number of storms.

TABLE 3.—Relation of Rainfall and Run-Off, Dayton, Ohio.

Date.	Average rainfall, in inches, above Dayton.	Run-off, in inches.	Run-off, in percentage of rainfall.
March 23-27, 1913	9.60	8.76	91.0
March 28-29, 1924	2.06	1.16	56.0
June 8, 1924	2.22	1.73	78.0
Sept. 12-13, 1925	3.20	0.04	1.25
Nov. 12, 1925	1.43	0.66	46.0
Nov. 26-27, 1925	1.29	0.20	15.6
April 7-8, 1926	1.23	0.84	68.0
March 19-21, 1927	3.44	2.31	67.0

The rainfall is the average of all the rainfall stations on the drainage area above Dayton. This method does not give the true average, but is sufficiently accurate for practical purposes. Rainfall contour maps are prepared for all storms of importance.

The run-off is determined by reducing the daily gauge heights at Dayton to discharges, in cubic feet per second, and platting a curve of the daily discharges for the storm period. The area included between this curve and a line showing the normal discharge is then measured and the result reduced to inches of rainfall over the drainage area.

A study of the records of the storms of September 12 and 13, 1925, and March 19 to 21, 1927, discloses a remarkable contrast. The average rainfall above Dayton for the two storms was 3.20 and 3.44 in., respectively. The run-off for the storm of September 12 and 13, 1925, was 0.04 in., or 1.25%

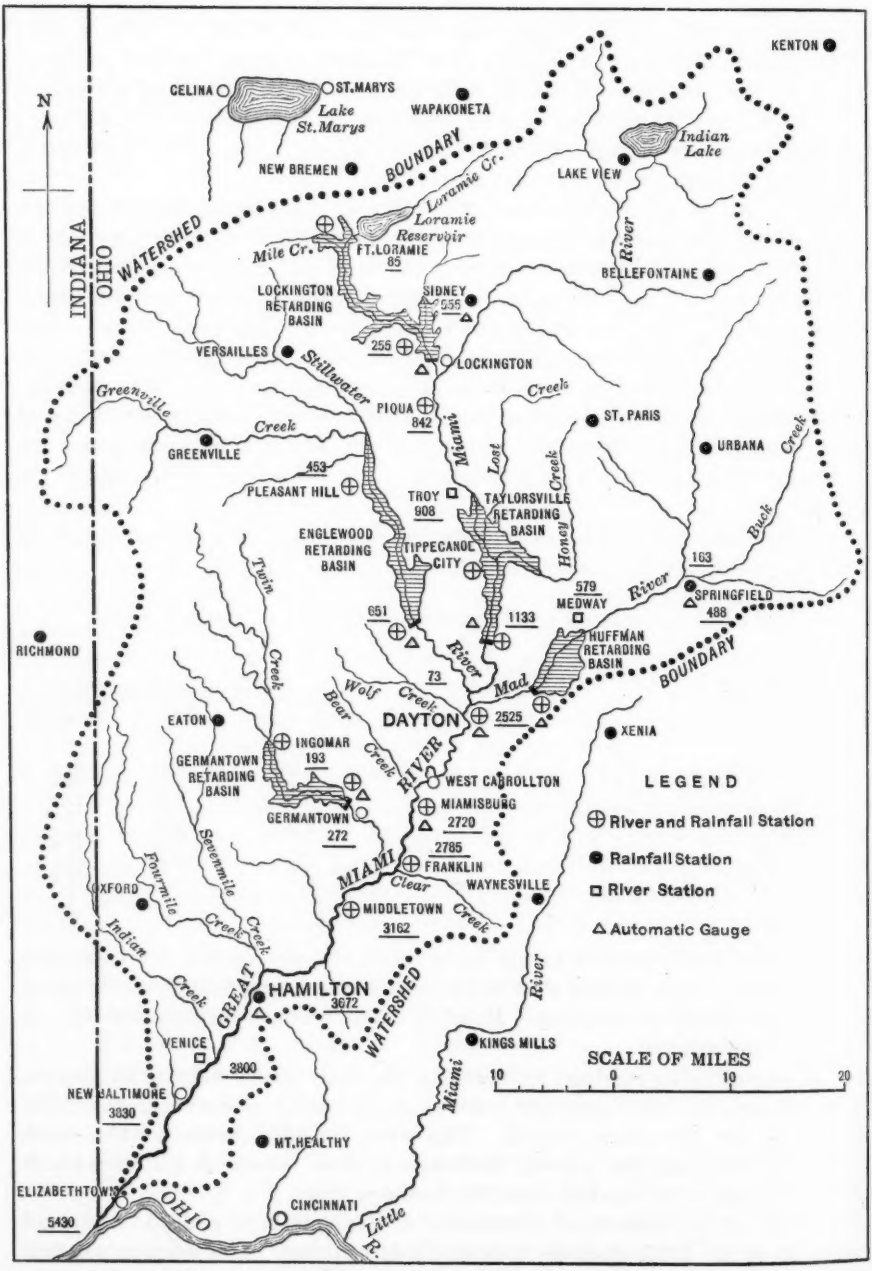

FIG. 12.—DRAINAGE AREA, SHOWING GAUGING STATIONS, MIAMI RIVER, OHIO.

of the rainfall. For the storm of March 19 to 21, 1927, the run-off was 2.31 in., or 67% of the rainfall.

The storm of September 12 and 13, 1925, was of shorter duration. The precipitation for 24 hours at several stations was greater than for any 24-hour period of the 1913 flood. Other things being equal, this would have caused the run-off for this storm to be the greater of the two. However, there was no back-water at any of the dams and the maximum stage reached at Dayton was 2.9 ft.

After the storm of March 19 to 21, 1927, all the dams had back-water, the depths being, at Englewood 48 ft.; Taylorsville, 26 ft.; Huffman, 16 ft.; Lockington, 32 ft.; and Germantown, 29 ft. The maximum stage recorded at Dayton was 12.8 ft. and at Hamilton, 14.3 ft.

The rainfall and run-off curves for both storms are shown in Fig. 13. Unquestionably the difference in run-off was due to the difference in soil conditions. The storm of September, 1925, came at the end of the growing season and following a dry period during which the transpiration and evaporation had been abnormally great. At the beginning of this period there was a deficiency in rainfall of about 5 in. These factors combined to give the soil a tremendous storage capacity. Preceding the storm of March, 1927, there had been little evaporation and there was no growing vegetation to consume moisture. To November 1, 1926, there had been an excess of 5.79 in. in rainfall since the beginning of the year.

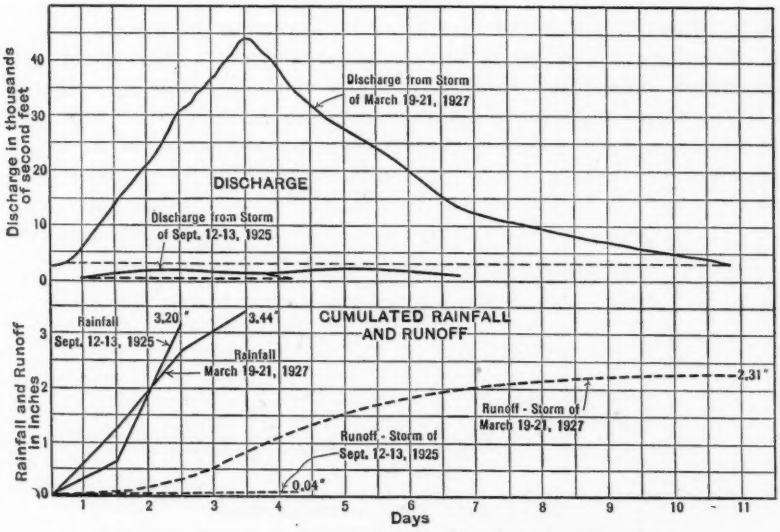

FIG. 13.—STUDY OF RAINFALL AND RUN-OFF, DAYTON, OHIO.

On October 2, 1925, an unusually heavy rain occurred over the drainage area of Clear Creek. This creek, which enters the Miami River just below Franklin, has a drainage area of 52 sq. miles. Following the storm numerous inquiries were made from residents and the information obtained showed the average rainfall for the area to be 4.3 in., with a maximum of 5½ in. The

rainfall occurred between midnight and 9:00 A. M. Surface slopes were determined from high-water marks west of the Dixie Highway Bridge, at Franklin, and the maximum discharge was calculated to be 10 000 sec-ft., or about 200 sec-ft. per sq. mile. It was not possible to obtain sufficient data to determine the total amount or percentage of run-off.

After high water under present conditions the question is often asked: "What would have happened under the old conditions?" The answer involves an interesting study. It is assumed that the maximum stage at Dayton would be produced by the maximum combination of the inflows into the Englewood, Taylorsville, and Huffman Basins. The run-off from the area below these basins, which would affect the maximum stage at Dayton, will, in most cases, be negligible. In practically all storms this run-off passes Dayton before the maximum stage is reached. It is, therefore, disregarded in these calculations.

During high water the back-water gauges at the dams are read every 2 hours. From the contour maps of the basins a tabulation of the storage capacities of each basin, for various elevations, has been made. From these and the 2-hourly readings, together with the outflow record, the total amount of inflow for each 2-hour period can be determined. From this the maximum inflow, in second-feet, is obtained.

From former studies the rate of flow in these rivers was found to be from 3½ to 4 miles per hour. The distances along the rivers from the three dams to Dayton, of course, are known.

With these data it is then necessary to determine the maximum combination of inflows which would have reached Dayton. This would give the discharge in second-feet at Dayton under the old conditions. The gauge height can then be obtained from the old rating curve.

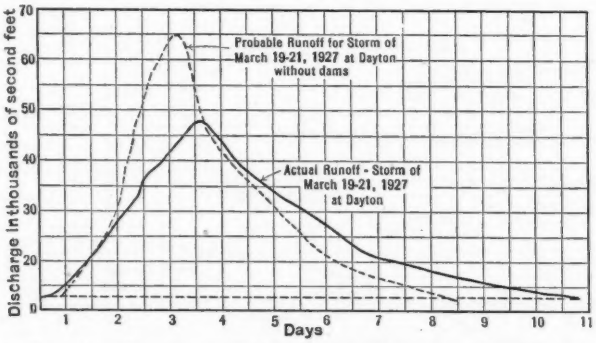

FIG. 14.—EFFECT OF RIVER IMPROVEMENTS FOR STORM OF MARCH 19-21, 1927, AT DAYTON, OHIO.

This method was applied to the storm of March 19 to 21, 1927, the results being shown in Fig. 14. The solid line shows the run-off as it actually occurred at Dayton; the dotted line represents the run-off as it would have taken place before the flood-control works were built. It is to be noted that the peak discharge under former conditions would have been 45% greater and would have occurred about 5 hours earlier.

The maximum gauge height at Dayton was 12.8 ft. In the old channel the actual elevation of the water surface would have been 6.9 ft. higher. The difference in the two peaks at Dayton will be much more pronounced in a larger flood for the reason that the velocities and discharges through the conduits at the dams vary approximately as the square roots of the heads.

Fig. 15 shows the inflow, outflow, and elevation curves for the March, 1927, storm at Englewood. These curves show that at this point alone the peak discharge was reduced from 22 300 to 9 000 sec-ft.

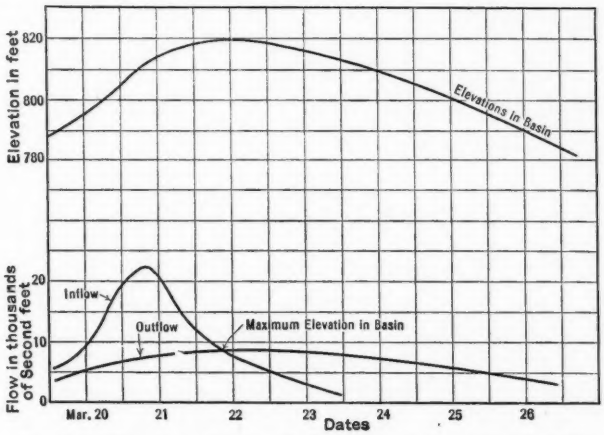

FIG. 15.—FLOOD OF MARCH 19-21, 1927, AT ENGLEWOOD, OHIO.

As stated at the beginning of this paper, the experience gained thus far confirms the correctness of the design. However, sufficient time has not elapsed to make definite conclusions possible along all lines. Investigations are to be continued in the future. The opportunities for securing the necessary data are limited to a large extent by the occurrence of floods. It is the purpose to make the most of such opportunities as they occur.

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PAPERS AND DISCUSSIONS

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MODERNIZING THE SUBURBAN TRANSIT OF THE METROPOLITAN DISTRICT

By Francis Lee Stuart,* M. Am. Soc. C. E.

To Be Presented September 5, 1928

In 1927 the suburban railroad traffic from the Metropolitan District into and through Manhattan, exclusive of subways and local ferries, was more than 38% of all the passengers entering railroad passenger cars in the whole United States, including Alaska. These passengers were handled by antiquated methods of transfer and at congested terminal stub stations. These methods can be modernized by the simple remedies proposed herein, which tend to round out the various sectors into a homogeneous whole and which later could be expanded as needed to meet the growth of the Metropolitan District.

Subway distribution and suburban transit relief are separate problems and require separate solutions. Any extension of the present New York City Subway Systems into suburban territory or New Jersey should be opposed on account of their inadequacy and the discomfort for long riders. The subways should be used where most effective for rapidly distributing local city traffic.

A proper solution of the suburban commuter problem of Manhattan should include intercommunication between all sectors of the Metropolitan District. Further, it should make provision for a distribution system in and through Manhattan without transfer, which would be supplemental to, and have no physical track connection with, the present city transportation system.

Provision should be made so that residents of the New Jersey, Westchester, Long Island, and Staten Island Sectors of the Metropolitan District may be routed to and through Manhattan when desired, from any one sector of the suburban area to another sector, thus securing the benefit of through routing and effectuating a two-way, interstate, passenger traffic movement.

Note.—This paper is issued before the date set for presentation and discussion. Correspondence is invited and may be sent by mail to the Secretary. Discussion on this paper will be closed in November, 1928.

* Cons. Engr., New York, N. Y.

The plan should be all-embracing and give all sectors common advantages. There should be Hudson River and East River crossings in the vicinity of 57th Street and connections to Westchester; and at the Battery, other crossings of the Hudson and East Rivers, connecting with east and west routes to Staten Island.

Preference should be given to the 57th Street Bridge for crossings to New Jersey in that vicinity, because of its location and other advantages, and because of the fact that it would be much cheaper per track than a tunnel crossing. Preference should be given to tunnel crossings of the Hudson and East Rivers at the Battery because this would serve the lower end of Manhattan and would divide the outlying suburban territory of New Jersey and Long Island to an economic advantage.

The distributing system in Manhattan should be through terminal tunnels deep enough so as not to interfere with local subways, possibly on a third level. To make a minimum use of the local street distributing means in Manhattan, which are already crowded, the system should consist of two terminal tunnel routes extending from the Battery to 57th Street, separated several blocks apart, one on the East Side and one on the West Side, with four or five station stops in each line for suburban transit, at which no local station-to-station traffic in Manhattan would be allowed.

There should be a connection between these two terminal tunnel routes at the Battery and at 57th Street; and possibly an inside terminal loop between 57th Street, 23d Street, and Madison and 7th Avenues. This in effect would give a choice of routes from all sectors and would divide the system into simple east side and west side terminal routes easily understood by the general public, with an inside terminal loop to cut down unproductive mileage and facilitate operation when desired.

All tunnels should be of sufficient size to accommodate the standard railroad suburban equipment, which now represents a big investment of the railroads. Probably many of these coaches could be converted for multiple-unit operation at relatively small cost.

There are residential sections enough and diversified industry enough in the Metropolitan District for all to find pleasant homes sufficiently close to their work, where the cost of transportation is within reach of their pocketbooks. The great mass of commuters comfortably delivered within walking distance of their destinations in Manhattan, without transfer, would gladly pay the cost of such improved service.

To accomplish the benefits sought for the public would take from five to eight years and would require the co-operation and co-ordinated effort of all the railroads and of the New Jersey, Westchester, Long Island, and Staten Island Sectors. Individual effort would be ineffective and could not succeed.

The railroads have the collecting and distributing means already available in the suburban territory for intrastate and interstate purposes; but they could not be expected voluntarily to assume further financial obligations in such a re-organization of the present unprofitable commuter business except with the co-operation of all concerned and with the assurance of the proper support of the public rate-making bodies.

As a practical method of inaugurating such a plan, an operating company should be formed by all the railroads, which company would lease, for passenger purposes only, existing rights of way of such suburban railroads as may be needed. It would operate through the tunnel terminal routes on a fare which would be sufficient to pay operating costs, lease rentals, taxes, interest, amortization, etc., on the entire cost of the system.

Such a suburban transit system could be built gradually and would cost approximately $1 250 000 000 to $1 500 000 000. The system by the time it could be completed would probably be carrying 700 000 000 passengers yearly and the charges might add 10 to 15 cents to each railroad fare to or from Manhattan, depending on the amount of benefit assessment levied.

The securities of this system, supported by the needs of such a vast and growing population and based on a proper agreement as to a rate of fare required to stabilize the financing of its cost, approved by the Interstate Commerce Commission, by the two States, and by the City, would be a very desirable offering to investors and could be marketed on unprecedented terms, probably 3½%, or lower.

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THE DESIGN OF TALL BUILDING FRAMES TO RESIST WIND*

By Albert Ward Ross, Jr.,† Jun. Am. Soc. C. E., and
Clyde T. Morris,‡ M. Am. Soc. C. E.

Synopsis

The purpose of this paper is to present a method of design for the wind-resisting bents of tall buildings, which approaches accuracy and economy, and which is practical for use. A brief review of present methods and previous investigations is given, the calculations of the wind stresses in the 48-story tower of the American Insurance Union Building, in Columbus, Ohio, by the Slope Deflection Method is outlined, and the results are tabulated and compared with other methods.

The subject of economy is considered and the conditions requisite for an ideal bent are presented, the design of the ideal bent being based on the slope deflection theory. Variations from the ideal as encountered in practice are considered, and a set of curves developed for determining the effect of these variations on the stresses in the bent. A proposed method of design is then presented, with a routine outline of procedure to be used in practice, and numerical examples are given to illustrate the method.

Introduction

The present-day agitation in favor of increasing the allowable working unit stresses used in the design of steel structures is based on the assumption that the actual stresses which will exist in the finished structure are known within a reasonable margin. The modern office building frame presents one of the most indeterminate structures in common use, as far as

* The major portion of this paper is based on a dissertation prepared under the direction of Professor Morris, and presented by Mr. Ross as a thesis for the degree of Master of Science, at the Ohio State University in June, 1926. The paper was presented at the meeting of the Structural Division, Columbus, Ohio, October 13, 1927.

† Mount Vernon, Ohio.

‡ Prof. of Structural Eng., Ohio State Univ., Columbus, Ohio.

stresses produced by lateral forces are concerned, and the usual methods of calculating wind stresses are, at best, mere approximations. As the height of structures increases, these stresses become of major importance, and a more accurate method of calculation that is simple enough for use in practice, is imperative.

The development of the tall building and its increasing importance have caused various theories to be advanced for the calculation of the stresses in frameworks due to wind forces. In 1913, Mr. R. Fleming collected and summarized a number of approximate methods of calculation, which he published.* Later, this and several other of Mr. Fleming's papers on wind stresses were published in book form.†

All these approximate methods are based on assumptions as to the distribution of shears among the columns, and the location of the points of contraflexure of the members. There have been many attempts, by various investigators, to develop a more accurate method of analysis which would be practical for use in design.‡ The most satisfactory of these exact methods of analysis is known as the Slope Deflection Method. The application of this to tall building frames was devised by W. M. Wilson, M. Am. Soc. C. E., and Mr. G. A. Maney, and published by them.§

The amount of labor involved in these more exact methods of analysis is so great as to render their use in design impractical. After presenting the theory, Messrs. Wilson and Maney give the calculations for a symmetrical three-span bent, twenty stories high. These calculations are summarized in twelve tables covering thirty-one pages. The calculations involve the solution of sixty simultaneous equations, with sixty unknowns, each equation involving from three to six of the unknowns. This method requires a thorough knowledge of mathematics and mechanics on the part of the designer, but is perfectly feasible. In their conclusions, Messrs. Wilson and Maney state:

"While the Slope Deflection Method is long, it could be used in the actual design of a building, but its greatest value is as a standard by means of which the accuracy of various approximate methods may be determined."

The determination of the stresses in a frame with quadrangular panels requires that the relative sizes of the various members be known before the stresses can be calculated. Therefore, some method of arriving at a preliminary design must be used before the slope deflection method can be applied and the true stresses determined.

Messrs. Wilson and Maney give a table‖ showing the relative accuracy of three approximate methods given by Fleming and compare them to the slope deflection method. In their conclusions, they state:

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* *Engineering News*, March 13, 1913.

† "Six Monographs on Wind Stresses", by R. Fleming.

‡ "The Theory of Frameworks with Rectangular Panels", by Ernest F. Jonson, *Transactions*, Am. Soc. C. E., Vol. LV (1905), p. 413; "Wind Stresses in the Frames of Office Buildings", by Albert Smith, *Journal*, Western Soc. of Engrs., Vol. 20, No. 4, p. 341; and "Stresses in Tall Buildings", by Cyrus A. Melick, *Bulletin No. 8*, Coll. of Eng., Ohio State Univ.

§ *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station.

‖ *Loc. cit.*, Table 23.

"Fleming's Methods Nos. II and III are so inaccurate that they should never be used. Fleming's Method No. I gives quite accurate results in some cases, but may give results which are seriously in error."

The erection in Columbus, Ohio, of a 48-story tower building for the home offices of the American Insurance Union, called for a careful consideration of wind stresses. One of the writers was employed as Consulting Engineer on the building, and used Fleming's Method No. I, referred to, in checking the wind stresses. However, it was his desire to have a more positive check on the distribution of stresses in the lower stories, where certain architectural requirements made the arrangement of girders somewhat irregular, and the slope deflection theory was applied to the lower eight stories of the tower. From a study of this solution, and a number of set problems, a method of design has been developed which it is believed will be found practical, and which is more nearly in accord with the true stresses than any of the approximate methods now in use. As this method of design is based on the slope deflection theory, it is desirable that the fundamental equations of that theory be reviewed herein.

Slope Deflection Method

The theory of the slope deflection method is developed by the principle of area moments and is based on the following five assumptions*:

1.—The connections between the columns and the girders are perfectly rigid.
2.—The change in length of a member, due to the direct stress in it, is negligible.
3.—The lengths of the members are the distances between the intersections of their neutral axes.
4.—The deflection of a member due to internal shear is negligible.
5.—The wind load is resisted entirely by the steel frame.

Professor Wilson and Professor H. F. Moore, of the University of Illinois, have made an extended investigation of the rigidity of riveted joints of steel structures.† Their tests and analyses show that, for the usual girder to column connection in which there is no spreading of the connection by gusset-plates, the slip in the connection is so great that it cannot be considered as rigid in analyzing the stresses. However, in a building which is tall enough to render the wind stresses important, the bending moments at the connections are so great that, if properly taken care of, the connections will have to be spread by gusset-plates. These tests showed that such connections were sufficiently rigid so that the results would not be affected seriously.

Application to a Definite Problem

Fig. 1 is a view of the American Insurance Union Building, and Fig. 2 shows the structural steel frame of the lower nine stories. The arched entrance is three stories high and this makes necessary the omission of the middle girder at the mezzanine and second floors. It was on account of

* The fundamental equation of the slope deflection method is based on the principle of area moments and its derivation is given completely in *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station.

† *Bulletin No. 104*, Univ. of Illinois Eng. Experiment Station.

this irregularity and the importance of wind stresses in the lower stories, that it was thought desirable to investigate the stresses by the slope deflection method.

The tower is practically square and has six lines of columns in each direction. The major portion of the wind shear is carried by the exterior frames. A careful investigation of the relative stiffness of the six frames showed that each interior frame would carry about 3% of the total shear, leaving 44% for each exterior frame. The exterior frames are not exactly symmetrical, due to one of the columns being a corner column; but in order to simplify the calculations, the frame was assumed to be symmetrical and the properties of the outside columns were taken as the average of the two actual columns. Fig. 3(*a*) is a diagram of the lower eight stories.

It would be too great a task to apply the slope deflection method to the entire height of the tower, so the assumption was made that the values of θ at the eighth floor were equal to those at the seventh floor, and that the value of R for the seventh story was equal to that of the sixth story. This assumption makes it possible to solve the lower eight stories without those above, and gives results which are correct for the part below about the fifth floor.

In a structure with rigid joints, acted upon by forces applied at the joints only, the shape of the elastic lines of the members will be reversed curves. Fig. 4 shows the elastic line of such a member, greatly exaggerated, and will serve to illustrate the notation used.

Notation.—

M_{AB} = moment at A in Member $A\ B$.
M_{BA} = moment at B in the same member.
θ_A = change in slope of the elastic line at A, due to external forces.
θ_B = change in slope of the elastic line at B, due to external forces.
d = total deflection of A from its original position, A'.
$\Delta_A = d - \theta_B\, l$ = deflection of A with respect to the tangent at B.
$\Delta_B = d - \theta_A\, l$ = deflection of B with respect to the tangent at A.
I = moment of inertia of the member.

$$K = \frac{I}{l}.$$

$$R = \frac{d}{l}.$$

As applied to the present problem, this notation is used, specifically as follows:

A, B, C, etc., are used to designate the columns, *A* being the exterior columns, for a symmetrical bent. *A′, B′, C′,* etc., are used for the other half of the bent. (See Fig. 3(*a*).)

a, b, e, etc., are used to designate the bays of a bent, *a* being the exterior bay. The letter, *e,* is used instead of *c,* to avoid confusion with *C* when used as a subscript.

$A_1\ A_2\ A_3$ etc., are used to designate the intersections of the neutral axes of the girders of the first, second, third, etc., floors with the neutral axis of Column *A*.

$B_1\ B_2\ B_3$ etc., are used to designate the intersections of the girders with Column *B*.

d = the deflection of a column in a story height.
E = the modulus of elasticity of the material.

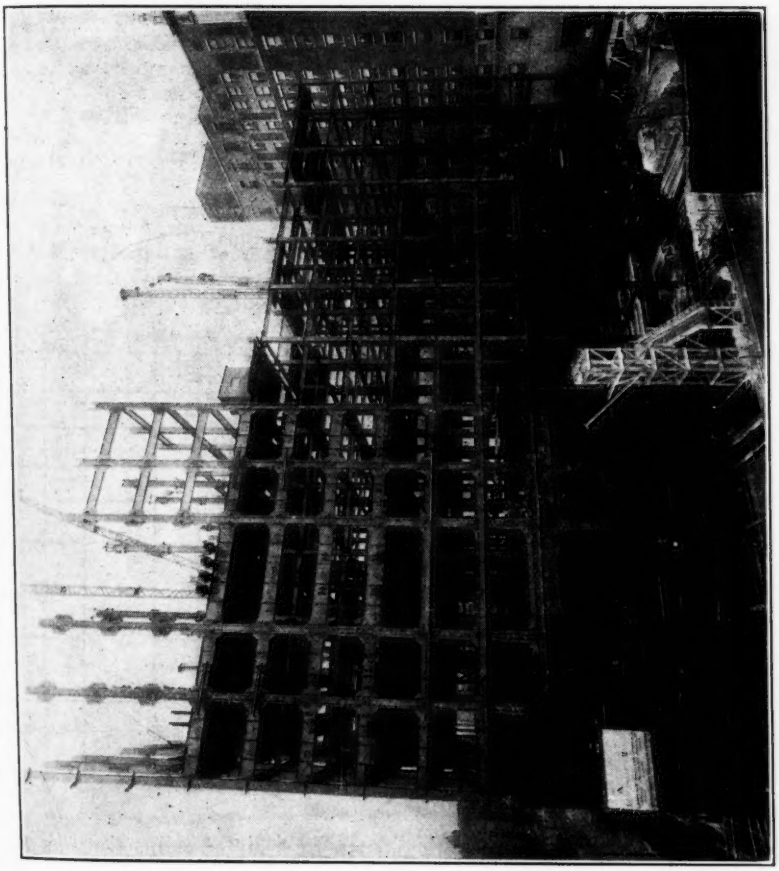

FIG. 2.—VIEW SHOWING STRUCTURAL STEEL FRAME OF LOWER NINE STORIES, AMERICAN INSURANCE UNION BUILDING.

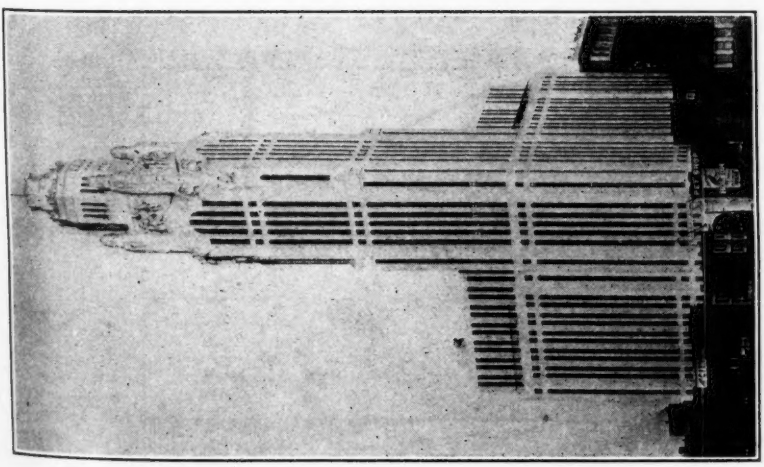

FIG. 1.—VIEW OF COMPLETED AMERICAN INSURANCE UNION BUILDING.

A
each s
values

h = the length of a column measured from neutral axis to neutral axis of girders.

L = the length of a girder measured from neutral axis to neutral axis of columns.

I = the moment of inertia of the girder or column section.

$K = \frac{I}{L}$ for girders, and $K = \frac{I}{h}$ for columns.

$R = \frac{d}{h}$ for any story.

M = bending moment.

W = total horizontal wind shear in a bent in any story.

θ = change in the slope of the tangent to the elastic curve.

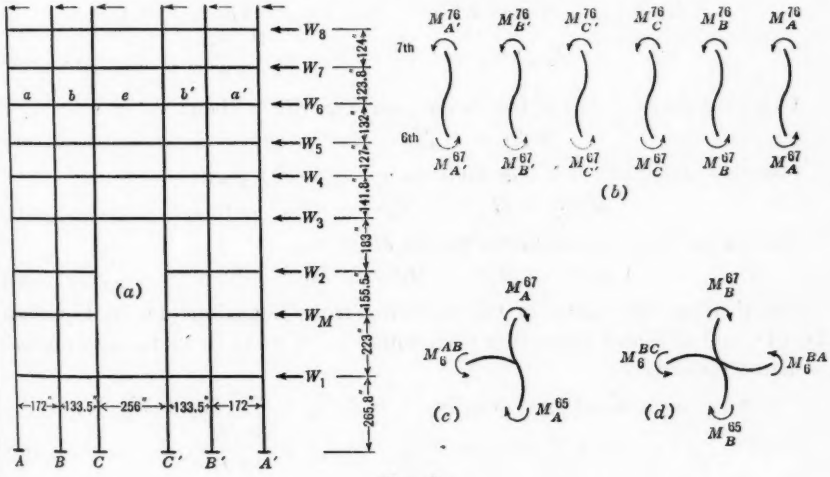

FIG. 3.

A moment equation may be written for each joint of the frame and for each story. Equations (3), (4) and (5) are typical for the sixth story. The values of the moments are substituted from Equation (1), and the four

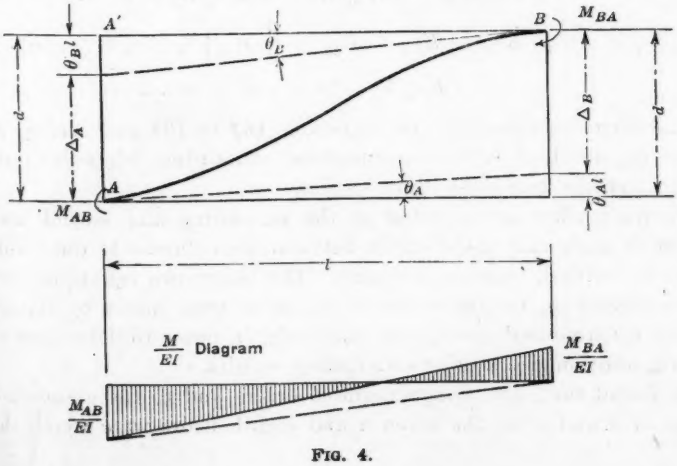

FIG. 4.

equations, (6), (7), (8), and (9), are derived accordingly. These will apply to any other story by changing the subscripts. In this way as many equations may be written as there are unknowns (values of θ and R).

Preliminary Equations

The fundamental equation of the slope deflection method is,

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B - 3\,R) \ldots\ldots (1)$$

Consider the sixth story as a free body (see Fig. 3(*b*)):

$$2\,(M_A^{67} + M_A^{76} + M_B^{67} + M_B^{76} + M_C^{67} + M_C^{76}) = W_6\,h_6$$

Substituting for M_{AB} in Equation (1) and simplifying:

$$K_{A6}\,(\theta_{A6} + \theta_{A7} - 2\,R_6) + K_{B6}\,(\theta_{B6} + \theta_{B7} - 2\,R_6) + K_{C6}\,(\theta_{C6} + \theta_{C7} - 2\,R_6) = \frac{W_6\,h_6}{12\,E} \ldots\ldots (2)$$

Consider Joint A_6 as a free body (see Fig. 3(*c*)) then:

$$M_A^{67} + M_A^{65} + M_6^{AB} = 0 \ldots\ldots (3)$$

Consider Joint B_6 as a free body (see Fig. 3(*d*)) then:

$$M_6^{BC} + M_B^{67} + M_6^{BA} + M_B^{65} = 0 \ldots\ldots (4)$$

Similarly, from a consideration of Joint C_6,

$$M_6^{CC} + M_C^{67} + M_6^{CB} + M_C^{65} = 0 \ldots\ldots (5)$$

Substituting the values of the moments from Equation (1), in Equations (3), (4), and (5), and expanding each equation (2 to 5) in turn, the equations become, respectively:

$$K_{A6}\,\theta_{A7} + K_{A6}\,\theta_{A6} + K_{B6}\,\theta_{B7} + K_{B6}\,\theta_{B6} + K_{C6}\,\theta_{C7} + K_{C6}\,\theta_{C6} - 2\,R_6\,(K_{A6} + K_{B6} + K_{C6}) = \frac{W_6\,h_6}{12\,E} \ldots\ldots (6)$$

$$K_{A6}\,\theta_{A7} + 2\,\theta_{A6}\,(K_{A6} + K_{A5} + K_{a6}) + K_{A5}\,\theta_{A5} + K_{a6}\,\theta_{B6} - 3\,K_{A6}\,R_6 - 3\,K_{A5}\,R_5 = 0 \ldots\ldots (7)$$

$$K_{B6}\,\theta_{B7} + 2\,\theta_{B6}\,(K_{B6} + K_{B5} + K_{a6} + K_{b6}) + K_{B5}\,\theta_{B5} + K_{a6}\,\theta_{A6} + K_{b6}\,\theta_{C6} - 3\,K_{B6}\,R_6 - 3\,K_{B5}\,R_5 = 0 \ldots\ldots (8)$$

$$K_{C6}\,\theta_{C7} + 2\,\theta_{C6}\left(K_{C6} + K_{C5} + K_{b6} + \frac{3}{2}\,K_{c6}\right) + K_{C5}\,\theta_{C5} + K_{b6}\,\theta_{B6} - 3\,K_{C6}\,R_6 - 3\,K_{C5}\,R_5 = 0 \ldots\ldots (9)$$

By changing the subscripts in Equations (6) to (9) and setting $\theta_8 = \theta_7$ and $R_7 = R_6$, the final thirty-two equations, containing thirty-two unknowns, may be written for the lower eight stories.

Where the girders are omitted at the mezzanine and second floors, the assumption is made that the distance between the columns at the point where the girder is omitted, remains constant. The thirty-two equations, involving thirty-two unknowns, for the lower eight stories were solved by the use of a calculating machine and also by the slide-rule, in order to determine whether a slide-rule solution would give satisfactory results.

It was found that, due to accumulated errors and to the assumption that the values of θ and R at the seventh and eighth floors were equal, the total

shears did not check with those determined directly from the wind. Therefore, the shears and, subsequently, the moments, were corrected.

The total direct stresses in the columns could not be determined from the solution because the distribution of the direct stresses in the columns above the seventh floor was not known. In order to obtain the proportion of the direct stress carried by each column above the seventh floor, the total increments in direct stress in the columns as determined by the girder shears below the seventh floor, were plotted as shown in Fig. 5. The ratios thus determined were assumed to be constant above the seventh floor, and by means of a moment equation, the direct stresses were calculated.

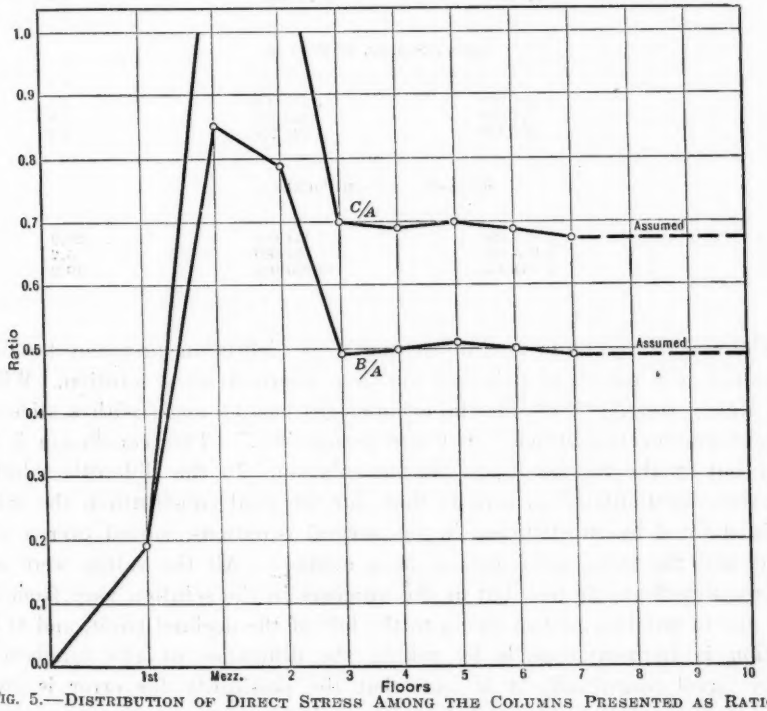

FIG. 5.—DISTRIBUTION OF DIRECT STRESS AMONG THE COLUMNS PRESENTED AS RATIOS.

In order to get some positive check on the results of the slope deflection solution, the moments, shears, and direct stresses at the bases of the columns were measured on a cardboard model by the use of the Beggs deformator gauges.* Table 1 shows a comparison of the results obtained. The discrepancy in the moments is probably due to some lost motion where the bottoms of the columns were clamped, thus rendering the bases in the model not perfectly fixed, as is assumed in the algebraic solution.

Tables 2 to 5 give a summary of the wind stresses below the eighth floor as determined by the slope deflection method using the calculating machine, the same method, and the slide-rule, and by the Fleming Method No. I. All the results in Tables 2 to 5 have been adjusted so that the total shears check.

* For a description of the use of the Beggs' deformator gauges, see, *Proceedings*, Am. Concrete Inst., 1922.

This adjustment eliminates considerable error. Where the different methods give a change in sign, the total error is given.

TABLE 1.

Column No.	Deformator gauge.	Slope deflection.	Percentage of variation.
	SHEAR, IN POUNDS.		
A	48 220	48 800	1.3
B	53 850	53 100	1.3
C	77 420	78 500	1.8
	DIRECT STRESS, IN POUNDS.		
A	1 271 000	1 257 400	1.1
B	614 950	630 800	2.6
C	— 924 500	— 943 700	2.0
	MOMENTS, IN INCH-POUNDS.		
A	5 800 000	7 134 000	22.9
B	6 930 000	7 293 000	5.2
C	—9 860 000	—10 870 000	10.2

The use of the slide-rule in solving slope deflection problems has been suggested as a means of reducing the time required for a solution. Wilson and Maney state,* "* * * the calculations can be made with a slide-rule without greater inaccuracies than are permissible." This conclusion is not borne out by the results of the present solution. In the slide-rule solution, the errors accumulated so rapidly that, for the final substitution, the values obtained for θ by substituting in the several equations, varied over a wide range, and the value used was, at best, a guess. All the values were read to three significant figures, but as the numbers in the solution vary from less than one to numbers of two places to the left of the decimal point, and as the solution is frequently made by taking the difference of two numbers of nearly equal magnitude, it is seen that the possibility for error is great. Even if the results show a fair agreement for some particular problem, the recognized possibilities for error would limit the use of such a solution, since in the practical application there would be no check available. Two solutions carried on simultaneously might give sufficiently accurate results, but one solution with a calculating machine will give a positive check and the total man-hours would be considerably less. It is concluded, therefore, that a slide-rule solution of a slope deflection problem is in no case advisable, particularly when the number of equations is comparatively large.

The stresses obtained by the slope deflection method are for the actual design used in the building. It is seen that the errors of the Fleming Method No. I are considerable, even for the upper stories where the arrangement of members is regular.

* *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station.

TABLE 2.—MOMENTS, IN 1 000 INCH-POUNDS.

Story.	Column A.				Column B.				Column C.			
	Slope deflection by calculating machine.	Slope deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.	Slope deflection by calculating machine.	Slope deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.	Slope deflection by calculating machine.	Slope deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.
0-1	7 134	7 122	0	2 643	−63	7 293	7 192	− 1	6 352	−13	10 870	10 730	− 1	13 678	26
	5 833	6 070	4		−55	6 811	6 823	0		− 7	9 994	10 056	0		37
1-*M*	4 005	8 430	111	2 991	−25	8 016	14 891	86	6 295	−21	10 783	7 249	−33	9 246	14
	3 318	6 466	95		−10	6 785	6 085	+10		− 7	6 701	3 674	−45		36
M-2	3 008	3 445	13	5 885	+96	6 809	6 732	1	9 474	−39	2 690	4 017	+49	2 853	18
	3 334	3 089	7		+76	7 017	6 554	7		+35	3 906	3 023	−20		40
2-3	2 392	2 478	4	2 405	+ 1	5 962	5 590	6	4 721	−10	4 271	3 931	8	8 558	−100
	3 616	3 581	1		−35	6 789	6 930	2		−30	8 261	8 019	3		4
3-4	2 985	3 000	3	1 585	−46	4 705	4 644	1	3 558	−24	5 704	5 656	1	6 450	13
	1 877	2 181	16		−16	3 601	3 466	4		− 1	4 205	4 125	2		53
4-5	1 862	2 202	18	1 865	−27	3 832	3 622	5	3 066	−20	4 463	4 249	4	5 557	25
	1 840	2 063	12		−26	3 778	3 704	2		−19	4 868	4 296	2		27
5-6	1 959	.....	.....	1 857	−29	3 993	.....	.....	3 113	−20	4 519	.....	.....	5 640	25
	1 885	.....	.....		−33	3 826	.....	.....		−19	4 395	.....	.....		28
6-7	1 820	.....	.....	1 277	−30	3 498	.....	.....	2 876	−18	4 089	.....	.....	5 197	27
	1 764	.....	.....		−28	3 463	.....	.....		−18	4 028	.....	.....		26
7-8	1 741	.....	.....	1 249	−28	3 461	.....	.....	2 807	−19	4 010	.....	.....	5 084	27
	.....	.....	.....		....	.....	.....	....		....	.....	.....	....		....

TABLE 2.—(*Continued*).

Floor.	Girder *a*.				Girder *b*.				Girder *c*.			
	Slope deflection by calculating machine.	Slope deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Slope deflection by calculating machine.	Slope deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Slope deflection by calculating machine.	Slope deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.
1	9 838	14 500	+47	5 633	7 441	11 748	+58	7 014	12 845	10 718	—17	15 910
	7 381	9 961	+35		7 915	6 581	+17					
M	6 326	9 911	+57	8 875	7 919	7 440	+6	6 894	.....	.....	....	.....
	5 671	5 359	+6		7 387	7 693	—3					
2	5 726	5 867	+3	7 989	7 079	7 082	—0	6 206	.....	.....	....	.....
	5 201	5 056	+3		8 176	6 952	+18					
3	6 551	6 581	0	3 669	6 304	6 285	+0	4 589	6 772	6 650	— 2	10 419
	5 189	5 298	+2		7 194	7 096	+1					
4	3 739	4 383	+17	2 950	4 057	3 950	+3	3 674	4 407	4 239	— 4	8 833
	3 378	3 140	+7		4 248	4 104	+3					
5	3 799	4 140	+9	2 752	4 208	3 806	+9	3 427	4 624	4 190	— 7	7 770
	3 471	3 129	+10		4 361	4 034	+7					
6	3 655	.....	....	2 657	4 001	.....	....	3 307	4 326	.....	....	7 501
	3 324	.....	....		4 160	.....	....					
7	3 505	.....	....	2 528	3 777	.....	....	3 153	4 008	.....	....	7 155
	3 148	.....	....		3 936	.....	....					
8	.....	.....	....	.....	.....	.....	....	.....	.....	.....	....	.....

Girder *a*.				Girder *b*.				Girder *c*.			
Slope deflection by calculating machine.	Slope deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Slope deflection by calculating machine.	Slope deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Slope deflection by calculating machine.	Slope deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.
9 838	14 500	+47	5 633	7 441	11 748	+58	7 014	12 845	10 718	—17	15 910
7 381	9 961	+35		7 915	6 581	+17					
6 326	9 911	+57	8 875	7 919	7 440	+6	6 894	.....	.....	....	.....
5 671	5 359	+6		7 387	7 693	—3					
5 726	5 867	+3	7 989	7 079	7 082	—0	6 206	.....	.....	....	.....
5 201	5 056	+3		8 176	6 952	+18					
6 551	6 581	0	3 669	6 304	6 285	+0	4 589	6 772	6 650	— 2	10 419
5 189	5 298	+2		7 194	7 096	+1					
3 739	4 383	+17	2 950	4 057	3 950	+3	3 674	4 407	4 239	— 4	8 833
3 378	3 140	+7		4 248	4 104	+3					
3 799	4 140	+9	2 752	4 208	3 806	+9	3 427	4 624	4 190	— 7	7 770
3 471	3 129	+10		4 361	4 034	+7					
3 655	.....	....	2 657	4 001	.....	....	3 307	4 326	.....	....	7 501
3 324	.....	....		4 160	.....	....					
3 505	.....	....	2 528	3 777	.....	....	3 153	4 008	.....	....	7 155
3 148	.....	....		3 936	.....	....					
.....	.....	....	.....	.....	.....	....	.....	.....	.....	....	.....

Girder *a*.			Girder *b*.			Girder *c*.		
Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.
—43	5 633	+47	7 014	+58	7 014	—6	15 910	+24
—24		+35		+17		—11		
+40	8 875	+57	6 894	+6	6 894	—18	.....	....
+57		+6		—18		—27		
+40	7 989	+3	6 206	0	6 206	—12	.....	....
+54		+3		—15		—24		
—44	3 669	0	4 589	0	4 589	—27	10 419	+54
—29		+2		+1		—36		
—21	2 950	+17	3 674	+3	3 674	—9	8 833	+89
—13		+7		+3		—14		
—28	2 752	+9	3 427	+9	3 427	—18	7 770	+72
—21		+10		+7		—31		
—27	2 657	.....	3 307	.....	3 307	—17	7 501	+73
—20		.....		.....		—21		
—28	2 528	.....	3 153	.....	3 153	—17	7 155	+75
—20		.....		.....		—20		
....	.....	.....	.....	.....	.....	....	.....	....
....						....		

TABLE 3.—SHEARS, IN THOUSANDS OF POUNDS.

Story.	Column A.				Column B.				Column C.			
	Slope-deflection by calculating machine.	Slope-deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.	Slope-deflection by calculating machine.	Slope-deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Slope-deflection by calculating machine.	Slope-deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.
0-1	46.8	49.6	+ 2	19.9	−59	53.1	52.7	1	47.8	−10	78.5	78.1	102.8	31
1-M	32.9	66.8	103	26.8	−19	66.4	61.8	7	56.5	−15	78.4	49.0	82.9	6
M-2	40.8	41.9	3	75.6	+85	88.9	85.1	4	121.8	+37	42.4	45.1	80.2	−171
2-3	32.8	33.1	1	23.0	−30	65.9	68.4	3	51.6	−22	68.5	65.7	93.5	37
3-4	33.9	36.5	8	22.3	−34	58.6	57.1	4	50.1	−15	69.9	68.9	90.9	30
4-5	29.2	33.6	15	21.5	−26	59.9	57.7	....	48.3	−19	69.5	67.3	87.5	26
5-6	28.7	....	.....	21.0	−27	58.5	....	....	47.2	−19	67.5	....	85.5	27
6-7	29.0	....	.....	20.6	−29	56.3	....	....	46.4	−18	65.6	....	83.8	28
7-8	27.7	....	.....	20.2	−27	55.6	....	....	45.3	−19	64.3	....	82.0	28

TABLE 4.—DIRECT STRESSES.

Story.	Column A.				Column B.				Column C.			
	Slope-deflection by calculating machine.	Slope-deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.	Slope-deflection with slide-rule.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.	Slope-deflection by calculating machine.	Slope-deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.
0–1	1 257.4	1 361.1	+ 8	1 032.8	— 18	630.8	478.8	— 24	504.5	—20	—943.7	—934.3	— 1	50.7	—105
1–M	1 157.3	1 218.9	+ 5	967.3	— 16	615.9	483.7	— 21	465.0	—25	—929.0	—880.7	— 5	81.4	—103
M–2	1 087.5	1 130.1	+ 4	864.1	— 21	556.0	459.1	— 17	465.0	—15	—799.4	—767.3	— 4	184.6	—117
2–3	1 024.0	1 068.3	+ 4	771.2	— 25	505.8	415.8	— 18	465.0	— 8	—685.1	—662.2	— 3	227.5	—133
3–4	955.8	999.3	+ 5	728.3	— 24	472.4	384.6	— 18	439.2	— 7	—636.9	—614.0	— 4	214.8	—134
4–5	914.4	955.6	+ 5	694.0	— 24	451.6	366.0	— 18	418.5	— 7	—609.1	—586.8	— 4	204.7	—134
5–6	872.1	913.3	+ 5	662.0	— 24	429.7	351.6	— 18	399.2	— 7	—580.8	—560.8	— 3	195.3	—134
6–7	831.5	.....	....	631.1	— 24	409.2	.....	....	380.6	— 7	—553.0	.....	....	186.2	—134
7–8	792.8	.....	....	601.7	— 24	390.1	.....	....	362.8	— 7	—527.2	.....	....	177.5	—134

TABLE 5.—SHEARS.

Floor.	Girder *a*.				Girder *b*.				Girder *c*.			
	Slope deflection by calculating machine.	Slope-deflection with slide-rule.	Percentage of variation from slide-rule calculation.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.	Slope-deflection with slide-rule.	Slope deflection by calculating machine.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.	Slope-deflection with slide-rule.	Slope-deflection by calculating machine.	Fleming Method No. I.	Percentage of variation from slide-rule calculation.
1	100.1	142.2	+42	65.5	−35	187.3	115.0	105.0	−9	83.7	100.4	124.3	+24
M	69.8	88.8	+27	108.2	+48	118.4	129.6	108.2	−20	....	.....	.....	....
2	63.5	61.6	−3	92.9	+46	105.1	114.3	92.9	−19	....	.....	.....	....
3	68.3	69.0	+1	42.9	−37	100.2	101.1	68.7	−32	52.0	52.9	81.4	+54
4	41.4	43.7	+6	34.3	−17	60.3	62.2	55.0	−12	33.1	34.4	65.1	+89
5	42.3	42.3	0	32.0	−24	58.7	64.3	51.3	−20	32.7	35.3	60.7	+72
6	40.6	.....	....	30.9	−24	.....	61.1	49.5	−19	....	38.8	58.6	+73
7	38.7	.....	....	29.4	−24	.....	57.8	47.2	−18	....	32.0	55.9	+75

Use of the Slope Deflection Method in Design

The slope deflection method is a method of analysis of the stresses in a bent of definite proportions. Any change in the sizes of the members, to make them conform more nearly to the stresses they have to carry will cause a redistribution of stress and a new solution will give other stresses. Thus, it is seen that the slope deflection method is not suitable for use in practice, aside from the excessive amount of labor involved.

Wilson and Maney have proposed* an approximate method in which they assume that the θ at any given floor is equal to the θ's at the floors immediately above and below, and that the R's of the stories immediately above and below the floor are equal. These assumptions make it possible to solve for the stresses in any story, independent of the balance of the bent. This is equivalent to assuming that the contraflexure points in the columns are at their mid-points. This approximate method also requires a bent of definite proportions for a solution, and any change in the sizes of the members will cause a redistribution of stresses.

In order to determine the possibility of using the approximate or the slope deflection method in designing, the nineteenth floor of the American Insurance Union Building was selected and the stresses in the girders were determined for the bent as constructed, by the approximate slope deflection method. The first solution gave a decreased moment for the long center span and for the outside span (see Fig. 3(a)) while the moment for the short span was considerably greater than that for which it was designed. The sections of the girders were reproportioned for these stresses and a second calculation of stresses made. The variation was in the same direction and more marked. A third solution showed the same tendency. In these solutions the sections of the columns were not changed, but it is evident that if the sections of the girders were changed as indicated by the stresses, the column moments would be greatly altered also.

Such an example serves to show that the design of a bent for stresses determined by the slope deflection method merely creates a new problem and that the short span invariably carries more stress than that for which it was designed. The only safe procedure in designing by the slope deflection method is to determine those members which are over-stressed, and then to increase all the members by a proportional amount, in order not to change the stress distribution. This would make no attempt at economy, and the final design would depend entirely upon the preliminary design.

Economy in Design

Next to safety, economy in design should be the main consideration. That design which requires the least steel to carry safely the given loads is generally the best. There are several factors that affect the proportioning of the members. Columns and girders must be proportioned to carry bending, shearing, and direct stresses. Of these, bending moments exert the greatest

* *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station.

influence on the size of the members; the direct stresses in the columns are important; and the shears seldom play a part in the design.

The total moment in any story of a bent (sum of the moments at both ends of all the columns of a story) is constant, and equals the total shear in the story times the story height. As the maximum moment is the most important factor in determining the column cross-sections, the maximum economy is obtained if the bending moments can be divided equally between the ends of the columns, regardless of the distribution of the moment among the various columns. Therefore, if the points of contraflexure can be made to fall at the middle of all the columns, the maximum economy is obtained as far as moment in the columns is concerned.

Since the total moment in each story is constant, the sum of the moments at the ends of all the girders at a floor level is constant, because the girders must resist the column moments. (See Fig. 6). Therefore, the maximum economy in the girders will obtain if their points of contraflexure are at their mid-points.

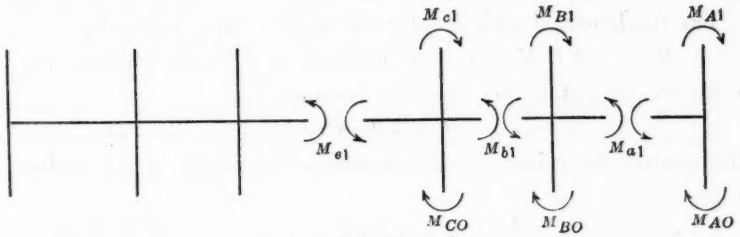

FIG. 6.

The direct stresses in the columns resist the external moment on the bent due to the wind; consequently, that design which distributes the greatest portion of the direct stress to the outside columns is the most economical. This is shown by a simple moment equation.

This discussion brings out the following requirements for the ideal bent:

(1) The points on contraflexure of all the members are at their mid-points; and
(2) All the direct stress is taken by the outside columns.

Is it possible to design a bent which meets these requirements? As for the first requirement, if the contraflexure points are all at their mid-points of the members, the θ's will all be equal. Fig. 6 shows a floor of a bent with the moments indicated. From Equation (1):

$$\left.\begin{aligned} &M_{a1} = 2\,E\,K_{a1}(3\,\theta);\ M_{b1} = 2\,E\,K_{b1}\,(3\,\theta);\ M_{c1} = 2\,E\,K_{c1}\,(3\,\theta);\\ &M_{A1} = 2\,E\,K_{A1}\,(3\,\theta - 3\,R);\ M_{B1} = 2\,E\,K_{B1}\,(3\,\theta - 3\,R);\\ &M_{C1} = 2\,E\,K_{C1}\,(3\,\theta - 3\,R);\ M_{A0} = 2\,E\,K_{A0}\,(3\,\theta - 3\,R);\\ &M_{B0} = 2\,E\,K_{B0}\,(3\,\theta - 3\,R);\ M_{C0} = 2\,E\,K_{C0}\,(3\,\theta - 3\,R) \end{aligned}\right\} \ldots(10)$$

Also, from Fig. 6,

$$M_{a1} = M_{A1} + M_{A0};\ M_{a1} + M_{b1} = M_{B1} + M_{B0};\ M_{b1} + M_{e1} = M_{C1} + M_{C0} \ldots(11)$$

Substituting the values of the moments (Equations (10)) in Equations (11):

$$2\,E\,K_{a1}\,(3\,\theta) = 2\,E\,K_{A1}\,(3\,\theta - 3\,R) + 2\,E\,K_{A0}\,(3\,\theta - 3\,R) \ldots\ldots (12)$$

$$2\,E\,K_{a1}\,(3\,\theta) + 2\,E\,K_{b1}\,(3\,\theta) = 2\,E\,K_{B1}\,(3\,\theta - 3\,R) + 2\,E\,K_{B0}\,(3\,\theta - 3\,R) \ldots\ldots (13)$$

$$2\,E\,K_{b1}\,(3\,\theta) + 2\,E\,K_{e1}\,(3\,\theta) = 2\,E\,K_{C1}\,(3\,\theta - 3\,R) + 2\,E\,K_{C0}\,(3\,\theta - 3\,R) \ldots\ldots (14)$$

Dividing Equation (12) by Equation (13) and Equation (13) by Equation (14), and transposing:

$$\frac{K_{a1}}{K_{A1} + K_{A0}} = \frac{K_{a1} + K_{b1}}{K_{B1} + K_{B0}} = \frac{K_{b1} + K_{e1}}{K_{C1} + K_{C0}} \ldots\ldots (15)$$

or in words, if the θ's are equal, the sum of the K's of a column above and below the floor must be proportional to the sum of the K's of the girders directly connected to it.

As for the conditions that affect the distribution of the direct stresses in the columns: If the direct stress due to the external wind moment is carried entirely by the outside columns, the direct stresses in the interior columns are all zero, and the shears are equal in all the girders at any floor level. The fundamental equation for the moment in a girder is,

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B) \text{ and } (R = 0 \text{ for the girders}) \ldots\ldots (16)$$

If the θ's are all equal, this equation becomes,

$$M = 2\,E\,K\,(3\,\theta) \ldots\ldots (17)$$

This assumes the points of contraflexure at the middle of the girders. Then,

$$M = \text{shear} \times \frac{L}{2} = 2\,E\,K\,(3\,\theta) \text{ and shear} = \frac{4\,E\,K\,(3\,\theta)}{L} \ldots\ldots (18)$$

From this it follows that, if the shears in the girders are constant for any floor, the values of $\frac{K}{L}$ must be constant, or, in other words, the K's of the girders must be proportional to their lengths. Therefore, for economy as to wind stresses, the ideal bent should have the following proportions:

(1) The sum of the K's of a column, above and below a floor, must be proportional to the sum of the K's of the girders directly connected to it.

(2) The K's of the girders must be proportional to their lengths.

Variations from the Ideal

In practice, it will seldom occur that the theoretical proportions of the ideal bent can be maintained; therefore, that design which conforms to these proportions will be referred to as "theoretically proportioned". In ordinary bents, the interior columns will follow the theoretical proportions very closely, because the direct stresses due to dead and live load will vary approximately, as the lengths of the girders; and for columns of similar cross-sections and equal widths, the moments of inertia will vary, approximately, as the direct stresses. The K's will follow the same relation since the lengths of all the

columns in a story are the same. This will not be true of the outside columns, however, due to the added wall load and the direct stress due to wind, as well as to the fact that they are usually corner columns which must be designed as members of two wind-resisting bents at right angles to each other. The interior columns may also vary from the theoretical proportions because of special loads, central towers, and the use of different shapes of cross-section.

Any variation from the theoretical proportions tends to change the θ's of the joints. Increases in the column K's tend to increase the angle, and reductions tend to decrease it. The shear and moment in the column and adjacent girders are, therefore, increased with increases in the size of the column over theoretical proportions, as a glance at Equation (1) will show. The increase of the θ of a joint depends upon two things: The ratio of the actual K of the column being considered to the K of the same column as determined by theoretical proportions, and the ratio of the K's of the theoretically proportioned column to the sum of the K's of the adjacent girders.

Variations in these relations and their effect upon the θ of a joint are plotted on Fig. 7. These curves were determined from the results of the solution of thirty-five bents of varying spans and proportions, calculated by the approximate slope deflection method. Errors of not more than 3% are noted for the range taken, which it is believed will cover any practical case. This variation is reduced for shears, as the relation between the two horizontal scales on the diagram will show. For instance, variations in the moment in the girder at the end near a variable column equals one and one-third times the corresponding variation in the shear and the moment at the other end is equal to two-thirds the corresponding variation in shear. The data on the bents and the results of the calculations are given in Table 6. It will be noted that the curves apply only to increases in the size of a column over the theoretical proportion. It is always possible to make these variations positive by selecting the proper column on which to base the theoretical proportion. This is apparent when a solution is examined.

The increase in the θ of any column increases the relative shear in the adjacent girders by one-half the percentage of increase in the θ, and changes the maximum moment by two-thirds the amount. The moment at the other end of the affected girder is increased by one-third the percentage. An examination of Equation (16) will show this. For theoretical proportions, $\theta_A = \theta_B$, and the result is Equation (17).

If θ_A is increased 100%, it becomes twice θ_B, and,

$$M_{AB} = 2\,E\,K\,(4\,\theta_B + \theta_B)$$

which is an increase of $66\frac{2}{3}$%, and,

$$M_{BA} = 2\,E\,K\,(2\,\theta_B + 2\,\theta_B)$$

which is an increase of $33\frac{1}{3}$ per cent.

The shear for theoretical proportions would be (see Equation (18)),

$$\text{Shear} = \frac{4\,E\,K\,(3\,\theta_B)}{L} = \frac{12\,E\,K\,\theta_B}{L} \quad \ldots\ldots\ldots\ldots\ldots (19)$$

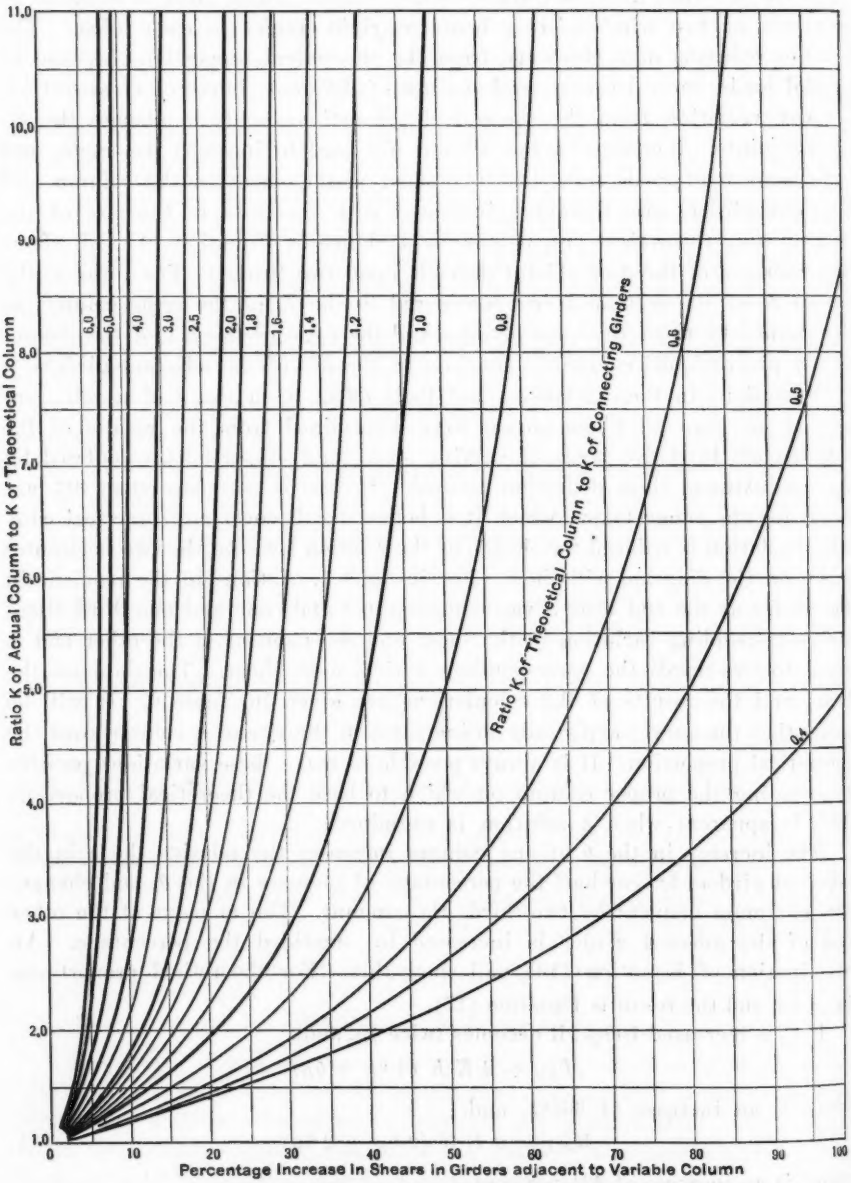

FIG. 7.—VARIATIONS IN MOMENTS AND SHEARS.

For a 100% increase in θ, the shear would be,

$$\text{Shear} = \frac{2\,E\,K\,(4\,\theta_B) + 2\,E\,K\,(5\,\theta_B)}{L} = \frac{18\,E\,K\,\theta_B}{L}$$

which is an increase of 50 per cent.

TABLE 6.—SUMMARY OF BENTS USED TO OBTAIN CHART OF VARIATIONS

Bent No.	Ratio of spans.	Ratio of Actual K to Theoretically Proportioned K of Columns.				Values of K for columns and girders.	Relative Angles.				θ of column to average of others.*
		Z.	A.	B.	C.		θ_Z	θ_A	θ_B.	θ_C	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	1:1: 1:1	1	1	1	1	.....	1.0	1.0	1.0	1.0	1.0
2	2:1: 3:6	1	1	1	1	.....	1.0	1.0	1.0	1.0	1.0
3	1:1: 1:1	2	1	1	1	1	1.435	1.0	1.044	1.040	1.407
4	1: 1:1	.....	2	1	1	1	.....	1.435	1.0	1.043	1.413
5	1: 1:1	.....	1.5	1	1	1	.....	1.264	1.0	1.026	1.251
6	1: 2:3	.....	2	1	1	2	.....	1.226	1.0	1.010	1.221
7	1:1	.....	2	1	.....	1	.....	1.429	1.0	.....	1.429
8	1:1	.....	1.5	1	.....	1	.....	1.261	1.0	.....	1.261
9	1: 1:1	.....	1	1	1.5	2	.....	1.101	1.0	1.140	1.135
10	1: 1:1	.....	1	1.5	1	2	.....	1.0	1.158	1.010	1.153
11	2: 1:2	.....	1.5	1	1	2	.. ..	1.166	1.0	1.020	1.156
12	2:15:3	.....	1.5	1	1.22	2	.....	1.153	1.0	1.082	1.153 1.082
13	1: 1:1	.....	3	1	1	1	.....	1.642	1.0	1.062	1.610
14	3: 1:3	.....	1.33	1	1.25	2	.....	1.114	1.0	1.085	1.114 1.085
15	1:1	.....	2	1	.....	4	.....	1.120	1.0	.....	1.120
16	1: 1:1	.....	2	1	1	3	.....	1.158	1.0	1.008	1.154
17	1: 1:1	.....	2	1	1	4	.....	1.120	1.0	1.004	1.118
18	1: 1:1	.....	2	1	1	5	.....	1.714	1.0	1.071	1.680
19	1: 1:1	.....	1.5	1	1	2	.....	1.147	1.0	1.009	1.142
20	1: 1:1	.....	1.5	1	1	4	.....	1.078	1.0	1.008	1.077
21	1:1: 1:1	2	1	1	1	2	1.232	1.0	1.015	1.014	1.232
22	1:1: 1:1	1.5	1	1	1	2	1.147	1.0	1.009	1.008	1.141
23	1:1: 1:1	2	1	1	1	4	1.120	1.0	1.004	1.004	1.117
24	1: 1:1	.....	1.5	1	1	.5	.....	1.444	1.0	1.064	1.412
25	3: 1:1	.....	1.33	1	1	2	.....	1.111	1.0	1.010	1.106
26	1: 1:1	.....	1	1	1.5	2	.....	1.010	1.0	1.140	1.135
27	1: 1:1	.....	1	1	2	2	.....	1.015	1.0	1.223	1.206
28	1: 1:1	.....	1	1	1.5	1	.....	1.027	1.0	1.244	1.231
29	1: 1:2	.....	1	1	2	2	.....	1.015	1.0	1.220	1.213
30	2: 1:1	.....	1	1.33	1	2	.....	1.007	1.115	1.0	1.112
31	1:1	.....	3	1	.....	4	.....	1.165	1.0	.....	1.165
32	1: 1:1	.....	4	1½	1	1	.....	1.734	1.0	1.072	1.698
33	1: 1:1	.....	4	1	1	2	.....	1.372	1.0	1.022	1.361
34	1: 1:1	.....	4	1	1	3	.....	1.248	1.0	1.012	1.242
35	1: 1:1	.....	4	1,	1	4	.....	1.187	1.0	1.006	1.184

*The θ of the column referred to in Column (12) is the θ of that column which varies from the theoretical proportions.

When more than one column varies from the theoretical proportion the effect of each may be determined and the results combined.

A Proposed Method of Design

The method of design herein developed is based on the approximate slope deflection theory as proposed by Wilson and Maney, and assumes that the points of contraflexure are at the mid-points of the columns. Table 7 gives the

proportionate height of the point of contraflexure above the lower end of the column sections, as determined by the slope deflection method, for the lower nine stories of the American Insurance Union Building, and for the twenty stories of the bent, analyzed by Wilson and Maney.*

TABLE 7.—PROPORTIONATE HEIGHT OF CONTRAFLEXURE POINTS IN COLUMNS.

AMERICAN INSURANCE UNION BUILDING.				WILSON AND MANEY, 20-STORY BENT.					
Story.	Column *A*.	Column *B*.	Column *C*.	Story.	Column *A*.	Column *B*.	Story.	Column *A*.	Column *B*.
0	0.550	0.516	0.519	1	0.604	0.551	11	0.466	0.476
1	0.547	0.541	0.617	2	0.487	0.514	12	0.451	0.550
M	0.475	0.493	0.408	3	0.475	0.490	13	0.453	0.478
2	0.398	0.437	0.341	4	0.500	0.496	14	0.444	0.475
3	0.609	0.566	0.576	5	0.495	0.500	15	0.451	0.475
4	0.503	0.504	0.505	6	0.526	0.521	16	0.430	0.471
5	0.516	0.504	0.506	7	0.511	0.504	17	0.429	0.466
6	0.508	0.504	0.503	8	0.604	0.594	18	0.398	0.459
7	0.508	0.503	0.504	9	0.497	0.498	19	0.348	0.441
..	... .	.....	.....	10	0.477	0.480	20	0.222	0.394

For that portion of the bent in which the arrangement of the columns and girders is uniform, the points of contraflexure fall very near to the middle of the column sections, even if the theoretical proportions of the new method do not exist. The new method will take care of these small variations, approximately, where the girders depart from the theoretical proportions, by the use of the curves of the chart of variations (Fig. 7). The moments at the top and bottom of the affected column may be calculated from the respective floors, and the shift in the point of contraflexure determined.

The variations for tall buildings are smaller than for bents of only a few stories, and the variation reduces as the distance from the top increases and larger sections become necessary. This is gratifying, in that the accuracy of the method increases with the relative importance of the problem.

The lower story columns present a distinct problem, because it is assumed that the anchorage at the bases of the columns is sufficient to hold them rigid, and this makes all the θ's of the bases of the columns zero. Therefore, the point of contraflexure in the lower story must fall above the middle of the column section.

Assuming that $\theta_0 = 0$, $M_{01} = 2\,E\,K\,(2\,\theta_0 + \theta_1 - 3\,R)$ and $M_{10} = 2\,E\,K\,(2\,\theta_1 + \theta_0 - 3\,R)$. Subtracting these two equations,

$$M_{01} - M_{10} = -2\,E\,K\,\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Thus, the difference in the moments at the top and bottom of the column is known if θ_1 is known. An approximate value of θ_1 can be obtained from the moment equation for the first floor girders, and as the sum of the moments at the top and bottom of all the lower story columns is equal to the wind

* *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station, Fig. 5.

shear times the story height, the individual moments can be found. The equation for the difference in the two moments, as given, is negative because the moments themselves are negative. The moment at the base is, of course, larger than that at the top.

Frequently, due to some architectural feature, such as the entrance to the American Insurance Union Building, it may be necessary to have an irregular arrangement of members of the bent, or the relation between the K's of the girders which makes them proportional to the girder lengths, cannot be maintained. These extreme variations can be handled, approximately, by the use of the chart of variations. Although the condition that the K's of the girders must be proportional to their lengths is used, this condition does not affect the curves of Fig. 7. This is significant because the proper variation of the θ's, shears, and moments can be obtained for any relation between the K's of the girders. For the lower story columns, and for cases of great irregularity in design, two or three determinations may be necessary. When a member is omitted, or too great irregularity occurs, the results cannot be very close. Even the slope deflection solution for the lower stories of the building, where the girders at two floors are omitted, may be considerably in error.

In order to pick the proper percentages from the curves, the ratio of the K's of the actual columns to those of the theoretically proportioned columns, and the ratio of the K's of the columns to the K's of the adjacent girders must be known. This requires a preliminary design. As a summary of the preceding discussion, and in order to render the method of more service to the practicing designer, a routine outline of procedure for use in the design of a bent is presented herewith. In cases of regular bents, it is believed that an adherence to this outline will produce a preliminary design that will have to be altered but little, and that the resulting design will be economical.

Routine Outline to Be Used in Design

1.—Calculate the wind shear in each story from the assumed wind loads.

2.—Calculate the external wind moments, and the direct stresses in the outside columns, assuming all of it to be there and that the points of contraflexure of the columns are at their mid-points.

3.—Calculate the column shears and moments, assuming that all the girder shears are equal and that the contraflexure points are at the middle of the members.

4.—Calculate the dead and live load stresses in the columns.

5.—Proportion the columns to carry the dead and live load from Item (4), together with the wind bending and direct stresses from Items (2) and (3).

6.—With the girder shears from Item (3), calculate the wind moments in the girders and design the key girder at each floor for its wind moment combined with its dead and live load moment. The key girder is usually the shortest one. (Note that the maximum wind moment and maximum dead and live load moments do not occur at the same point.)

7.—Proportion the other girders by their relative K's, which are proportional to the girder lengths taken directly from the dimensions of the bent.

The sizes thus determined should be checked to see that the allowed unit stresses are not exceeded for dead and live load.

The preliminary design is now complete and it will usually be found to differ materially from theoretical proportions. Therefore, the stresses as found must be revised.

8.—The relative "theoretically proportioned" K's of the columns are determined by adding the K's of the adjacent girders.

9.—Determine the ratio of the sum of the actual K's above and below each floor to the theoretically proportioned K's of the columns. The one with the least ratio will then be termed "theoretically proportioned."

10.—Determine the ratio of the sum of the K's of the theoretically proportioned column above and below each floor, to the sum of the K's of the connecting girders.

11.—With the ratios of the actual column K's to the theoretical column K's as determined for the columns which vary from theoretical proportions in Item (9), and the ratios of the theoretical column K's to the K's of the adjacent girders, as determined in Item (10), the relative variation in the shears and moments in the girders can be determined from Fig. 7.

12.—For equal θ's the relative moments in the girders are proportional to the girder K's.

13.—Combining the relative moments found in Items (11) and (12), the resulting proportionate moments for the girders are found.

14.—From the relative moments found in Item (13) the actual girder moments may be found, because the sum of the moments at the ends of all the girders at a floor is equal to the shear in the story above times half its story height, plus the shear in the story below times half its story height. (This is approximate because the contraflexure points in the columns may not be at the mid-height.)

15.—The moments in the columns may now be found by assuming that the ratio of the moment above a floor to that below is the same as the ratio of the story shears multiplied by the story heights. From Fig. 6, it then follows:

$$M_A{}^{12} = \frac{M_1{}^{AB}}{1 + \frac{S_0\ L_0}{S_1\ L_1}};\quad M_B{}^{12} = \frac{M_1{}^{BA} + M_1{}^{BC}}{1 + \frac{S_0\ L_0}{S_1\ L_1}};\quad M_C{}^{12} = \frac{M_1{}^{CB} + M_1{}^{CC}}{1 + \frac{S_0\ L_0}{S_1\ L_1}};$$

$$M_A{}^{21} = \frac{M_2{}^{AB}}{1 + \frac{S_2\ L_2}{S_1\ L_1}};\ M_B{}^{21} = \frac{M_2{}^{BA} + M_2{}^{BC}}{1 + \frac{S_2\ L_2}{S_1\ L_1}};\ \text{and, } M_C{}^{21} = \frac{M_2{}^{CB} + M_2{}^{CC}}{1 + \frac{S_2\ L_2}{S_1\ L_1}} \quad \ldots(21)$$

16.—The girder shears are determined from the equation,

$$S_{AB} = \frac{M_{AB} + M_{BA}}{L} \quad \ldots(22)$$

From the girder shears the column direct stresses may be determined.

17.—The basement story column moments must be calculated from Equation (20). The method of doing this is given in detail in the illustrative problem in Table. 8.

Illustrative Problems

To illustrate the method of procedure, the wind stresses in the lower twelve stories of the American Insurance Union Building are worked out in detail (see Table 8 and Fig. 8). This problem was selected because it departs radically from theoretical proportions, especially in the lower stories where the middle girders are omitted. Its solution illustrates the method of procedure fully. For symmetrical bents with four lines of columns, there will be only one varying column and the solution is much simplified. For unsymmetrical bents, the solution must be carried entirely across the bent. The moments are in thousands of foot-pounds. Table 9, with Fig. 9, gives the solution of the stresses in the twenty-story bent which was solved by the slope deflection method.*

Calculation of Column Moments in the Basement Story.—If the columns were fixed at both top and bottom, the sums of the moments at the two ends of the columns would be proportional to the moments of inertia of the columns. If both ends of the columns were partly fixed as they are in the other stories, the sums of the moments at the top and bottom would be distributed among the columns in about the same ratio as in the other stories.

Proportioning the moment sums by the moments of inertia of the columns (the K's are used, being derivatives of the moments of inertia),

$$M_A^{01} + M_A^{10} = \frac{42\,98}{285.0} \times 355.2 \times 22.15 = 1\,186.8$$

$$M_B^{01} + M_B^{10} = \frac{39.65}{285.0} \times 355.2 \times 22.15 = 1\,094.5$$

$$M_C^{01} + M_C^{10} = \frac{59.87}{285.0} \times 355.2 \times 22.15 = 1\,652.6$$

Proportioning the moment sums by those in the third story, which is the nearest story comparable to the basement story,

$$M_A^{01} + M_A^{10} = \frac{362.6}{3\,815.2} \times 7\,867.7 = \quad 747.8$$

$$M_B^{01} + M_B^{10} = \frac{737.0}{3\,815.2} \times 7\,867.7 = 1\,519.8$$

$$M_C^{01} + M_C^{10} = \frac{808.0}{3\,815.2} \times 7\,867.7 = 1\,666.2$$

The actual conditions in the basement story lie somewhere between these two and the results therefore will be averaged.

$$M_A^{01} + M_A^{10} = \quad 967.3 \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

$$M_B^{01} + M_B^{10} = 1\,307.1 \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

$$M_C^{01} + M_C^{10} = 1\,659.4 \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

From the slope deflection equation for the girders of the first floor (Equation (1), $R = 0$ for all girders),

$$M_1^{AB} = 2\,E\,K_{a1}\,(2\,\theta_{A_1} + \theta_{B_1}) = 747\,800 \times 12$$

$$M_1^{BA} = 2\,E\,K_{a1}\,(2\,\theta_{B_1} + \theta_{A_1}) = 581\,600 \times 12$$

* *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station.

TABLE 8.—AMERICAN INSURANCE UNION BUILDING, ROSS METHOD.

Routine Outline.		(1)		(2)			(3)							(4)(5)(6)	(7)
Story.	Floor.	Wind load.	Wind shear.	External moment.	Direct Stress. Column A.	Direct Stress. Column C.	Girder shears.	Column Shears. H_A.	Column Shears. H_B.	Column Shears. H_C.	Column Moments. M_A.	Column Moments. M_B.	Column Moments. M_C.	The design of this building was made according to the Fleming Method No. I. Column and girder sizes given on Fig. 8.	Relative "theoretically proportioned" girder K's, $K_a = 1.288\ K_b$. $K_a = 1.918\ K_b$.
11	..	....	263.9	50 605	700.4	0	.....	26.15	46.49	59.31	135.1	240.2	306.5		
..	11	7.2	.....	.....	.....	....	38.3	.....	.....	.....	.....	.....	.....		
10	..	....	271.1	53 369	738.7	0	.....	26.90	47.76	60.90	139.0	246.8	314.6		
..	10	7.2	.....	.....	.....	....	39.3	.....	.....	.....	.....	.....	.....		
9	..	....	278.3	56 207	778.0	0	.....	27.60	49.02	62.52	142.6	253.3	323.0		
..	9	7.2	.....	.....	.....	....	40.3	.....	.....	.....	.....	.....	.....		
8	..	....	285.5	59 120	818.3	0	.....	28.33	50.31	64.11	146.4	259.9	331.2		
..	8	7.2	.....	.....	.....	....	41.3	.....	.....	.....	.....	.....	.....		
7	..	....	292.7	62 107	859.6	0	.....	29.06	51.58	65.72	150.1	266.5	339.5		
..	7	7.3	.....	.....	.....	....	42.4	.....	.....	.....	.....	.....	.....		
6	..	....	300.0	65 169	902.0	0	.....	29.80	52.85	67.35	154.0	273.1	348.0		
..	6	7.4	.....	.....	.....	....	44.8	.....	.....	.....	.....	.....	.....		
5	..	....	307.4	68 408	946.8	0	.....	30.51	54.16	69.03	167.8	297.9	379.7		
..	5	7.6	.....	.....	.....	....	46.4	.....	.....	.....	.....	.....	.....		
4	..	....	315.0	71 765	993.2	0	.....	31.28	55.49	70.72	165.6	293.8	374.4		
..	4	7.8	.....	.....	.....	....	49.6	.....	.....	.....	.....	.....	.....		
3	..	....	322.8	75 339	1042.8	0	.....	31.95	56.88	72.56	188.8	336.2	428.8		
..	3	9.5	.....	.....	.....	....	61.3	.....	.....	.....	.....	.....	.....		
2	..	....	332.3	79 779	1104.1	0	.....	33.02	58.54	74.60	251.8	446.4	568.8		
..	2	9.8	.....	.....	.....	....	93.33	.....	.....	.....	.....	.....	.....		
M	..	....	342.1	84 528	1197.4	— 93.3	.....	64.35	114.40	—7.70	416.9	741.2	—49.9		
..	*M*	10.8	.....	.....	.....	....	107.96	.....	.....	.....	.....	.....	.....		
1	..	....	352.9	90 365	1305.4	—201.3	.....	38.38	68.10	69.96	356.6	632.6	650.2		
..	1	2.3	.....	.....	.....	....	99.85	.....	.....	.....	.....	.....	.....		
B	..	....	355.2	97 577	1405.2	—201.3	.....	32.40	57.62	87.58	358.8	638.0	969.9		
..	0	....	.....	.....	.....	....	.....	.....	.....	.....	.....	.....	.....		

TABLE 8.—(*Continued*).

Routine Outline.		(8)					(9)							(10)	(11)
Story.	Floor.	Wind load.	Wind shear.	External moment.	Direct Stress. Column A.	Direct Stress. Column C.	Girder shears.	Column Shears. H_A.	Column Shears. H_B.	Column Shears. H_C.	Column Moments. M_A.	Column Moments. M_B.	Column Moments. M_C.	The design of this building was made according to the Fleming Method No. II. Column and girder sizes given on Fig. 8.	Relative "theoretically proportioned" girder K's, $K_a = 1.288\ K_b$. $K_a = 1.918\ K_b$.
11	..	....	263.9	50 605	700.4	0	.....	26.15	46.49	59.31	135.1	240.2	306.5		
..	11	7.2	.....	.....	.....	....	38.3	.....	.....	.....	.....	.....	.....		
10	..	....	271.1	53 369	738.7	0	.....	26.90	47.76	60.90	139.0	246.8	314.6		
..	10	7.2	.....	.....	.....	....	39.3	.....	.....	.....	.....	.....	.....		
9	..	....	278.3	56 207	778.0	0	.....	27.60	49.02	62.52	142.6	253.3	323.0		
..	9	7.2	.....	.....	.....	....	40.3	.....	.....	.....	.....	.....	.....		
8	..	....	285.5	59 120	818.3	0	.....	28.33	50.31	64.11	146.4	259.9	331.2		
..	8	7.2	.....	.....	.....	....	41.3	.....	.....	.....	.....	.....	.....		
7	..	....	292.7	62 107	859.6	0	.....	29.06	51.58	65.72	150.1	266.5	339.5		
..	7	7.3	.....	.....	.....	....	42.4	.....	.....	.....	.....	.....	.....		
6	..	....	300.0	65 169	902.0	0	.....	29.80	52.85	67.35	154.0	273.1	348.0		
..	6	7.4	.....	.....	.....	....	44.8	.....	.....	.....	.....	.....	.....		
5	..	....	307.4	68 408	946.8	0	.....	30.51	54.16	69.03	167.8	297.9	379.7		
..	5	7.6	.....	.....	.....	....	46.4	.....	.....	.....	.....	.....	.....		
4	..	....	315.0	71 765	993.2	0	.....	31.28	55.49	70.72	165.6	293.8	374.4		
..	4	7.8	.....	.....	.....	....	49.6	.....	.....	.....	.....	.....	.....		
3	..	....	322.8	75 339	1042.8	0	.....	31.95	56.88	72.56	188.8	336.2	428.8		
..	3	9.5	.....	.....	.....	....	61.3	.....	.....	.....	.....	.....	.....		
2	..	....	332.3	79 779	1104.1	0	.....	33.02	58.54	74.60	251.8	446.4	568.8		
..	2	9.8	.....	.....	.....	....	93.33	.....	.....	.....	.....	.....	.....		
M	..	....	342.1	84 528	1197.4	— 93.3	.....	64.35	114.40	—7.70	416.9	741.2	—49.9		
..	*M*	10.8	.....	.....	.....	....	107.96	.....	.....	.....	.....	.....	.....		
1	..	....	352.9	90 365	1305.4	—201.3	.....	38.38	68.10	69.96	356.6	632.6	650.2		
..	1	2.3	.....	.....	.....	....	99.85	.....	.....	.....	.....	.....	.....		
B	..	....	355.2	97 577	1405.2	—201.3	.....	32.40	57.62	87.58	358.8	638.0	969.9		
..	0	....	.....	.....	.....	....	.....	.....	.....	.....	.....	.....	.....		

TABLE 8.—(*Continued*).

Routine Outline.								(9)				
									(10)			
Story.	Floor.	Column K's.			Girder K's.			$\Sigma \frac{K_A}{K_a}$	$\Sigma \frac{K_B}{K_a + K_b}$ *	$\Sigma \frac{K_C}{K_b + K_c}$	Actual.	Theoretical.
		K_A.	K_B.	K_C.	K_a.	K_b.	K_c.				Column *A*.	Column *C*.
11	..	29.04	35.01	48.98	.....	.....	.....	.....	.....	.....	....	....
..	11	.....	.....	.....	12.13	15.63	20.36	4.79	2.53	2.72	1.89	1.08
10	..	29.04	35.01	48.98	.....	.....	.....	.....	.....	.....	....	....
..	10	.....	.....	.....	12.13	15.63	20.36	4.79	2.53	2.72	1.89	1.08
9	..	29.04	35.01	48.98	.....	.....	.....	.....	.....	.....	....	....
..	9	.....	.....	.....	28.81	37.12	37.14	2.22	1.14	1.40	1.95	1.23
8	..	35.01	40.41	55.11	.....	.....	.....	.....	.....	.....	....	....
..	8	.....	.....	.....	28.81	37.12	37.14	2.43	1.23	1.48	1.97	1.20
7	..	35.01	40.41	55.11	.....	.....	.....	.....	.....	.....	....	....
..	7	.....	.....	.....	28.81	37.12	37.14	2.43	1.23	1.48	1.97	1.20
6	..	35.01	40.41	55.11	.....	.....	.....	.....	.....	.....	....	....
..	6	.....	.....	.....	28.81	37.12	37.14	2.62	1.31	1.60	2.00	1.22
5	..	40.64	46.07	63.68	.....	.....	.....	.....	.....	.....	....	....
..	5	.....	.....	.....	28.81	37.12	37.14	2.88	1.42	1.75	2.03	1.23
4	..	42.24	47.89	66.19	.....	.....	.....	.....	.....	.....	....	....
..	4	.....	.....	.....	28.81	37.12	37.14	2.78	1.38	1.69	2.02	1.22
3	..	37.84	42.91	59.30	.....	.....	.....	.....	.....	.....	....	....
..	3	.....	.....	.....	135.85	191.88	160.71	0.74	0.37	0.54	2.00	1.46
2	..	62.78	77.67	131.52	.....	.....	.....	.....	.....	.....	....	....
..	2	.....	.....	.....	53.60	69.06	0	2.55	1.38	4.14	1.85	3.00
M	..	78.88	91.40	154.78	.....	.....	.....	.....	.....	.....	....	....
..	*M*	.....	.....	.....	53.60	69.06	0	2.34	1.26	3.80	1.86	3.02
1	..	51.52	63.74	107.93	.....	.....	.....	.....	.....	.....	....	....
..	1	.....	.....	.....	135.85	191.88	293.81	0.70	0.32	0.35	2.19	1.09
B	..	42.98	39.65	59.87	.....	.....	.....	.....	.....	.....	....	....
..	0	.....	.....	.....	.....	.....	.....	.....	.....	.....	....	....

* Column *B* is assumed theoretically proportioned, having the smallest ratio.

TABLE 8.—(*Continued*).

Routine Outline.		(11)							(12)			(13)					
Story.	Floor.	Shear Percentage.		Girder Moments from Percentages.					Moments for Equal θ's.			Resulting Proportionate Moment.					
		a.	*b* and *e*.	M_{AB}.	M_{BA}.	M_{BC}.	M_{CB}.	M_{CC}.	M_a.	M_b.	M_e.	M_{AB}.	M_{BA}.	M_{BC}.	M_{CB}.	M_{CC}.	ΣM.
11	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	11	8.4	1.2	1.112	1.056	1.016	1.008	1.024	1.000	1.288	1.678	1.112	1.056	1.309	1.298	1.718	6.493
10	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	10	8.4	1.2	1.112	1.056	1.016	1.008	1.024	1.000	1.288	1.678	1.112	1.056	1.309	1.298	1.718	6.493
9	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	9	18.0	6.3	1.240	1.120	1.084	1.042	1.126	1.000	1.288	1.289	1.240	1.120	1.396	1.342	1.451	6.549
8	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	8	16.2	5.1	1.216	1.108	1.068	1.034	1.102	1.000	1.288	1.289	1.216	1.108	1.376	1.332	1.420	6.452
7	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	7	16.2	5.1	1.216	1.108	1.068	1.034	1.102	1.000	1.288	1.289	1.216	1.108	1.376	1.332	1.420	6.452
6	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	6	15.9	5.1	1.212	1.106	1.068	1.034	1.102	1.000	1.288	1.289	1.212	1.106	1.376	1.332	1.420	6.446
5	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	5	15.2	5.1	1.203	1.101	1.068	1.034	1.102	1.000	1.288	1.289	1.203	1.101	1.376	1.332	1.420	6.432
4	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	4	15.6	5.1	1.208	1.104	1.068	1.034	1.102	1.000	1.288	1.289	1.208	1.104	1.376	1.332	1.420	6.440
3	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	3	(?) 45.0	(?) 27.0	1.600	1.300	1.360	1.180	1.540	1.000	1.412	1.183	1.600	1.300	1.920	1.666	1.822	8.308
2	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	2	14.1	22.5	1.188	1.094	1.300	1.150	0	1.000	1.288	0	1.188	1.094	1.674	1.481	0	5.437
M	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	*M*	15.0	23.4	1.200	1.100	1.312	1.156	0	1.000	1.288	0	1.200	1.100	1.690	1.489	0	5.479
1	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	1	(?) 60.0	6.9	1.800	1.400	1.092	1.046	1.138	1.000	1.412	2.163	1.800	1.400	1.542	1.477	2.461	8.680
B	..	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
..	0	....	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

TABLE 8.—(*Continued*).

Routine Outline.		(14)		(15)	(16)	(17)				
Story.	Floor.	Shear Percentage.		Moments for Equal θ's.		Resulting Proportionate Moment.				
		a.	*b* and *e*.	M_a.	M_b.	M_{AB}.	M_{BA}.	M_{BC}.	M_{CB}.	M_{CC}.
11	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	11	8.4	1.2	1.000	1.288	1.112	1.056	1.309	1.298	1.718
10	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	10	8.4	1.2	1.000	1.288	1.112	1.056	1.309	1.298	1.718
9	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	9	18.0	6.3	1.000	1.288	1.240	1.120	1.396	1.342	1.451
8	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	8	16.2	5.1	1.000	1.288	1.216	1.108	1.376	1.332	1.420
7	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	7	16.2	5.1	1.000	1.288	1.216	1.108	1.376	1.332	1.420
6	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	6	15.9	5.1	1.000	1.288	1.212	1.106	1.376	1.332	1.420
5	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	5	15.2	5.1	1.000	1.288	1.203	1.101	1.376	1.332	1.420
4	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	4	15.6	5.1	1.000	1.288	1.208	1.104	1.376	1.332	1.420
3	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	3	(?) 45.0	(?) 27.0	1.000	1.412	1.600	1.300	1.920	1.666	1.822
2	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	2	14.1	22.5	1.000	1.288	1.188	1.094	1.674	1.481	0
M	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	*M*	15.0	23.4	1.000	1.288	1.200	1.100	1.690	1.489	0
1	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	1	(?) 60.0	6.9	1.000	1.412	1.800	1.400	1.542	1.477	2.461
B	..	....	....	.....	.....	.....	.....	.....	.....	.....
..	0	....	....	.....	.....	.....	.....	.....	.....	.....

TABLE 8.—(*Continued*).

Routine Outline.		(14)						(15)			(16)			(17)		
		Actual Girder Moments						Column Moments.			Girder Shears.			Column Direct.		
Story.	Floor.	$\Sigma M.$	$M_{AB}.$	$M_{BA}.$	$M_{BC}.$	$M_{CB}.$	$M_{CC}.$	*A.*	*B.*	*C.*	*a.*	*b.*	*c.*	*A.*	*B.*	*C.*
11	..	.....	.....	.....	.....	.....	.....	116.8	248.4	316.7	....	....	....	653.3	317.3	−445.5
..	11	1382.1	236.7	224.8	278.7	276.3	365.7	.....	.....	.....	32.2	49.9	34.3	.....	.....	.....
10	..	.....	.....	.....	.....	.....	.....	{ 119.9	255.1	325.3 }	....	....	....	685.5	335.0	−461.1
..	10	1419.3	243.1	230.8	286.2	283.7	375.5	{ 119.9	255.1	325.3 }	33.1	51.2	35.2	.....	.....	.....
9	..	.....	.....	.....	.....	.....	.....	.....	.....	.....	....	....	....	718.6	353.1	−477.1
..	9	1456.5	275.8	249.1	310.5	298.5	322.7	{ 123.2	261.9	333.9 }	36.6	54.8	30.3	.....	.....	.....
8	..	.....	.....	.....	.....	.....	.....	{ 136.1	276.1	306.5 }	....	....	....	755.2	371.3	−501.6
..	8	1493.7	281.6	256.5	318.6	308.4	328.8	{ 139.7	283.5	314.7 }	37.6	56.4	30.8	.....	.....	.....
7	..	.....	.....	.....	.....	.....	.....	{ 139.1	284.0	314.7 }	....	....	....	792.8	390.1	−527.2
..	7	1531.1	288.6	263.0	326.6	316.1	337.0	.....	.....	.....	38.5	57.8	31.6	.....	.....	.....
6	..	.....	.....	.....	.....	.....	.....	{ 142.5	291.1	322.5 }	....	....	....	831.3	409.4	−553.4
..	6	1620.3	304.7	278.0	345.9	334.8	357.0	{ 142.5	291.1	322.5 }	40.7	61.2	33.5	.....	.....	.....
5	..	.....	.....	.....	.....	.....	.....	.....	.....	.....	....	....	....	872.0	429.9	−581.1
..	5	1678.5	314.0	287.3	359.1	347.6	370.6	{ 146.7	298.5	330.6 }	42.0	63.6	34.7	.....	.....	.....
4	..	.....	.....	.....	.....	.....	.....	{ 145.7	298.4	330.9 }	....	....	....	914.0	451.5	−610.0
..	4	1787.0	335.2	306.4	381.8	369.6	394.0	.....	.....	.....	44.8	67.6	36.9	.....	.....	.....
3	..	.....	.....	.....	.....	.....	.....	{ 159.0	325.5	360.9 }	....	....	....	958.8	474.3	−640.7
..	3	2220.8	427.7	347.5	513.2	445.3	487.0	{ 158.1	325.5	361.7 }	54.1	86.2	45.7	.....	.....	.....
2	..	.....	.....	.....	.....	.....	.....	.....	.....	.....	....	....	....	1012.9	506.4	−681.2
..	2	2375.3	519.0	478.0	731.4	647.0	0	{ 155.9	320.9	356.5 }	69.6	124.0	0	.....	.....	.....
M	..	.....	.....	.....	.....	.....	.....	{ 156.3	320.9	356.1 }	....	....	....	1082.5	560.8	−805.2
..	*M*	2747.6	601.8	551.6	847.5	746.8	0	.....	.....	.....	80.5	143.4	0	.....	.....	.....
1	..	.....	.....	.....	.....	.....	.....	{ 178.9	367.3	407.5 }	....	....	....	1163.0	623.7	−948.6
..	1	3606.1	747.8	581.6	640.6	613.6	1022.3	{ 183.7	369.7	400.5 }	92.6	112.8	95.9	.....	.....	.....
B	..	.....	.....	.....	.....	.....	.....	.....	.....	.....	....	....	...	1255.6	643.9	−965.5
..	0	.....	.....	.....	.....	.....	.....	{ 244.0	491.0	531.8 }	....	....	....	.....	.....	.....
								{ 276.8	645.0	345.0 }						
								.....	.....	.....						
								{ 242.2	564.4	302.0 }						
								{ 212.8	564.4	301.3 }						
								.....	.....	.....						
								{ 359.0	834.7	445.5 }						
								{ 408.0	606.7	892.4 }						
								.....	.....	.....						
								{ 435.4	633.3	795.0† }						
								{ 531.9	673.7	864.4 }						
								.....	.....	.....						

† See text for basement-story columns, p. 1419.

TABLE 9.—SOLUTION OF ILLUSTRATIVE PROBLEM USED BY WILSON AND MANEY.

Story.	Floor.	Wind load.	Wind shear.	Column K's.		Girder K's.		$\Sigma \frac{K_A}{K_a}$.	$\Sigma \frac{K_B^*}{K_a + K_b}$.	Column $\frac{A \text{ Actual}}{\text{Theoretical}}$.	Percentage increase in shear, Girder a.
				A.	B.	a.	b.				
..	20	360	.....	....	....	7.7	9.4	1.7	0.77	2.21	30
20	..	....	360	13.1	13.2	....	....	....	....	....	....
..	19	360	.....	....	....	7.7	9.4	3.4	1.54	2.21	15.6
19	..	....	720	13.1	13.2	....	....	....	....	....	....
..	18	360	.....	....	....	7.7	9.4	3.4	1.54	2.21	15.6
18	..	....	1 080	13.1	13.2	....	....	....	....	....	....
..	17	360	.....	....	....	7.7	9.4	3.4	1.54	2.21	15.6
17	..	....	1 440	13.1	13.2	....	....	....	....	....	....
..	16	360	.....	....	....	7.7	9.4	3.56	1.62	2.20	14.7
16	..	....	1 800	14.3	14.6	....	....	....	....	....	....
..	15	360	.....	....	....	7.7	9.4	3.72	1.71	2.18	13.8
15	..	....	2 160	14.3	14.6	....	....	....	....	....	....
..	14	360	.....	....	....	7.7	9.4	4.23	1.94	2.18	12.3
14	..	....	2 520	18.3	18.6	....	....	....	....	....	....
..	13	360	.....	....	....	7.7	9.4	4.75	2.18	2.18	11.7
13	..	....	2 880	18.3	18.6	....	....	....	....	....	....
..	12	360	.....	....	....	7.7	9.4	4.82	2.26	2.13	10.5
12	..	....	3 240	18.8	19.9	....	....	....	....	....	....
..	11	360	.....	....	....	7.7	9.4	4.88	2.33	2.10	10.2
11	..	....	3 600	18.8	19.9	....	....	....	....	....	....
..	10	360	.....	....	....	7.7	9.4	5.18	2.69	1.93	8.1
10	..	....	3 960	21.1	26.1	....	....	....	....	....	....
..	9	360	.....	....	....	7.7	9.4	5.48	3.05	1.80	6.6
9	..	....	4 320	21.1	26.1	....	....	....	....	....	....
..	8	360	.....	....	....	7.7	9.4	6.47	3.28	1.97	7.2
8	..	....	4 680	28.7	30.0	....	....	....	....	....	....
..	7	360	.....	....	....	12.8	15.7	4.48	2.11	2.12	11.4
7	..	....	5 040	28.7	30.0	....	....	....	....	....	....
..	6	390	.....	....	....	14.1	17.2	4.12	1.93	2.14	12.3
6	..	....	5 430	29.4	30.4	....	....	....	....	....	....
..	5	420	.....	....	....	19.5	23.8	3.02	1.41	2.14	16.2
5	..	....	5 850	29.4	30.4	....	....	....	....	....	....
..	4	420	.....	....	....	19.5	26.2	3.32	1.44	2.30	17.1
4	..	....	6 270	35.4	35.5	....	....	....	....	....	....
..	3	420	.....	....	....	21.4	26.2	3.31	1.49	2.22	15.9
3	..	....	6 690	35.4	35.5	....	....	....	....	....	....
..	2	450	.....	....	....	21.4	29.2	3.32	1.41	2.36	18.0
2	..	....	7 140	35.6	35.6	....	....	....	....	....	....
..	1	570	.....	....	....	30.5	37.3	2.01	0.90	2.23	25.8
1	..	....	7 710	25.8	25.8	....	....	....	....	....	....
..	0	....	.....	....	....	....	....	....	....	....	....

* Theoretically proportioned.

TABLE 9.—(*Continued*).

Story.	Floor.	Wind load.	Wind shear.	Girder Moments from Percentages.			Girder Moments, θ.		Proportionate Girder Moments.			
				M_{AB}.	M_{BA}.	M_{BB}.	M_a.	M_b.	M_{AB}	M_{BA}.	M_{BB}.	ΣM.
..	20	360	.....	1.40	1.200	1.00	1.00	1.220	1.400	1.200	1.220	3.820
20	..	....	360	.....	.....	....	....	.....	.....	.....	.....	.....
..	19	360	.....	1.208	1.104	1.00	1.00	1.220	1.208	1.104	1.220	3.532
19	..	....	720	.....	.....	....	....	.....	.....	.....	.....	.....
..	18	360	.....	1.208	1.104	1.00	1.00	1.220	1.208	1.104	1.220	3.532
18	..	....	1 080	.....	.....	....	....	.....	.....	.....	.....	.....
..	17	360	.....	1.208	1.104	1.00	1.00	1.220	1.208	1.104	1.220	3.532
17	..	....	1 440	.....	.....	....	....	.....	.....	.....	.....	.....
..	16	360	.....	1.196	1.098	1.00	1.00	1.220	1.196	1.098	1.220	3.514
16	..	....	1 800	.....	.....	....	....	.....	.....	.....	.....	.....
..	15	360	.....	1.184	1.092	1.00	1.00	1.220	1.184	1.092	1.220	3.496
15	..	....	2 160	.....	.....	....	....	.....	.....	.....	.....	.....
..	14	360	.....	1.164	1.082	1.00	1.00	1.220	1.164	1.082	1.220	3.466
14	..	....	2 520	.....	.....	....	....	.....	.....	.....	.....	.....
..	13	360	.....	1.156	1.078	1.00	1.00	1.220	1.156	1.078	1.220	3.454
13	..	....	2 880	.....	.....	....	....	.....	.....	.....	.....	.....
..	12	360	.....	1.140	1.070	1.00	1.00	1.220	1.140	1.070	1.220	3.430
12	..	....	3 240	.....	.....	....	....	.....	.....	.....	.....	.....
..	11	360	.....	1.136	1.068	1.00	1.00	1.220	1.136	1.068	1.220	3.424
11	..	....	3 600	.....	.....	....	....	.....	.....	.....	.....	.....
..	10	360	.....	1.108	1.054	1.00	1.00	1.220	1.108	1.054	1.220	3.382
10	..	....	3 960	.....	.....	....	....	.....	.....	.....	.....	.....
..	9	360	.....	1.088	1.044	1.00	1.00	1.220	1.088	1.044	1.220	3.352
9	..	....	4 320	.....	.....	....	....	.....	.....	.....	.....	.....
..	8	360	.....	1.096	1.048	1.00	1.00	1.220	1.096	1.048	1.220	3.364
8	..	....	4 680	.....	.....	....	....	.....	.....	.....	.....	.....
..	7	360	.....	1.152	1.076	1.00	1.00	1.220	1.152	1.076	1.220	3.448
7	..	....	5 040	.....	.....	....	....	.....	.....	.....	.....	.....
..	6	390	.....	1.164	1.082	1.00	1.00	1.220	1.164	1.082	1.220	3.466
6	..	....	5 430	.....	.....	....	....	.....	.....	.....	.....	.....
..	5	420	.....	1.216	1.108	1.00	1.00	1.22	1.216	1.108	1.220	3.544
5	..	....	5 850	.....	.....	....	....	.....	.....	.....	.....	.....
..	4	420	.....	1.228	1.114	1.00	1.00	1.34	1.228	1.114	1.343	3.685
4	..	....	6 270	.....	.....	....	....	.....	.....	.....	.....	.....
..	3	420	.....	1.212	1.106	1.00	1.00	1.22	1.212	1.106	1.220	3.538
3	..	....	6 690	.....	.....	....	....	.....	.....	.....	.....	.....
..	2	450	.....	1.240	1.120	1.00	1.00	1.36	1.240	1.120	1.364	3.724
2	..	....	7 140	.....	.....	....	....	.....	.....	.....	.....	.....
..	1	570	.....	1.344	1.172	1.00	1.00	1.22	1.344	1.172	1.220	3.736
1	..	....	7 710	.....	.....	....	....	.....	.....	.....	.....	.....
..	0	....	.....	.....	.....	....	....	.....	.....	.....	.....	.....

TABLE 9.—(*Continued*).

Story.	Floor.	Wind load.	Wind Shear.	Actual Girder Moments.				Column *M*.		Girder Shears.		Column Direct.	
				Σ *M*.	M_{AB}.	M_{BA}.	M_{BB}.	M_A.	M_B.	*a*.	*b*.	*A*.	*B*.
..	20	360	.....	1 080	0.40	0.34	0.34	.....	.....	34	38	.....	.....
20	..	....	360	.....	.....	.....	.....	0.40 0.37	0.68 0.71	.....	.....	34	4
..	19	360	.....	3 240	1.11	1.01	1.12	.....	.....	96	125	.....	.....
19	..	....	720	.....	.....	.....	.....	0.74 0.74	1.42 1.42	.....	.....	130	33
..	18	360	.....	5 400	1.85	1.69	1.86	.....	.....	161	207	.....	.....
18	..	....	1 080	.....	.....	.....	.....	1.11 1.11	2.13 2.13	.....	.....	291	79
..	17	360	.....	7 560	2.59	2.36	2.61	.....	.....	225	290	.....	.....
17	..	....	1 440	.....	.....	.....	.....	1.48 1.47	2.84 2.85	.....	.....	516	144
..	16	360	.....	9 720	3.31	3.04	3.37	.....	.....	289	374	.....	.....
16	..	....	1 800	.....	.....	.....	.....	1.84 1.83	3.56 3.58	.....	.....	805	229
..	15	360	.....	11 880	4.02	3.71	4.15	.....	.....	351	461	.....	.....
15	..	....	2 160	.....	.....	.....	.....	2.19 2.18	4.28 4.30	.....	.....	1 156	339
..	14	360	.....	14 040	4.72	4.38	4.94	.....	.....	414	549	.....	.....
14	..	....	2 520	.....	.....	.....	.....	2.54 2.52	5.02 5.04	.....	.....	1 570	474
..	13	360	.....	16 200	5.42	5.06	5.72	.....	.....	476	635	.....	.....
13	..	....	2 880	.....	.....	.....	.....	2.90 2.87	5.74 5.77	.....	.....	2 046	633
..	12	360	.....	18 360	6.10	5.73	6.53	.....	.....	538	725	.....	.....
12	..	....	3 240	.....	.....	.....	.....	3.23 3.23	6.49 6.50	.....	.....	2 584	820
..	11	360	.....	20 520	6.81	6.40	7.31	.....	.....	600	812	.....	.....
11	..	....	3 600	.....	.....	.....	.....	3.58 3.55	7.21 7.27	.....	.....	3 184	1 032
..	10	360	.....	22 680	7.43	7.07	8.18	.....	.....	659	909	.....	.....
10	..	....	3 960	.....	.....	.....	.....	3.88 3.86	7.98 8.01	.....	.....	3 843	1 282
..	9	360	.....	24 840	8.06	7.74	9.04	.....	.....	718	1 004	.....	.....
9	..	....	4 320	.....	.....	.....	.....	4.20 4.22	8.77 8.73	.....	.....	4 561	1 568
..	8	360	.....	27 000	8.80	8.41	9.79	.....	.....	782	1 088	.....	.....
8	..	....	4 680	.....	.....	.....	.....	4.58 4.69	9.47 9.35	.....	.....	5 343	1 874
..	7	360	.....	29 160	9.74	9.10	10.32	.....	.....	856	1 147	.....	.....
7	..	....	5 040	.....	.....	.....	.....	5.05 5.07	10.07 10.05	.....	.....	6 199	2 165
..	6	390	.....	34 125	11.46	10.65	12.01	.....	.....	1 005	1 334	.....	.....
6	..	....	5 430	.....	.....	.....	.....	6.39 6.53	12.61 12.49	.....	.....	7 204	2 494
..	5	420	.....	39 480	13.55	12.34	13.59	.....	.....	1 177	1 510	.....	.....
5	..	....	5 850	.....	.....	.....	.....	7.02 6.82	13.44 13.63	.....	.....	8 381	2 847
..	4	420	.....	42 420	14.14	12.82	15.46	.....	.....	1 225	1 716	.....	.....
4	..	....	6 270	.....	.....	.....	.....	7.32 7.52	14.65 14.43	.....	.....	9 606	3 318
..	3	420	.....	45 360	15.54	14.18	15.64	.....	.....	1 351	1 736	.....	.....
3	..	....	6 690	.....	.....	.....	.....	8.02 7.80	15.39 15.62	.....	.....	10 957	3 703
..	2	450	.....	51 975	17.30	15.63	19.04	.....	.....	1 497	2 115	.....	.....
2	..	....	7 140	.....	.....	.....	.....	9.50 10.28	19.05 18.30	.....	.....	12 454	4 321
..	1	570	.....	70 965	25.53	22.26	23.18	.....	.....	2 172	2 575	.....	.....
1	..	....	7 710	.....	.....	.....	.....	13.88 22.00	21.79 27.14	.....	.....	14 626	4 724
..	0	....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

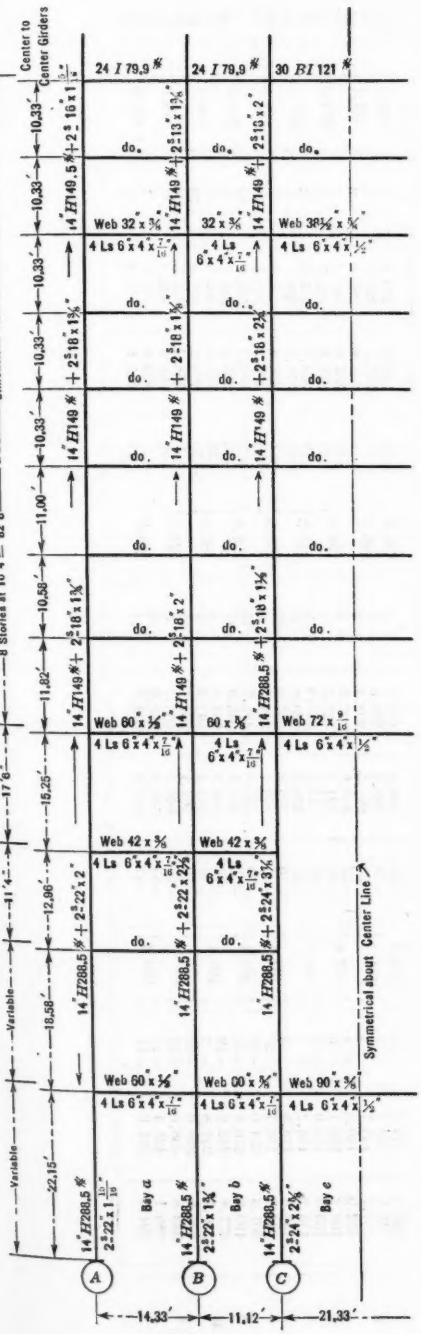
Center to Center Girders
24 I 79,9 #
24 I 79,9 #
30 BI 121 #
do.
Web 32" x 3/8"
32" x 3/8"
Web 38 3/8" x 5/8"
4 Ls 6" x 4" x 7/16"
Web 60" x 1/2"
60" x 5/8"
Web 72" x 9/16"
Web 42" x 3/8"
Web 90" x 5/8"
Dimensions Floor to Floor
8 Stories at 10'4" = 82'8"
Variable
Variable
Symmetrical about Center Line
Bay a
Bay b
Bay c
A
B
C
14,33'
11,12'
21,33'

FIG. 8.

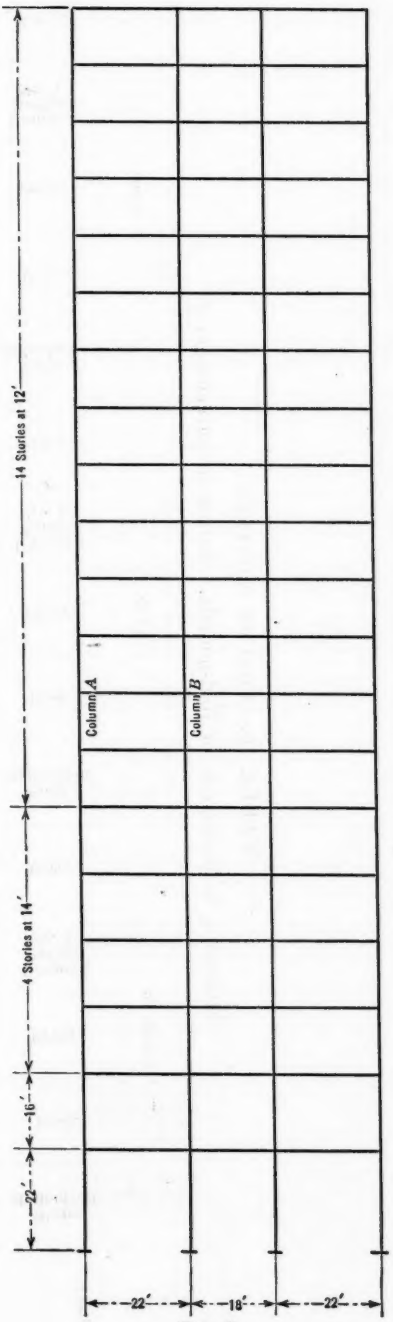
14 Stories at 12'
4 Stories at 14'
Column A
Column B
22'
16'
22'
22'
18'
22'

FIG. 9.

TABLE 10.—COLUMN MOMENTS.

(Moments, in thousands of foot-pounds. Errors in percentages.)

Story.	Column A.				Column B.				Column C.			
	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.
7	145.1	142.5	— 2	104.1	288.4	291.1	+ 1	238.9	334.2	322.5	— 4	422.0	27
6	147.0	146.1	— 1	106.4	288.6	298.5	+ 3	239.7	335.7	330.6	— 2	433.1	29
5	151.7	145.7	— 4	115.6	291.5	298.4	+ 2	250.4	340.8	330.9	— 3	470.0	27
4	152.9	159.0	+ 4	118 8	316.8	325.5	+ 2	255.5	366.2	360.9	— 1	463.1	28
3	163.2	158.1	— 3	132.1	324.4	325.5	0	296.5	376.6	361.7	— 4	587.5	25
2	153.3	155.9	+ 2	200.4	314.8	320.9	+ 2	398.4	364.0	356.5	— 2	713.2	27
M	155.2	156.3	+ 1	490.4	319.3	320.9	0	789.5	371.1	356.1	— 4	196.1	25
1	156.4	178.9	+14	249.2	300.1	367.8	+22	524.6	350.4	407.5	+16	770.5	53
0	244.6	183.7	—25	132.1	392.1	369.7	— 6	296.5	475.3	400.5	—16	587.5	13
	301.3	244.0	—19	200.4	555.8	491.0	—13	398.4	688.4	531.8	—23	713.2	4
	199.3	276.8	+39	490.4	488.5	645.0	+47	789.5	355.9	345.0	— 3	196.1	+100
	277.8	242.2	—18	249.2	584.8	564.4	— 4	524.6	325.5	302.0	— 7	770.5	— 40
	250.7	242.8	— 3	132.1	567.4	564.4	— 1	296.5	224.2	301.3	+34	587.5	— 13
	276.6	359.0	+30	200.4	565.4	594.7	+48	398.4	558.4	445.5	—20	713.2	— 36
	333.8	408.0	+22	490.4	668.0	666.7	0	789.5	898.6	892.4	— 1	196.1	+ 14
	486.1	435.4	—10	249.2	567.6	638.8	+12	524.6	832.8	795.0	— 5	770.5	— 37
	594.5	531.9	—11	220.2	607.8	678.7	+11	529.8	905.8	864.4	— 5	1 199.8	— 26

TABLE 11.—COLUMN DIRECT STRESSES.

(Stresses in thousands of pounds. Errors in percentages.)

Story.	Column A.				Column B.				Column C.			
	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Slope deflection	Ross.	Error.	Fleming Method No. I.	Error.
7	792.8	792.8	0	601.7	390.1	390.1	0	362.8	—527.2	—527.2	0	177.5	—66
6	881.5	881.3	0	631.1	409.2	409.4	0	380.6	—558.0	—553.4	0	186.2	—66
5	872.1	872.0	0	662.0	429.7	429.9	0	399.2	—580.3	—581.1	0	195.3	—66
4	914.4	914.0	0	694.0	451.6	451.5	0	418.5	—600.1	—610.0	0	204.7	—66
3	955.8	958.8	0	728.3	472.4	474.3	0	439.2	—636.9	—640.7	+1	214.8	—66
2	1 024.0	1 012.9	—1	771.2	505.3	506.4	0	465.0	—685.1	—681.2	—1	227.5	—67
M	1 067.5	1 062.5	—1	864.1	556.0	560.8	+1	465.0	—799.4	—805.2	+1	184.6	—83
1	1 157.3	1 168.0	+1	967.3	615.9	623.7	+1	465.0	—929.0	—948.6	+2	31.4	—97
0	1 257.4	1 255.6	—1	1 032.8	630.8	648.9	+2	504.5	—943.7	—965.5	+2	50.7	—94

TABLE 12.—Girder Moments.

(Moments in thousands of foot-pounds. Errors in percentages.)

Floor.	Girder *a*.					Girder *b*.					Girder *e*.				
	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.
7	292.1	288.6	— 1	210.7	—28	314.8	326.6	+ 4	262.8	—17	341.5	337.0	— 1	596.2	+75
	262.3	263.0	0		—20	328.0	316.1	— 4		—20					
6	304.6	304.7	0	221.4	—27	333.4	345.9	+ 4	275.6	—17	360.5	357.0	— 1	625.1	+73
	277.0	278.0	0		—20	346.7	334.8	— 3		—21					
5	316.6	314.0	— 1	229.3	—28	350.2	359.1	+ 3	285.6	—18	377.0	370.6	— 2	647.5	+72
	289.2	287.3	— 1		—21	363.4	347.6	— 4		—21					
4	311.6	335.2	+ 8	245.8	—21	338.1	381.8	+13	306.2	— 9	367.2	394.0	+ 7	694.4	+89
	281.5	306.4	+ 9		—13	354.0	369.6	+ 4		—14					
3	545.9	427.7	—22	307.4	—44	525.3	513.2	— 2	352.4	—27	564.3	487.0	—14	868.2	+54
	432.4	347.5	—20		—29	599.5	445.3	—26		—36					
2	477.2	519.0	+ 9	665.8	+40	589.9	731.4	+24	517.2	—12	.....	.....	....	.....	....
	433.4	478.0	+10		+54	681.3	647.0	— 5		—24					
M	527.2	601.8	+14	739.6	+40	659.9	847.5	+28	574.5	—13	.....	.....	....	.....	....
	472.6	551.6	+17		+57	782.2	746.8	— 4		—27					
1	819.8	747.8	— 9	469.4	— 43	620.8	640.6	+ 3	584.5	— 6	1 070.4	1 022 3	— 5	1 325.8	+24
	615.1	581.6	— 5		—24	659.6	613.6	— 7		—11					

TABLE 13.—Girder Shears.

(Shears in thousands of pounds. Errors in percentages.)

Floor.	Girder *a*.					Girder *b*.					Girder *e*.				
	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.
7	38.7	38.5	0	29.4	—24	57.8	57.8	0	47.2	—18	32.0	31.6	— 2	55.9	+75
6	40.6	40.7	0	30.9	—24	61.1	61.2	0	49.5	—19	33.8	33.5	— 1	58.6	+73
5	42.3	42.0	— 1	32.0	—24	64.2	63.6	— 1	51.3	—20	35.3	34.7	— 2	60.7	+72
4	41.4	44.8	+ 1	34.3	—17	62.2	67.6	+ 9	55.0	—12	34.4	36.9	+ 7	65.1	+89
3	68.3	54.1	—21	42.9	—37	101.1	86.2	—15	68.7	—32	52.9	45.7	—14	81.4	+54
2	63.5	69.6	+10	92.9	+46	114.3	124.0	+ 8	92.9	—19	.....	....	....	.....	....
M	69.8	80.5	+15	103.2	+48	129.6	143.4	+11	103.2	—20	.....	....	....	.....	....
1	100.1	92.6	— 7	65.5	—35	115.0	112.8	— 2	105.0	— 9	100.4	95.9	— 4	124.3	+24

TABLE 14.—Column Moments.

Story	Column A.					Column B.				
	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.
12	3.45	3.23	− 6	3.18	− 8	6.79	6.49	− 4	6.59	− 3
	2.84	3.23	+14		+12	6.26	6.50	+ 4		+ 5
11	3.96	3.58	−10	3.49	−12	7.42	7.21	− 3	7.34	− 1
	3.46	3.55	+ 3		+ 1	7.60	7.27	− 4		− 3
10	3.87	3.88	0	3.83	− 1	8.50	7.98	− 6	8.07	− 5
	3.53	3.86	+ 9		+ 9	7.85	8.01	+ 2		+ 3
9	4.38	4.20	− 4	4.28	− 2	8.68	8.77	+ 1	8.71	0
	4.32	4.22	− 2		− 1	8.61	8.73	+ 1		+ 1
8	3.70	4.58	+24	4.66	+26	8.10	9.47	+17	9.42	+16
	5.64	4.69	−17		−17	10.62	9.35	−12		−11
7	4.80	5.05	+ 5	4.93	+ 3	10.17	10.07	− 1	10.17	0
	5.00	5.07	+ 1		− 1	10.33	10.05	− 3		− 2
6	6.25	6.39	+ 2	6.19	− 1	11.94	12.61	+ 6	12.75	+ 7
	6.93	6.53	− 6		−11	12.97	12.49	− 4		− 2
5	7.04	7.02	0	6.74	− 4	13.52	13.44	− 1	13.59	+ 1
	6.90	6.82	− 1		− 2	13.50	13.63	+ 1		+ 1
4	7.32	7.32	0	7.26	− 1	14.67	14.65	0	14.61	− 0
	7.32	7.52	+ 3		− 1	14.46	14.43	0		+ 1
3	8.36	8.02	− 4	7.62	− 9	15.62	15.39	− 1	15.63	0
	7.56	7.80	+ 3		+ 1	15.17	15.62	+ 3		+ 3
2	9.42	9.50	+ 1	9.25	− 2	18.87	19.05	+ 1	19.05	+ 1
	8.96	10.28	+15		+ 3	19.92	18.30	− 8		− 4
1	14.90	13.88	− 6	13.91	− 7	20.92	21.79	+ 4	28.60	+37
	22.67	22.00	− 3		−89	25.68	27.14	+ 6		+11

TABLE 15.—Column Direct Stresses.

Story.	Column A.					Column B.				
	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.
11	3.20	3.18	....	3.21	....	1.04	1.03	....	0.93	....
10	3.86	3.84	....	3.89	....	1.29	1.28	....	1.13	....
9	4.56	4.56	....	4.63	....	1.57	1.57	....	1.34	....
8	5.27	5.34	....	5.43	....	1.86	1.87	....	1.58	....
7	6.19	6.20	....	6.30	....	2.16	2.16	....	1.83	....
6	7.18	7.20	....	7.31	....	2.49	2.49	....	2.09	....
5	8.39	8.38	....	8.49	....	2.81	2.83	....	2.46	....
4	9.62	9.61	....	8.75	....	3.29	3.32	....	2.54	....
3	10.98	10.96	....	11.10	....	3.69	3.70	....	3.22	....
2	12.44	12.45	....	12.64	....	4.32	4.32	....	3.67	....
1	14.46	14.63	....	14.76	....	4.59	4.72	....	4.28	....

TABLE 16.—GIRDER MOMENTS.

Floor.	Moment M_{AB}.				Moment M_{BA}.				Moment M_{BB}.			
	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.	Slope deflection.	Ross.	Error.	Fleming Method No. I.	Error.
12	6.12	6.10	0	6.02	5.72	5.73	0	6.02	+5	6.52	6.53	0	6.32	−3
11	6.82	6.81	0	6.69	6.38	6.40	0	6.69	+5	7.29	7.31	0	7.16	−2
10	7.47	7.43	−1	7.26	7.08	7.07	0	7.26	+3	8.21	8.18	0	8.12	−1
9	7.92	8.06	+2	8.07	7.59	7.74	+2	8.07	+6	8.90	9.04	+2	8.69	−2
8	7.96	8.80	+10	9.06	7.67	8.41	+10	9.06	+17	8.98	9.79	+9	8.85	−11
7	10.49	9.74	−7	9.54	9.75	9.10	−7	9.54	−2	11.04	10.82	−7	10.13	−8
6	11.25	11.46	+2	11.10	10.47	10.65	+2	11.10	+6	11.79	12.01	+2	11.73	0
5	13.97	13.55	−3	12.90	12.67	12.84	+3	12.90	+2	13.83	13.59	−2	13.70	−1
4	14.21	14.14	0	14.10	12.80	12.82	0	14.10	+10	15.37	15.46	+1	14.31	−7
3	15.59	15.54	0	14.85	14.31	14.18	−1	14.85	+4	15.80	15.64	+1	15.66	−1
2	16.92	17.30	+2	16.65	15.33	15.63	+2	16.65	+9	18.83	19.04	+1	17.60	−7
1	23.98	25.33	+6	23.27	20.42	23.26	+9	23.27	+14	20.62	23.18	+12	24.58	+19

from which,

$$\theta_{A_1} = \frac{13\ 458}{E} \text{ and } \theta_{B_1} = \frac{6\ 116}{E}$$

Similarly, from Girder $B\,C$,

$$\theta_{B_1} = \frac{6\ 114}{E} \text{ and } \theta_{C_1} = \frac{6.958}{E}$$

The difference in the moments at the top and bottom of the columns may now be obtained from Equation (20),

$$M_A^{01} - M_A^{10} = 2\,E\,K_{A_0}\,\theta_{A_1} = 1\ 156\ 800 \text{ in-lb.} = 96\ 400 \text{ ft-lb} \ldots . (26)$$

$$M_B^{01} - M_B^{10} = 2\,E\,K_{B_0}\,\theta_{B_1} = 484\ 900 \text{ in-lb.} = 40\ 400 \text{ ft-lb} \ldots . (27)$$

$$M_C^{01} - M_C^{10} = 2\,E\,K_{C_0}\,\theta_{C_1} = 833\ 200 \text{ in-lb.} = 69\ 400 \text{ ft-lb} \ldots . (28)$$

Solving Equations (23) and (26),

$$M_A^{01} = 531.9;\ M_A^{10} = 435.4$$

Solving Equations (24) and (27),

$$M_B^{01} = 673.7;\ M_B^{10} = 633.3$$

Solving Equations (25) and (28),

$$M_C^{01} = 864.4;\ M_C^{10} = 795.0$$

Conclusions

The American Insurance Union Building was chosen as an illustrative problem because of the wide variation of the bents from "theoretical proportions." It would be expected, therefore, that the stresses obtained by the new method would not check closely with the slope deflection results. Tables 9, 10, 11, and 12, give a comparison of the results and also the results of the solution by Fleming's Method No. I. It is seen that for the stories above the fourth floor where the arrangement of the members is fairly regular (see Fig. 3(*a*)) the errors of the new method are very small, and that those for the lower stories are not excessive when compared to the errors resulting from the use of the old approximate solutions.

Tables 13, 14, 15, and 16 give a similar comparison of results for the lower twelve stories of the 20-story bent of Wilson and Maney. Here, likewise, the errors of the new method are seen to be comparatively small. The proportions of this bent happen to be such that the Fleming Method No. I approaches the true stresses more closely than it does in bents of other proportions. This may be seen by referring to the comparisons made by Wilson and Maney for their Fig. 13,* in which the errors of the Fleming Method No. I run as high as 70%; and referring also to the results for the American Insurance Union Building in Table 8.

The new method of calculation requires little more time than is required for any of the approximate methods in common use. The entire set of calculations for the Wilson and Maney 20-story bent given in Table 9 were made in 6 hours, including the time required to put them on tracing cloth. The tabular form of calculation, given in Tables 8 and 9, greatly facilitates the work.

* *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station.

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A PROPOSED FORMULA FOR COLUMNS

By T. F. Hickerson,* M. Am. Soc. C. E.

Synopsis

The function of columns used so extensively in bridges, buildings, and other engineering structures is to resist compressive loads with safety. The internal compressive stresses caused by the direct loads are quite determinate, but the additional stresses resulting from the possibility of incipient bending may reach the yield point of the material and produce a sudden collapse of the column. This dangerous bending tendency, which is characteristic of slender columns, is of an indeterminate nature, since the actual final deflection resulting from bending is unknown.

If the column is perfectly straight, the material absolutely homogeneous, and the load is applied exactly along the central axis, then, from the standpoint of pure flexure alone, it may be said that no bending at all will occur; but no such perfectly ideal conditions occur in practice. The column is likely to have a slight initial bend; there may be non-uniform initial stresses set up in the material due to riveting the parts together; the material may be non-homogeneous; the modulus of elasticity may not be constant; and the load may be applied slightly off center.

The effect of all these imperfections may be accounted for by an assumed initial curvature of the column axis and eccentricity of loading, which, if applied to the ideal column, will cause it to fail at the same load as the imperfect column of practice.

Let Fig. 1 represent a round-ended (or pivoted) column subjected to a load, P, which is applied e in. off center at the end. The column also has a slight initial bend. Before loading, its axis (Fig. 1(b)) is assumed to have the shape, $A\,C\,B$; after loading, it will take the shape, $A\,O\,B$. It will be

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assumed that the curve, $A\ C\ B$, is a smooth plane curve and an arc of a parabola, its exact shape not being of great importance. The curve, $A\ O\ B$, is the final shape of the line of resistance. This will first be considered as a parabola, and then corrected so as to coincide with a sine curve.

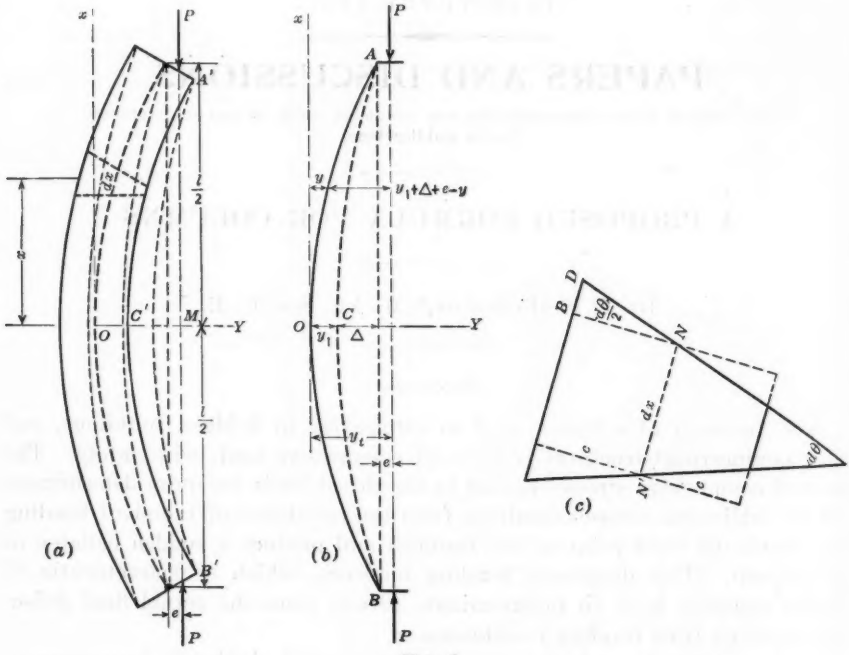

FIG. 1.

NOTATION

Let P = column load.

l = unsupported length.

r = radius of gyration about the axis of bending.

$\frac{l}{r}$ = slenderness ratio.

I = moment of inertia about the axis of bending.

A = cross-section.

e = accidental end eccentricity of loading.

Δ = initial deflection of the mid-section due to the initial bent shape of the column.

y_1 = additional deflection of the mid-section due to the load.

y_t = total maximum deflection, measured from the load line.

y_0 = total maximum deflection, measured from the load line (Salmon's formula).

c = distance from neutral axis of cross-section to outside fibers (Fig. 1(c)).

f = total maximum unit stress at the concave side of column.
f_b = maximum unit stress on the concave side due to bending only.
$\frac{P}{A}$ = average unit stress at the mid-section of column.
M = bending moment.

Using rectangular axes, the origin will be assumed at O (Fig. 1), the center of the bent column. The ordinate of Point A with reference to the $0X$-axis is the same relatively as the central deflection of the column with reference to the line, $A\,B$, joining its ends.

Consider an elementary slice of the column of length $(= d\,x)$. The strain of this small portion of the column through an angle, $d\,\theta$ (Fig. 1(c)) causes the end, A (Fig. 1(b)) to deflect (relatively) toward the right an amount equal to $\left(\frac{l}{2} - x\right) d\,\theta$.

Summing up these small deflections due to the successive strains of all the elementary parts from O to A gives the total deflection, $C\,O$, of the mid-section.

From Fig. 1(c), which is an enlarged view of an elementary portion of the column:

$$d\,\theta = \frac{2\,B\,D}{N\,B} = \frac{f\,d\,x}{E\,c} = \frac{M\,d\,x}{E\,I} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The bending moment at any section of the column is:

$$M = P\,(y_1 + \Delta + e - y) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

At the mid-section of the column, the moment is greatest. Hence,

$$M_{(\max.)} = P\,y_t = P\,(y_1 + \Delta + e) = f_b\,\frac{I}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Substituting Equation (2) in Equation (1):

$$d\,\theta = \frac{P\,(y_1 + \Delta + e - y)\,d\,x}{E\,I} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The elastic curve, $A\,O\,B$ (Fig. 1(b)), is assumed, for the time being, as a parabola; that is, $y = k\,x^2$, with the vertex at O, the origin.

If $x = \frac{l}{2}$, $y = y_1 + \Delta$; then,

$$y_1 + \Delta = k\left(\frac{l}{2}\right)^2$$

hence,

$$k = \frac{4\,(y + \Delta)}{l^2}$$

and the equation of parabola is:

$$y = \frac{4\,(y_1 + \Delta)}{l^2}\,x^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Now, the deflection of A relative to O due to bending is:

$$y_1 = \int_0^{\frac{l}{2}} \left(\frac{l}{2} - x\right) d\,\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Substituting Equation (4) for $d\theta$ and then integrating and reducing:

$$y_1 = \frac{\frac{P\,l^2}{E\,I}\left(\frac{5}{48}\,\Delta + \frac{e}{8}\right)}{1 - \frac{5}{48}\,\frac{P\,l^2}{E\,I}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Equation (7) shows that the coefficients of Δ and e differ in the ratio of about $\frac{1}{10}$ to $\frac{1}{8}$; that is, with equal magnitudes of Δ and e, the latter will produce the greater deflection; but many writers treat Δ and e as identical in deriving the well known secant formula.

Equation (7) shows that if both Δ and e are zero, then $y_1 = 0$; but if some deflection should occur when Δ and e are both zero, then y_1 is not zero; and Equation (7) must take the form, $y_1 = \frac{0}{0}$, which is an indeterminate quantity.

Since the denominator of Equation (7) is zero (while y_1 is not zero), then,

$$P = \frac{48}{5}\,\frac{E\,I}{l^2} = 9.6\,\frac{E\,I}{l^2}$$

which is practically the same as Euler's formula. If the bent axis, $A\,O\,B$, had been assumed to be a sine curve instead of a parabola, then the factor, 9.6, would be replaced by π^2 (or 9.87), thus agreeing exactly with Euler's formula, which is,

$$P = \pi^2\,\frac{E\,I}{l^2}$$

Hence, it is justifiable to substitute $\frac{1}{9.87}$ for $\frac{5}{48}\left(=\frac{1}{9.6}\right)$ in the denominator of Equation (7).

Accordingly, Equation (5) will take the form:

$$y_1 = \frac{\frac{P\,l^2}{E\,I}\left(\frac{\Delta}{9.6} + \frac{e}{8}\right)}{1 - \frac{1}{9.87}\,\frac{P\,l^2}{E\,I}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The accuracy of Equation (8) is verified by its agreement with a similar, but more complicated, expression derived by Salmon in a rigorously mathematical fashion, and presented as Equation (86) in his book.* On changing the symbols of Salmon's Equation (86) to those used in this paper, it reduces to:

$$y_0 = e\,\sec\frac{1}{2}\,\frac{l}{r}\sqrt{\frac{P}{A}\,\frac{1}{E}} + \frac{8\,E\,\Delta}{\left(\frac{l}{r}\right)^2\,\frac{P}{A}}\left[\sec\frac{1}{2}\,\frac{l}{r}\sqrt{\frac{P}{A}\,\frac{1}{E}} - 1\right] \ldots (9)$$

Thus, if $\frac{l}{r} = 50$, $\frac{e\,c}{r^2} = 0.1$, $\frac{\Delta\,c}{r^2} = 0.075$, and $\frac{P}{A} = 12\,920$; then,

$$y_t = (y_1 + \Delta + e) = 0.1995\,\frac{r^2}{c}$$

* "Columns," by E. H. Salmon, Lond., 1921, p. 42.

and,

$$y_0 = 0.1996 \frac{r^2}{c}$$

Hence, the difference in deflections of the mid-sections as given by the two formulas is,

$$y_t - y_0 = 0.0001 \frac{r^2}{c}$$

The corresponding maximum unit stress due to a combination of direct stress and bending, according to Equation (8), is:

$$f = \frac{P}{A} + \frac{M c}{I} = \frac{P}{A} + \frac{P y_t c}{A r^2} = \frac{P}{A} + \frac{P (y_1 + \Delta + e) c}{A r^2} \quad \ldots\ldots (10)$$

The same, according to Equation (9) is:

$$f = \frac{P}{A} + \frac{P y_0 c}{A r^2} \quad \ldots\ldots (11)$$

The difference in stress as given by Equations (10) and (11) is:

$$\left(\frac{P}{A}\right)\left(\frac{c}{r^2}\right)(y_0 - y_t)$$

Hence, the error in applying Equation (8) to this case is only,

$$12\,920 \left(\frac{c}{r^2}\right)\left(0.0001 \frac{r^2}{c}\right) = 1.3 \text{ lb. per sq. in.},$$

a negligible quantity.

If $\frac{l}{r} = 100$; $\frac{e c}{r^2} = 0.1$; $\frac{\Delta c}{r^2} = 0.15$; and $\frac{P}{A} = 8\,858$; then,

$$y_t = 0.3685 \frac{r^2}{c}$$

and,

$$y_0 = 0.3690 \frac{r^2}{c}.$$

The difference in deflections of the mid-sections is,

$$y_t - y_0 = 0.0005 \frac{r^2}{c}.$$

Hence, the difference in stress as given by Equations (10) and (11) is only,

$$8\,858 \left(\frac{c}{r^2}\right)\left(0.0005 \frac{r^2}{c}\right) = 4.4 \text{ lb. per sq. in.}$$

Also, if $\frac{l}{r} = 200$, $\frac{e c}{r^2} = 0.1$, $\frac{\Delta c}{r^2} = 0.3$, and $\frac{P}{A} = 2\,975$; then,

$$y_t = 0.6902 \frac{r^2}{c}$$

and,

$$y_0 = 0.6914 \frac{r^2}{c}$$

Hence, the difference in deflections of the mid-sections is,

$$y_t - y_0 = 0.0012 \frac{r^2}{c}.$$

In this case, the difference in stress as given by Equations (10) and (11) is only,

$$2\,975\left(\frac{c}{r^2}\right)\left(0.0012\,\frac{r^2}{c}\right) = 3.6 \text{ lb. per sq. in.}$$

Therefore, a column formula based on Equation (8) may be considered just as rational as one based on Equation (9). Furthermore, it is obvious that an expression involving Equation (9) is not directly solvable for $\frac{P}{A}$.

Substituting the value of y_1 from Equation (8) in Equation (10), reducing, and grouping terms, gives:

$$\left(\frac{P}{A}\right)^2\left[\frac{1}{9.87\,E}\left(\frac{l}{r}\right)^2\left(1 - 0.028125\,\frac{\Delta\,c}{r^2} - 0.23375\,\frac{e\,c}{r^2}\right)\right]$$
$$- \frac{P}{A}\left[1 + (\Delta + e)\,\frac{c}{r^2} + \frac{1}{9.87\,E}\,f\left(\frac{l}{r}\right)^2\right] + f = 0 \quad \ldots\ldots(12)$$

Equation (12) is a quadratic in $\frac{P}{A}$. It takes the form:

$$\left(\frac{P}{A}\right)^2 m - \left(\frac{P}{A}\right) n + f = 0 \quad \ldots\ldots(13)$$

in which,

$$m = \frac{1}{9.87\,E}\left(\frac{l}{r}\right)^2\left(1 - 0.028125\,\frac{\Delta\,c}{r^2} - 0.23375\,\frac{e\,c}{r^2}\right) \quad \ldots\ldots(14)$$

and,

$$n = 1 + (\Delta + e)\,\frac{c}{r^2} + \frac{1}{9.87\,E}\,f\left(\frac{l}{r}\right)^2 \quad \ldots\ldots(15)$$

Solving Equation (13) gives:

$$\frac{P}{A} = \frac{n \pm \sqrt{n^2 - 4\,f\,m}}{2\,m} \quad \ldots\ldots(16)$$

Rationalizing the numerator of Equation (16); that is, multiplying numerator and denominator of the fraction by $n \mp \sqrt{n^2 - 4\,f\,m}$, gives:

$$\frac{P}{A} = \frac{n^2 - (n^2 - 4\,f\,m)}{2\,m\left(n \mp \sqrt{n^2 - 4\,f\,m}\right)} = \frac{2\,f}{n \mp \sqrt{n^2 - 4\,f\,m}} \quad \ldots\ldots(17)$$

Restoring the original terms, Equation (17) becomes:

$$\frac{P}{A} = \frac{2\,f}{1 + (\Delta + e)\,\frac{c}{r^2} + \frac{f}{9.87\,E}\left(\frac{l}{r}\right)^2 \mp \sqrt{\left[1 + (\Delta + e)\,\frac{c}{r^2} + \frac{f}{9.87\,E}\right]^2 - \frac{4\,f}{9.87\,E}\left(\frac{l}{r}\right)^2\left(1 - 0.028125\,\frac{\Delta\,c}{r^2} - 0.23375\,\frac{e\,c}{r^2}\right)}} \quad (18)^*$$

Equation (18) presents a rational expression for the average stress occurring in the mid-section of a column composed of any material, it being sub-

* In Equation (18), the plus sign only should actually be used for the symbol, $\mp$, with the radical term.

jected to initial curvature and eccentric longitudinal loading. A study of this equation shows the following limiting conditions: If $\frac{l}{r}$ approaches zero; then, using the plus sign, $\frac{P}{A}$ approaches $\frac{f}{1 + (\Delta + e)\frac{c}{r^2}}$; using the minus sign, $\frac{P}{A}$ approaches $\frac{2f}{0}$ (infinity).

If $(\Delta + e)$ approaches zero; then, using the plus sign, $\frac{P}{A}$ approaches f; using the minus sign, $\frac{P}{A}$ approaches $\frac{9.87\,E}{\left(\frac{l}{r}\right)^2}$ (Euler's formula).

Evidently, in applying Equation (18), the plus sign only should be used with the radical term. Furthermore, the following conditions will be assumed: $\frac{e\,c}{r^2} = 0.1$; $\frac{\Delta\,c}{r^2} = 0.0015\,\frac{l}{r}$; $f = 36\,000$ lb. per sq. in. (the yield point of the material).

Thus, if $\frac{l}{r} = 100$, the following characteristics are found for the smaller column shown in Fig. 2(*a*):

Section:

One web-plate, 36 in. by $\frac{1}{2}$ in.	= 18.00 sq. in.
Four angles, 6 by 4 in. by $\frac{5}{8}$ in.	= 23.44 " "
Two cover-plates, 14 in. by $\frac{3}{4}$ in.	= 21.00 " "
Gross area.....	62.44 sq. in.
r (Axis 1 — 1)	= 16.11 in.
r (Axis 2 — 2)	= 2.96 in.
l = 296 in. = 24 ft. 8 in.	
c = 7 in.	

Hence,

$$\frac{e\,c}{r^2} = 0.1$$

and,

$$e = 0.1\,\frac{r^2}{c} = 0.125 \text{ in.} = \frac{1}{8} \text{ in.}$$

also,

$$\frac{\Delta\,c}{r^2} = 0.15$$

and,

$$\Delta = 0.1875 \text{ in.} = \frac{3}{16} \text{ in.}$$

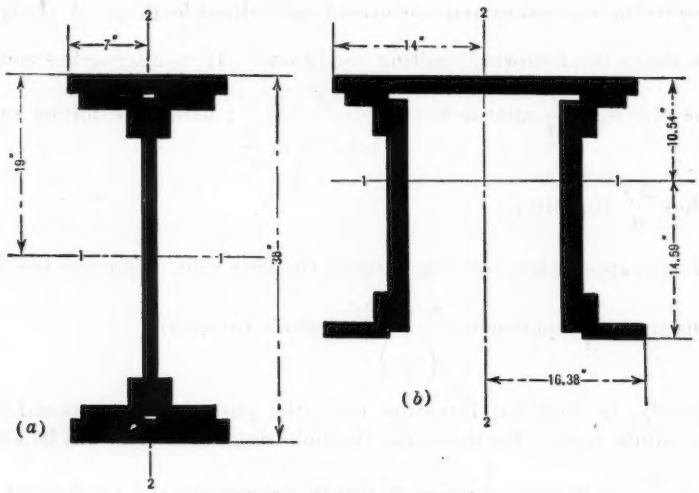

FIG. 2.—COLUMN SECTIONS.

If $\frac{l}{r} = 50$, the corresponding values for the larger column (Fig. 2(*b*)) are:

Section:

One web-plate, 28 in. by $\frac{5}{8}$ in.	=	18.13 sq. in.
Two angles, 4 by 4 in. by $\frac{7}{16}$ in.	=	6.62 " "
Two cover-plates, 24 in. by $\frac{3}{4}$ in.	=	36.00 " "
Two angles, 6 by 4 in. by $\frac{5}{8}$ in.	=	11.72 " "
Gross area.....		72.47 sq. in.

r (Axis 2 — 2) = 10.25 in.

r (Axis 1 — 1) = 9.47 in.

l = 39 ft. 5 in.

c = 10.54 in.

Hence,

$$e = 0.1\,\frac{r^2}{c} = 0.1 \times \frac{89.68}{10.54} = 0.851 \text{ in.} = \text{about } \frac{7}{8} \text{ in.}$$

also,

$$\frac{\Delta\, c}{r^2} = 0.075$$

and,

$$\Delta = 0.075\,\frac{r^2}{c} = 0.638 \text{ in.} = \text{about } \frac{5}{8} \text{ in.}$$

Substituting the assumed conditions of initial curvature and eccentricity of loading in Equation (18), dividing the numerator and the denominator by 2, and using a factor of safety = $2\frac{1}{4}$,

$$\frac{P}{A} = \frac{16\,000}{a + \sqrt{a^2 - b}} \qquad (19)$$

in which,

$$a = 0.55 + 0.00075\,\frac{l}{r} + 0.0000607902\left(\frac{l}{r}\right)^2$$

and,

$$b = 0.0001187386\left(\frac{l}{r}\right)^2 - 0.000000005129\left(\frac{l}{r}\right)^3$$

On plotting Equation (19), the curve is seen to be as represented in Fig. 3. A point of inflection occurs when $\frac{l}{r}$ is about 100. Therefore, it is a cubic or a higher powered parabola.

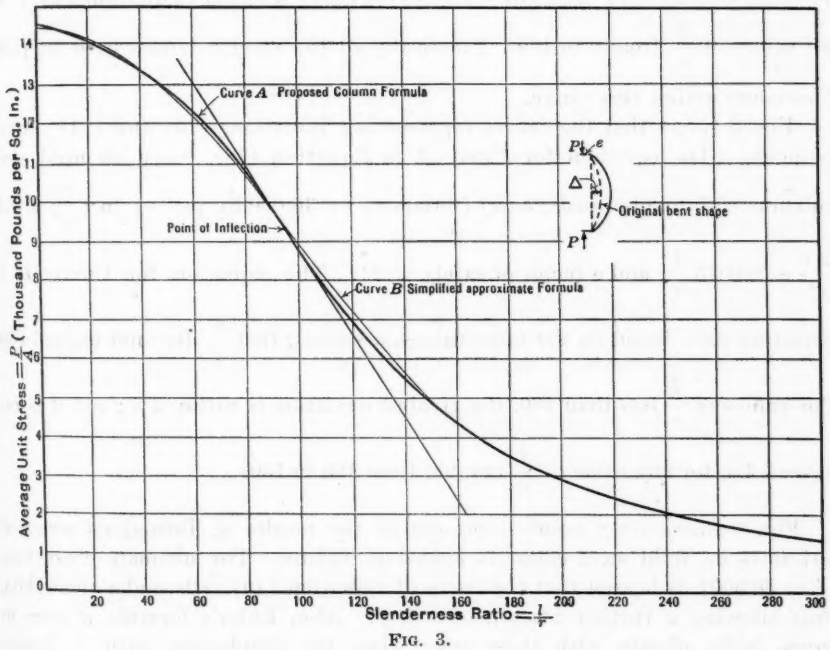

FIG. 3.

The curve will be considered as a cubic parabola of the form:

$$\frac{P}{A} = K + A\left(\frac{l}{r}\right) + B\left(\frac{l}{r}\right)^2 + C\left(\frac{l}{r}\right)^3 \quad \ldots\ldots\ldots\ldots (20)$$

From Equation (19), if,

$$\frac{l}{r} = 0, \frac{P}{A} = 14\,545;$$

$$\frac{l}{r} = 30, \frac{P}{A} = 13\,762;$$

$$\frac{l}{r} = 100, \frac{P}{A} = 8\,819; \text{ and if,}$$

$$\frac{l}{r} = 200, \frac{P}{A} = 2\,969.$$

Substituting these values in Equation (20) gives four equations from which the four unknowns, K, A, B, and C may be obtained, giving $K = 14\,545$; $A = -5.0$; $B = -0.781$; and $C = +0.00258$.

Substantially the same results occur by assuming that, for the sake of simplicity, $K = 14\,500$; $A = 0$; $B = -0.8$; $C = 0.0025$.

Then Equation (20) takes the form:

$$\frac{P}{A} = 14\,500 - \left(\frac{l}{r}\right)^2 \left[0.8 - 0.0025\ \frac{l}{r}\right] \ldots\ldots\ldots\ldots (21)$$

Equation (21) is, therefore, a simplified approximate formula which is in close agreement with the more complicated exact formula (Equation (19)) for all values of $\frac{l}{r}$ from 0 to 150. Practically all the steel columns used in practice come within this range.

Fig. 3 shows that the curves representing Equations (19) and (21) almost coincide. The equation for Curve A is Equation (18), based on mild steel columns with pivoted ends, and f (ultimate) $= 36\,000$ lb. per sq. in.; $\frac{c\,c}{r^2} = 0.1$; $\frac{\Delta\,c}{r^2} = 0.0015\ \frac{l}{r}$; and a factor of safety $= 2\frac{1}{4}$. The equation for Curve B is Equation (20), based on the same values, assuming that $\frac{l}{r}$ does not exceed 150. For values of $\frac{l}{r}$ less than 100, the greatest deviation is within 2%; and it is not beyond 4% for any value of $\frac{l}{r}$ ranging from 100 to 150.

Fig. 4 shows sixty points representing the results of Tetmajer's series of 104 tests on mild steel columns loaded to failure. For ultimate conditions ($f = 36\,000$), it is seen that the curve of Equation (19) falls under these dots, thus allowing a further margin of safety. Also, Euler's formula is seen to agree fairly closely with these tests when the slenderness ratio is larger than 120.

The curves of Fig. 5 show the proposed column formula (Curve 1) as compared with the following empirical formulas:

Curve 2, the Rankine formula:

$$\frac{P}{A} = \frac{16\,000}{1 + \frac{1}{13\,500}\left(\frac{l}{r}\right)^2} \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Curve 3, the American Railway Engineering Association formula:

$$\frac{P}{A} = 15\,000 - 50\ \frac{l}{r} \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

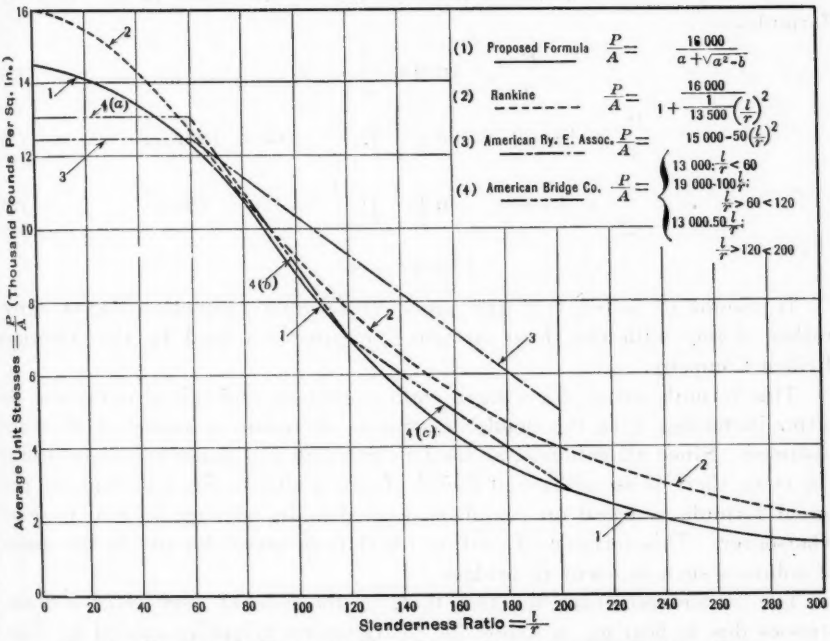

FIG. 4.—TETMAJER'S TESTS ON MILD STEEL COLUMNS WITH PIVOTED ENDS.

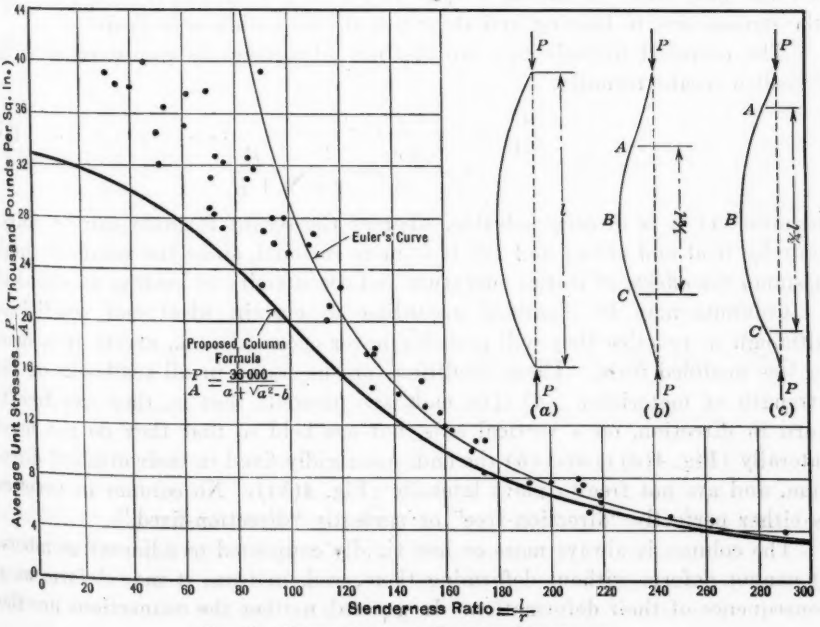

FIG. 5.—COMPARISON OF COLUMN FORMULAS.

Curve 4, (*a*), (*b*), and (*c*), respectively, the American Bridge Company formulas:

$$\frac{P}{A} = 13\,000; \frac{l}{r} < 60 \quad \ldots\ldots (24)$$

$$\frac{P}{A} = 19\,000 - 100\left(\frac{l}{r}\right); \frac{l}{r} > 60 < 120 \quad \ldots\ldots (25)$$

$$\frac{P}{A} = 13\,000 - 50\left(\frac{l}{r}\right); \frac{l}{r} > 120 < 200 \quad \ldots\ldots (26)$$

Conclusions

It should be noted that the proposed formula (Equation (18)) agrees rather closely with the three straight-line equations used by the American Bridge Company.

This formula allows for a small end eccentricity and initial curvature, the latter increasing with the slenderness ratio. A factor of safety of 2¼ is also assumed. Since all columns, as used in practice, are somewhat restrained at the ends, there is an additional factor of safety due to the fact that the proposed formula is based on round or pivoted ends, offering no end restraint whatsoever. This formula (Equation (18)) is proposed for use in the design of columns such as occur in bridges.

The writer recognizes the fact that, in the case of very short columns, stresses due to bearing, or those due to transverse crippling caused by shear, may be more serious than those due to flexure. However, since the maximum average unit stress occurring is only 14 500 lb. per sq. in., it is thought that the stresses due to bearing and shear are always within safe limits.

The proposed formula has two distinct advantages as compared with the so-called secant formula:

$$\frac{P}{A} = \frac{f}{1 + \frac{e\,c}{r^2} \sec \frac{l}{2\,r}\sqrt{\frac{P}{A\,E}}} \quad \ldots\ldots (27)$$

because (1) it is directly solvable, whereas the secant formula can be solved only by trial and error; and (2) it is more rational, since the secant formula assumes the effects of initial curvature and eccentricity of loading as identical.

Columns may be classified according to certain ideal end conditions, although in practice they will probably never occur as such, except in a more or less modified form. These conditions are as stated in all textbooks on the strength of materials: (*a*) The ends are pivoted; that is, they are free to turn in direction, on a vertical axis, but are held so that they do not move laterally (Fig. 4(*a*)); and (*b*) the ends are rigidly fixed in their original direction, and are not free to move laterally (Fig. 4(*b*)). No column in practice is either perfectly "direction-free" or perfectly "direction-fixed."

The column is always more or less rigidly connected to adjacent members. It cannot deform without deforming them, and, in turn, it may deform as a consequence of their deformation. In general, neither the connections nor the

adjacent members will be sufficiently rigid to fix the ends of the column perfectly.

A common method of making allowance for the end conditions is to determine what is called the "free length" of the column. The deflection curve of any column is a part of a curve of sines, and by choosing a suitable part of the axis to be the sine curve, the deflection curve for any end condition can be represented.

If the column has fixed ends, it is seen from Fig. 4(*b*) that the portion, $A\,C$, is a sine curve of length, $\frac{1}{2}\,l$. Hence, it may be said, other things being equal, that the ideal fixed-ended column has the same strength as the ideal pivoted column having one-half its length. The true end condition of columns used in practice lies somewhere between the extreme limits of the pivoted and the fixed state.

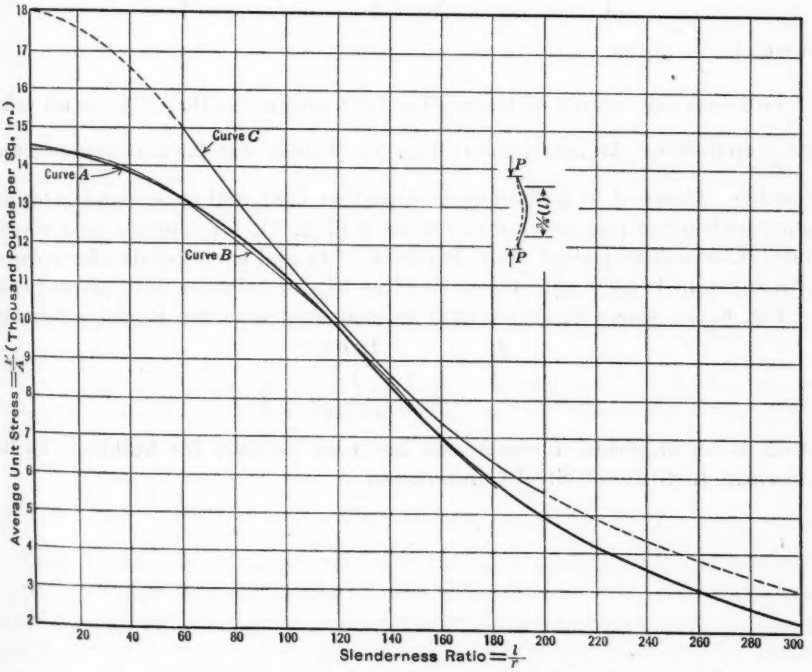

FIG. 6.—COMPARISON OF FORMULAS FOR COLUMNS WITH RESTRAINED ENDS.

In order to derive a formula that shall be appropriate for a column such as would usually occur in buildings, it will be assumed that the end restraint is sufficient to produce the same strength as the ideal pivoted column having three-fourths its length (Fig. 4(*c*)). The eccentricity of loading, initial curvature, yield point, and factor of safety, will be taken the same as for the case of the pivoted column. Accordingly, it will merely be necessary to sub-

stitute $\frac{3}{4}\,l$ for l in Equation (19). The resulting expression becomes:

$$\frac{P}{A} = \frac{16\,000}{a_1 + \sqrt{a_1^2 - b_1}} \qquad (28)$$

in which,

$$a_1 = 0.55 + 0.0005625\,\frac{l}{r} + 0.00003419449\left(\frac{l}{r}\right)^2$$

and,

$$b_1 = 0.00006679047\left(\frac{l}{r}\right)^2 - 0.000000002163873\left(\frac{l}{r}\right)^3$$

The simplified approximate formula $\left(\text{derived by substituting } \frac{3}{4}l \text{ for } l \text{ in Equation (20)}\right)$ gives:

$$\frac{P}{A} = 14\,500 - \left(\frac{l}{r}\right)^2\left[0.45 - 0.001\,\frac{l}{r}\right] \qquad (29)$$

in which, $\frac{l}{r} < 180$.

Equation (29) agrees with Equation (28) within less than 2% for all values of $\frac{l}{r}$ up to 180. An inspection of Fig. 6 will show that the two curves almost coincide. Curve A is plotted from Equation (28) and corresponds (for columns with restrained ends) with Curve A (Fig. 3), for columns with pivoted ends. Curve B is plotted from Equation (29) and corresponds (for columns with restrained ends) with Curve B (Fig. 3), for columns with pivoted ends.

Fig. 6 also shows Equation (28) as compared with the Rankine formula:

$$\frac{P}{A} = \frac{18\,000}{1 + \frac{1}{18\,000}\left(\frac{l}{r}\right)^2} \qquad (30)$$

which is an empirical formula that has been adopted for buildings by the American Institute of Steel Construction.

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EFFECT OF EARTH SHOCKS ON STRUCTURES

By M. de Bussy,* Esq.

Synopsis

This paper is a study of the stresses that affect simple structures as a result of movements caused by earthquake shocks. It was not the aim of the writer to find, immediately, rules for the design of structures so as to make them proof against earthquakes. His intention was, on the contrary, to take up the problem in general, attacking the easiest questions first with the methods of computation that are available, so as to attain a better understanding of the phenomena that occur; and after obtaining this result, to consider more complicated cases.

The writer also calls attention to other questions that are still unsolved. He hopes that engineers who agree with this inductive manner of attacking the problem, will become interested enough to study it further.

Motion of the Ground

Since the stresses in a moving body are caused by, and are proportional to, its inert mass and its acceleration, the acceleration must be considered as the most important measure of the intensity of an earthquake shock. Except near the "fault", where the wildest motion can be expected, the horizontal component of the acceleration is generally greater than the vertical component. Moreover, the vertical acceleration causes forces for which the structure is designed with a safety factor; therefore it has a far smaller destructive effect. The computation of this effect is not difficult as long as the acceleration is not downward and greater than that due to gravity. The effect is similar to that due to an increase or decrease of the weight of every part of the structure. It can, therefore, be excluded from the following considerations.

Note.—Written discussion on this paper will be closed in September, 1928.

* Hilversum, Holland.

An exception must be made for the case in which the vertical acceleration has different values, at the same moment, under different parts of the base of the structure. Under this condition a tilting of the structure occurs. This phenomenon (an undulation of the ground with a short wave length) has been reported by some eye-witnesses, in soft ground. It may be, however, that they were subject to an optical illusion.

The horizontal motion of the ground in the destructive part of an earthquake shock may be very complicated, and is seldom registered, since the seismographs, built to record the minute motions, are often disarranged, or their records fall partly outside the recording sheet. The movement during the shock in Tokyo, Japan (1894), was, approximately, forward, 37 mm.; back, 73 mm.; and forward again 42 mm. That during the first part of the shock in 1923 was as shown in Fig. 1*.

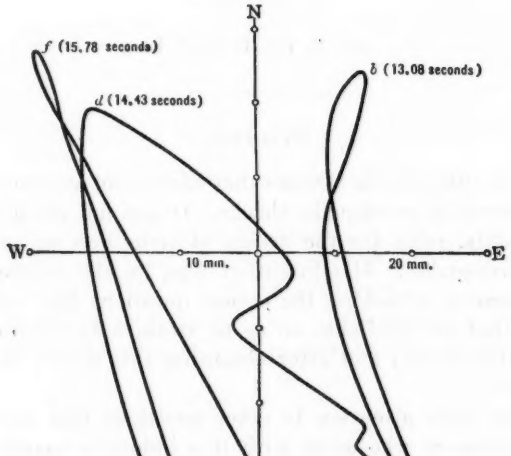

FIG. 1.—HORIZONTAL MOVEMENT AT COMMENCEMENT OF DESTRUCTIVE EARTHQUAKE, SEPTEMBER 1, 1923.

Witnesses describe the motion generally as an "oscillation". It is doubtful, however, if many (say, three or four) consecutive oscillations of nearly the same period and amplitude ever occur. Probably the movement can only be defined as a number of unequal displacements. The maximum acceleration as such has never been measured by instruments. It has been computed from the seismograms, in which the oscillations are assumed to be harmonic. It is evident that there is a need for better records.

* *Seismological Notes, No. 6*, Imperial Earthquake Investigation Committee, Tokyo, 1924.

Movement of Objects

Loose objects, like bottles, pieces of furniture, Japanese tombstones and lanterns, etc., will not break during an earthquake shock. They will slide, overturn, begin to rock, or remain upright and follow the motion of the ground. These phenomena are, perhaps, of no importance for the determination of stresses in structures. However, as the late Professor F. Omori pointed out, they are very important as a means of measuring the intensity of the shock. Engineers living in regions where earthquake shocks occur, should keep this in mind.

This is especially true about sliding. The experiments made by Professor Omori on the shaking table* show that sliding of objects actually occurs at a value of the maximum acceleration that can be predetermined. It is well known that the effect, on an object, of an acceleration of the ground is the same as that of a force that is the product of the mass and the acceleration and in the opposite direction. If it stands on a horizontal surface, the object will begin to slide when this force is greater than the friction between it and the material on which it rests; or, in other words, if the ratio of the acceleration of the ground to the acceleration due to gravity, is greater than the coefficient of friction. This coefficient of friction can usually be determined, very easily for research purposes, by weighing the body and measuring (with a spring scale) the force that makes it slide; or, for an object standing on a table, by tilting the table and measuring the critical angle. The estimate of the acceleration found in this manner may be somewhat rough, but when other measurements fail it will be of much value.

Professor Omori showed also that the presence of a vertical component in the acceleration affects only slightly the value found for the horizontal component.

For objects on an inclined surface (roofing tiles, for instance), a small computation must be made. This is easy after replacing the acceleration by a force in the opposite direction.

To overturn an object, the acceleration must be so great that the force computed, applied at the center of gravity and combined with the weight of the object, gives a resultant that intersects the ground outside the base. In addition, the acceleration must last a certain length of time. If it acts very briefly, the object falls back to its original position. The fact that an object falls over during an earthquake is not, therefore, a measure of the maximum acceleration. If an object does not fall over, the maximum acceleration may have been greater (for a very short moment) than that which is theoretically necessary to overturn it. This subject is treated by Mr. P. Kirkpatrick†, who, by slightly modifying the problem, found an exact formula for the oscillation required to cause overturning. He suggests a simple and cheap instrument that will indicate both amplitude and maximum acceleration of an earthquake.

* "Seismic Experiments on the Fracturing and Overturning of Columns," by F. Omori, *Bulletin*, Imperial Earthquake Investigation Committee, Vol. 4, Tokyo, 1910–12.

† *Science*, Vol. LXV, No. 1685, p. 379.

A rocking motion should not be confused, as is sometimes done, with vibration or oscillation of structures, which is due to their elastic deformation. In a rocking motion, the period varies with the amplitude, whereas in vibration or oscillation, the period is constant, as in any harmonic oscillation.

Motion of Buildings

Except in very rare cases, sliding, overturning, and rocking are factors that need not be considered, because these movements cause internal stresses that are so high that rupture occurs before the structure starts to move. Moreover, the elastic deformation can not be neglected, in comparison with the relatively small movements of the ground, and this complicates the phenomena. In this paper, attention will be given first to the stresses in buildings that are supposed to be perfectly rigid (and, therefore, do not undergo any deformation); and, second, to some phenomena resulting from the flexibility of the buildings.

Rigid Buildings.—If the motion of the ground is considered to be horizontal, the first assumption to be made for the motion of the building itself is that it moves horizontally also and does not have any other motion. Computations show that this will not explain failures of chimneys and damages to high buildings at a point high above the base, as so often occurs. Professor Omori, therefore, originated the idea of a motion like that of a free, nonconstrained rod, which shows, at about two-thirds of its length, a so-called "center of rotation", a point where the acceleration is zero. For such a motion Professor Joseph N. LeConte* computes shear and bending-moment stresses for a rod of constant cross-section.

Which motion the rigid building will make, however, depends entirely on its foundation, especially on the size and shape of its base and on the firmness of the ground. When such a building has a wide base, and stands on very hard ground, its motion will be very nearly a horizontal displacement without rotation. Tilting will practically not occur at all; or, in other words, the point of zero acceleration lies at infinity. When, on the other hand, the base is narrow, and the ground soft, it is not difficult to imagine a motion approximately like a free rod; that is, with a point of zero acceleration on its axis at about two-thirds of the height. For intermediate conditions, it is clear that a motion may occur with a point of zero acceleration somewhere between infinity and two-thirds of the height. Further study shows, however, that if the ground, to a certain degree, has elastic properties, the point of zero acceleration may have any location whatever on the axis, between plus and minus infinity.

If the earth develops a horizontal force on the base of the structure, this force will tend to cause the building to tilt; and at the very first moment of the shock there is nothing to counteract this tilting. As soon, however, as an inclination really exists, the ground will tend to bring the building back to its vertical position. This action of the ground, therefore, is not

* "The Stresses in a Free Prismatic Rod Under a Single Force Normal to Its Axis." *Transactions*, Am. Soc. C. E., Vol. 91 (December, 1927), pp. 968–990.

simultaneous with the tilting action of the horizontal earth shock; and since this shock acts alternately in one direction and then in the reverse direction, with very short intervals, it is clear that the resulting tilting motion of the building will be quite complicated.

Fig. 2 gives an illustration of this. It shows the successive inclinations of a building, during an earthquake shock, which moves it from the right to the left and back again to its original place, with a harmonic motion. If the ground were very soft, the building would move like a free rod, and Curve 1, which is a curve of sines, would represent its consecutive inclinations, all of them being to the right. Curves 2 and 3 are calculated for certain conditions of rigidity of the ground and dimensions of the bearing surface of the base. Curve 3 represents a larger base, or a more rigid ground, than Curve 2, and does not deviate so far from the zero line. For a very large base, or for a perfectly hard ground, the resulting curve would be still nearer the zero line. Curves 2 and 3 show inclinations to the left, differing from Curve 1 in this respect.

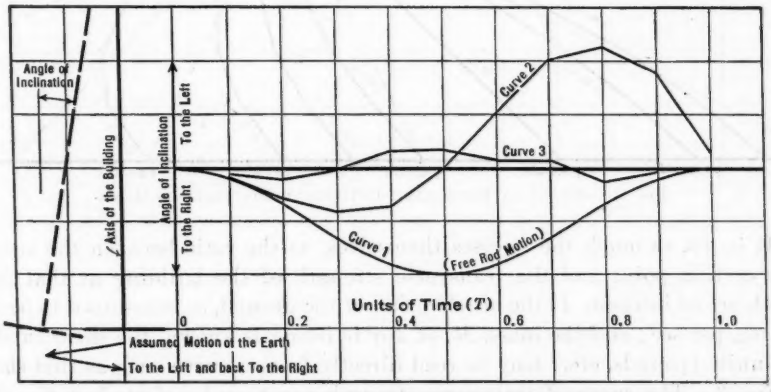

FIG. 2.—MOTION OF TALL STRUCTURES ON COMPRESSIBLE GROUND.

At three-quarters of a period the horizontal acceleration of the earth, and, therefore, of the base, is zero. At the same moment Curve 2 shows a great inclination of the axis of the building, which means that, at that moment, the ground develops a great resisting couple, which tends to bring the building back to a vertical position. This causes an angular acceleration of the building about its base. At that moment, then, the point of zero acceleration lies in the base. This illustrates that the point may lie lower than two-thirds of the height. Other points of Curves 2 and 3 give still different locations for the point of zero acceleration, and, in general, it may be said that this point may be found anywhere on the axis.

In case of an "undulation" of the ground, if the length of the waves is not many times the width of the base of the structure, computation shows that the location of the point of zero acceleration is also very low.

From a given momentary acceleration of the ground, and a given location of the point of zero acceleration at that moment, the determination of shear

and bending moment is very easy. In bents of frame buildings, the shear gives the intensity of the stresses in the material; in thin rods, chimneys, towers with rigid walls, etc., the bending moment must be considered principally.

Fig. 3 shows the shear curves for a rod with a uniformly distributed mass, a condition that practically prevails in high office buildings. The straight line, marked $z = \infty$, gives the shear line for a displacement of the structure without rotation; the curve marked $z = +\frac{2}{3}l$ gives the well known shear lines, for the free rod motion.

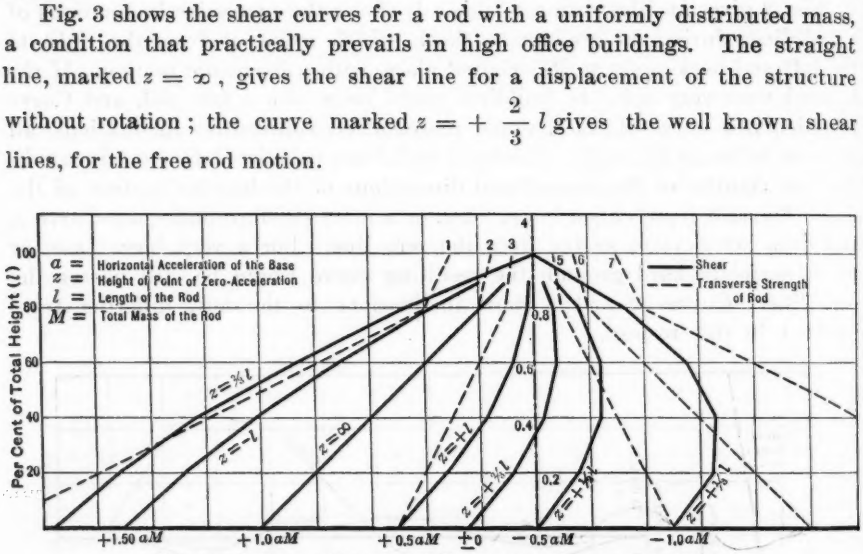

FIG. 3.—SHEAR IN RODS WITH UNIFORMLY DISTRIBUTED MASS.

It is not so much the stresses themselves, as the ratio between the stresses at a certain point and the transverse strength of the building at that point which are of interest. If the acceleration of the ground, a, is assumed to be 1 ft. per sec. per sec., and the mass, M, of any building is known, the shear in absolute units (pounds, etc.) may be read directly from a curve such as that shown in Fig. 3. The curve of transverse strength can then be plotted on the same diagram as shown. This is done schematically, for a theoretical high office building, designed to withstand a uniformly distributed wind load. Comparison can then be made between this curve and the curves for shear, all of which are for an acceleration of 1 ft. per sec. per sec. In order to make the comparison also possible for accelerations equal to 2 ft. and to 4 ft. per sec. per sec., the ordinates of the strength curve are reduced to one-half and one-fourth. Fig. 3 shows, for example, that when $a = 4$ (instead of unity) and $z = \frac{l}{2}$, the ultimate strength is surpassed at a distance from the base from 0.6 to 0.8 of the height, or, in other words, the mentioned conditions would cause rupture at that point.

It should be kept in mind that the accelerations referred to, are not the maximum accelerations of oscillations as in an earthquake; they are accelerations at a certain moment.

This diagram, therefore, gives a possible explanation of the damage to high buildings above the base. No point of the building can be said to be

free from danger; everything depends on the prevailing conditions of the earthquake shock.

The problem of building for safety against earthquakes, then, takes the following form in the special case of rigid buildings. Suppose, a certain amount of money can be spent to strengthen the building, that is, to bulge out the strength curve to a certain degree (see Fig. 3). Is it possible to indicate in which way that curve should be changed? By limiting the investigation to the beginning of an earthquake shock, for example, to the first half oscillation (from zero to 0.5 of the total time in Fig. 2), it is possible to find the greatest possible shearing stresses during that time.

These shearing stresses are shown in Fig. 4, for a maximum horizontal acceleration of the oscillation equal to a_m; they are much greater than the stresses produced by a horizontal displacement of the building as a whole, with an equal acceleration. It is clear that the stresses due to the horizontal force only (those of the free rod motion) are largely surpassed by those due to the angular acceleration of the building with respect to the ground. This is still more true at later periods of the oscillation, because of the fact that the angular motion about the base is, in principle an oscillatory one; earth shocks with a certain period are, therefore, apt to produce an effect of "resonance."

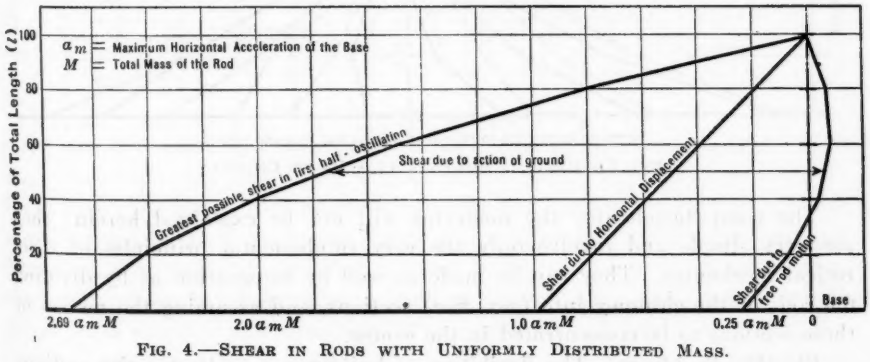

FIG. 4.—SHEAR IN RODS WITH UNIFORMLY DISTRIBUTED MASS.

From the foregoing it follows that (always in the special case of very rigid buildings) the curve representing the required strength of a building should preferably be more like that of the stresses due to an angular acceleration, than of those due to a uniform horizontal acceleration.

This conclusion is, theoretically, not restricted to the tall building with a narrow base on soft ground. It holds, also, for wide buildings and hard ground. In the latter case the deviations of the vertical axis from its vertical position will be very small, as already explained by Fig. 2; but due to the rigidity of the ground and the extension of the base, small deviations cause great angular accelerations. and, therefore, great stresses. It may be that because of the minuteness and quick damping of the angular motion, and the elastic yielding of the structure, the stresses will attain, in reality, only

a fraction of these theoretical values, but it is not possible to estimate offhand how great this fraction will be, and an investigation of a few actual cases should be made.

A diagram for the bending moments in a rod with uniformly distributed mass is not given, since it could not be applied in practice to any building.

Fig. 5 gives the bending moments for a chimney, that collapsed in the Japanese earthquake of 1923. It gives the moments, in foot-pounds, for a horizontal acceleration of the base equal to 1 ft. per sec. per sec. The strength curves give 0.1, 0.2, and 0.3 of the moments corresponding to a tensile strength in the masonry of about 28 lb. per sq. in., which is assumed to be the ultimate. It can be seen that, for low points of zero acceleration, practically all parts of the chimney (except the base and the top) are in equal danger of rupture. The point at which failure first takes place depends, then, entirely on accidental weaknesses in the structure.

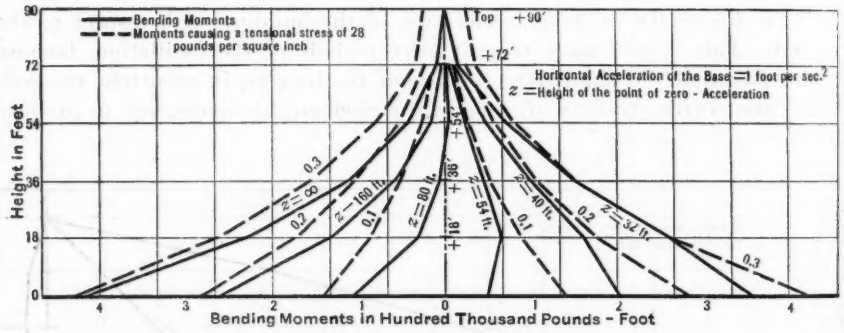

FIG. 5.—BENDING MOMENTS IN A BRICK CHIMNEY.

The computations for the diagrams will not be explained herein; they are very simple and involve only the very fundamental principles of theoretical mechanics. They can be made as well by integration as by dividing the rods or the chimney into (say, five) sections, and assuming the masses of these sections to be concentrated in the center.

Flexible Buildings.—The flexibility of buildings complicates the motions caused by an earthquake. It is the cause of a range of phenomena such as, elastic yielding, magnification of stresses due to the impulsive action of the ground, free oscillation, and forced oscillation. Few investigations have been made as yet, and these all refer to the special case of rods or buildings fixed rigidly to the ground, so that a tilting of the building cannot take place.

Figs. 6 to 11 illustrate the various phenomena that are due to the flexibility of structures. All these curves refer to the motion of a simple elastic system (Fig. 6), consisting of a heavy mass on top of an elastic rod of which the weight is neglected, and which is fixed rigidly into a solid body called the "ground". If the ground moves horizontally, the rod will show varying deflections, which may be easily calculated. The formula used involves a slight inaccuracy, which is of no importance, however, to the purely visual purpose of the investigation. The only elastic property entering the formula is the

proportionality between the deflection of the rod and the force with which it acts on the mass. The results, therefore, hold as well when the deflection is due to the bending of the rod, as when it is due to the deformation caused by shear; and also in the case of single or multiple portals, where the mass is distributed along the upper rod.

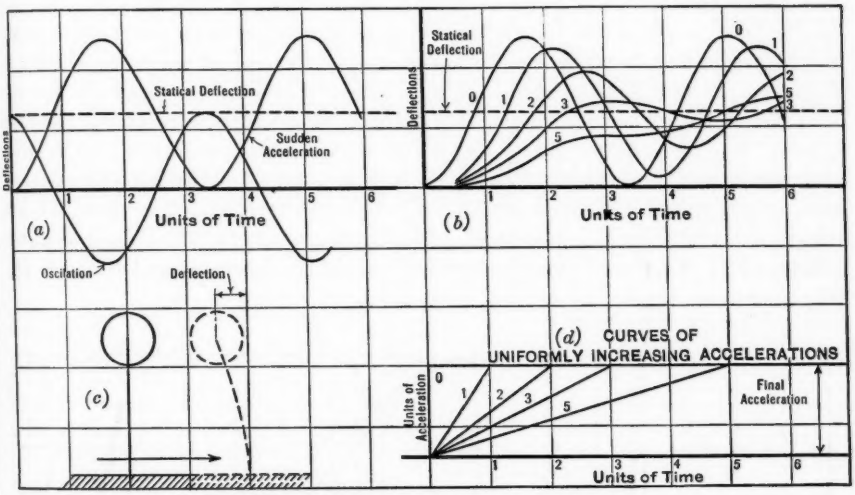

FIG. 6.—MOTIONS OF A SIMPLE ELASTIC BODY.

Fig. 6 (*a*) shows first the deflection assuming that the ground does not move, and that the mass is brought out of its position of equilibrium. The mass then starts an oscillation with a period that ought to be exactly π. It also shows the consecutive deflections caused by a sudden acceleration applied to the ground. The mass again begins to oscillate with its free period; the deflections vary between zero and a value which proves to be exactly twice the deflection of the rod under a statical force. This deflection is equal to the mass times the acceleration of the ground, and might be called the "statical deflection", for the assumed acceleration. The stresses in the rod are of course proportional to the deflections. This proves that the well-known law, which states that the stresses due to a suddenly applied load on any structure are twice those due to the same load at rest, applies also for sudden accelerations.

The effect of increasing an acceleration uniformly, but in a short time, to a certain value, is indicated in Figs. 6 (*b*) and 6 (*d*) which show the consecutive deflections for accelerations increasing from zero to the maximum value, equal for all cases, in various units of time. The longer the initial period, the less the magnitude of the deflections. Roughly speaking, the increase above the statical deflection is about 20%, if the length of the initial period is about one and one-half times the period of free oscillation. It is clear that when the acceleration has gained its final value, the curve of the deflection oscillates about the statical deflection line.

Accelerations in one direction for a long duration of time do not occur in earthquake shocks. The fundamental motion of importance can be taken to be one displacement, by which is meant a movement of the ground, in a straight line, from its original position of rest to another position, where it comes to rest once more. Necessarily, the ground must have first an acceleration, and, later, a retardation, or a negative acceleration.

The motion of the ground is usually taken to be a simple harmonic one. In this case the diagram for the acceleration during the time interval between the beginning and end of one displacement is a curve of sines, as shown in Fig. 7 (*a*). The question can be put, whether a different assumption for the variation of the acceleration would give a much different result for the stresses and deflections. In order to investigate this, two other acceleration arrangements are assumed, (Fig. 7 (*a*)), which need no further explanation. One might be called a rectangular and the other a triangular plan. In the diagram the accelerations are plotted on such a scale that the displacements are equal if the time intervals between the beginning and end of each displacement are equal.

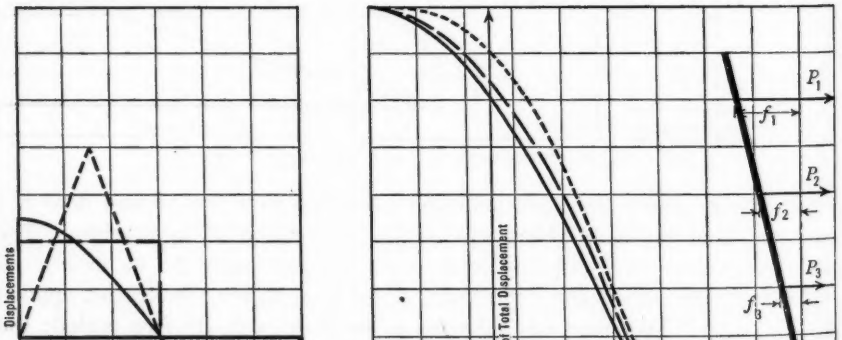

FIG. 7.

The assumption of motions different from the harmonic has an effect on the way the total distance is described, as shown in Fig. 7 (*b*). The differences are so small that it must be difficult to tell, from any given seismogram, the exact nature of the motion.

Figs. 8 (*a*) and 8 (*b*) give the deflections of the rod if the system is subject to displacements of two different time intervals; they allow a comparison of the effects of the assumed acceleration plans (Fig. 7 (*a*)). In both cases the harmonic motion gives the smallest maximum deflections and the triangular motion the greatest. The first could be called the "quietest", and

the latter the "wildest" motion. Probably the harmonic motion is the quietest possible motion. The differences are not great for a period of the displacement equal to about that of free oscillation of the system; they are still less for shorter durations. Considerable difference enters for the longer period. It cannot be said, however, that the maximum deflections are proportional to the acceleration, which, of course, must be the rule for very long durations.

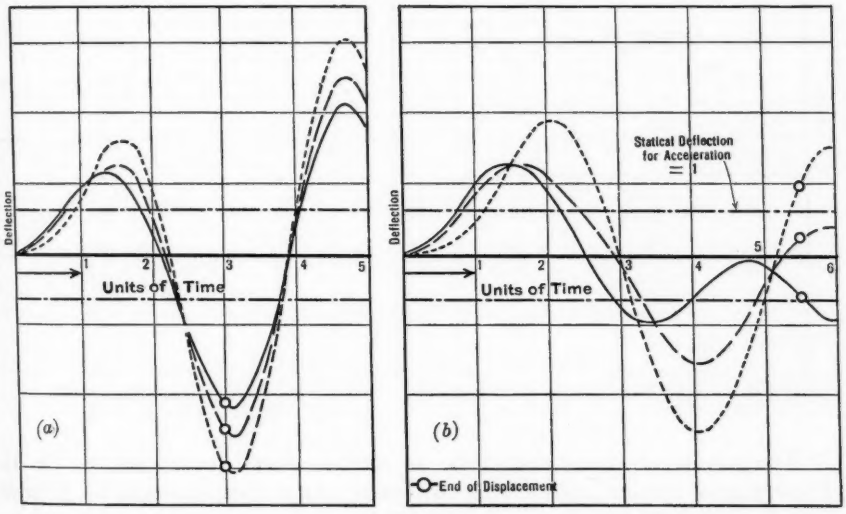

FIG. 8.

Fig. 9 shows, exclusively for the rectangular acceleration plan (Fig. 7 (*a*)), the deflection due to displacements of equal accelerations and different time intervals (and, therefore, of different amplitudes). They can, of course, all be compared to the statical deflection belonging to the assumed acceleration. As could be expected, the ratio between the duration and the period of free oscillation of the system now plays an important part. The great deflection in a time interval of three is evidently due to the fact that the sudden reversion of the acceleration occurs at the moment when the internal stresses in the rod also tend to throw the mass back.

This appears to be very similar to the magnification of stresses in the case of resonance between a forced oscillation and the free oscillation. However, there is a great difference. Since one full oscillation consists of two displacements, the "duration" of the displacement must be compared with the half period of an oscillation. In the case of "one displacement", the first maximum stresses occur, when the "duration" equals the period of free oscillation; and there is no apparent maximum when twice the duration equals that period.

For comparison, Fig. 10 shows the deflections caused by a continued forced harmonic oscillation with a period of three; it shows that only after one

and one-half oscillations, or after three displacements, the stresses are becoming greater than in some cases of one displacement.

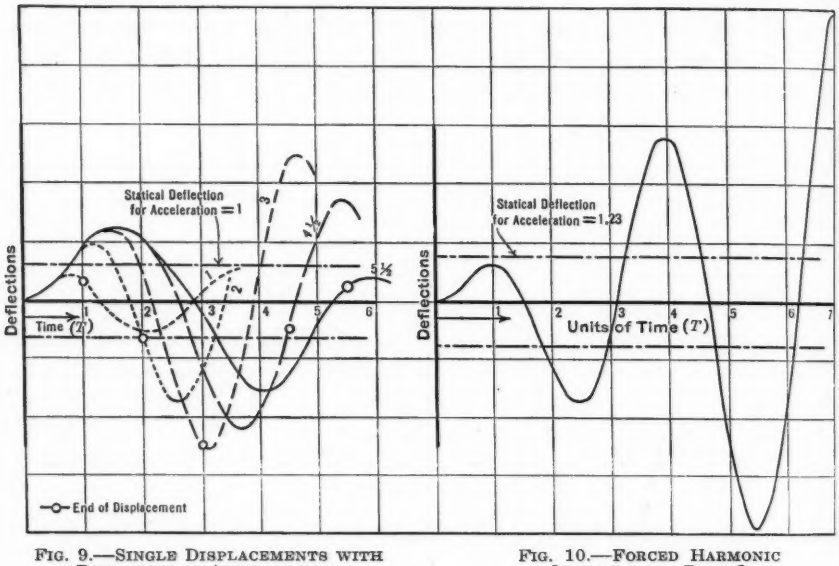

FIG. 9.—SINGLE DISPLACEMENTS WITH RECTANGULAR ACCELERATION.

FIG. 10.—FORCED HARMONIC OSCILLATION; $T = 3$.

The results of the investigation in another form are shown in Fig. 11. These curves refer to the stresses caused by equal displacements in different systems, the difference of the systems being respectively in the mass of the ball (Fig. 11 (*a*)), in the thickness of the assumed rod (Fig. 11 (*b*)), and in the shape of the assumed rectangular rod, of which the section modulus of the cross-section is constant (Fig. 11 (*c*)), so that the stress due to a "statical acceleration" has a constant value. In Fig. 11 (*c*), the rod is "designed for a certain acceleration", but has a varying rectangular cross-section. Each of these variations affects the period of free oscillation of the system, T, which varies from 0.6 to 3.3 times the duration of the displacement, D.

Only the maximum stresses, positive and negative, due to the maximum deflections to either side are plotted. The stresses are considered to be due to the bending of the rod, and shear is neglected. The stresses for the statical deflection are shown as dotted lines. Fig. 11 (*a*) shows that a greater mass causes a longer period of free oscillation and a greater stress. The increase in stress, however, is most rapid where the period equals the duration of the displacement. According to Fig. 11 (*b*) thinner rods give longer periods of oscillation and stresses increase rapidly till the period is one and one-half to two times the duration of the displacement. Fig. 11 (*c*) shows that the most unfavorable method of proportioning the ratio for a statical acceleration is that which makes the period of free oscillation equal to the duration of the displacement.

From the diagrams, the conclusion can be reached that in the case of one single displacement, stresses can be expected to surpass considerably (as much as four times) those due to a statical acceleration; and, further, that a ratio of about one between the period of free oscillation and the duration of the displacement has a very unfavorable effect on the resulting stresses. Probably these conclusions will hold also for rods and structures. It is dangerous, however, and not the intention of the writer to apply these results to more complicated bodies, although they may serve to formulate more definite questions on the subject of elastic structures.

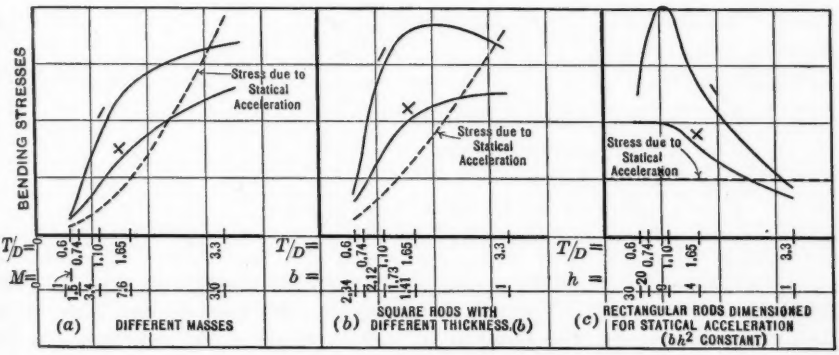
BENDING STRESSES
Stress due to Statical Acceleration
Stress due to Statical Acceleration
Stress due to Statical Acceleration
T/D = 0 0.6 0.74 1.10 1.65 3.3
M = ∞ 1.5 3.4 7.6 3.0
T/D = 0.6 0.74 1.10 1.65 3.3
b = 2.24 2.12 1.73 1.41 1
T/D = 0.6 0.74 1.10 1.65 3.3
h = 30 20 9 4 1
(a) DIFFERENT MASSES
(b) SQUARE RODS WITH DIFFERENT THICKNESS (b)
(c) RECTANGULAR RODS DIMENSIONED FOR STATICAL ACCELERATION (bh^2 CONSTANT)

FIG. 11.—EFFECT OF EQUAL DISPLACEMENTS ON DIFFERENT SYSTEMS.

The importance of the period of free oscillation of a structure is evident from the given considerations. It is still more so in the case of a forced oscillation; that is, in case the base of the structure is compelled, by the ground, to make a prolonged oscillatory motion. If the period of this forced motion is nearly equal to the period of free oscillation of the structure, and if the motion is continued, the stresses are very high and rupture is certain to occur. (See, also, Fig. 10).

It should be kept in mind that a rod has several periods of free oscillation, each corresponding to a certain mode of deformation: That without a node, that with one node, with two nodes, etc. The period decreases swiftly with the number of nodes.

The formulas, known for forced oscillation, apply only in the case of a thin rod of constant cross-section, fixed rigidly to the ground, and after the motion has become regular, that is, after an unknown number of oscillations. For all these reasons their application to the conditions of a building in an earthquake shock is somewhat unsafe. However, it is important that, in general, the stresses due to forced or free oscillation in the lower parts of the building are shown (by computation) to decrease less rapidly in the direction from the base to the top, than by a horizontal acceleration of the building as a whole, so that herein lies a possible explanation of rupture at great height.

It would be possible to gain an idea of the motion and stresses during the very first oscillations by considering a complex group of three or four

balls, attached to a light rod, instead of the single ball considered in this paper.

The need of methods to compute the period of free oscillation of an existing or a designed structure is clear. Computations can be checked and corrected by measurements of that period in existing buildings, as have been made already with the aid of seismographs. In a tall building with a narrow base, which may be subject to a rocking motion on compressible ground, the effect of this rocking should be eliminated.

A thin rod, with a constant cross-section, has a period of free oscillation without a node of:

$$T^2 = 0.32\,\pi^2\,\frac{L^3}{E\,I}\,M \qquad (1)$$

in which,

T = period of free oscillation;
M = total mass of the rod;
L = length of the rod;
I = moment of inertia of its cross-section;
E = modulus of elasticity.

The application of Equation (1) to buildings is dangerous, even for comparative purposes. In chimneys, for example, the mass is far from being uniformly distributed, the moment of inertia is far from constant, and the deformation is not only caused by the bending moments (as in the case of a thin rod), but also by the shear. In frame buildings the mass may be fairly uniformly distributed, but again the "rigidity" is not constant and the deformation is practically only determined by shear. In case of a deformation of frame bents due to shear, the character of the formula changes entirely, since then the length of the building does not appear in the third, but in the first, power.

If the deformation of a structure under a system of horizontal forces can be computed, then an estimate of the period of free oscillation is possible by the following method (see Fig. 7 (*c*)).

Divide the structure along its vertical axis in a number of (say, four or five) sections. By oscillation, each section executes a harmonic motion for which,

$$a = \frac{4\,\pi^2\,f}{T^2} \qquad (2)$$

in which, a = its acceleration at a certain moment; f = its lateral displacement from its position of equilibrium; and, T = period of oscillation, equal for all sections.

The acceleration, a, is equal to the force, P, acting on the section (due to the internal stresses in the structure), divided by the mass of the section. Therefore, for each section:

$$\frac{P}{M} = \frac{4\,\pi^2\,f}{T^2}$$

or,

$$f = \frac{P\,T^2}{4\,\pi^2\,M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

A second relation between the lateral displacements, f, of the sections, and the forces, P, is given by the elastic properties of the structure, and can be found, by ordinary engineering methods, by computing the deformation under the system of the horizontal (unknown) forces, P_1, P_2, etc., taken in the opposite direction. The result will have the form:

$$\left.\begin{aligned} f_1 &= a\,P_1 + b\,P_2 + c\,P_3 + \ldots\ldots \\ f_2 &= d\,P_1 + e\,P_2 + g\,P_3 + \ldots\ldots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which, a, b, c, etc., are numerical coefficients. Substituting f from Equation (3) in Equations (4), gives a set of equations with the unknowns, P_1, P_2, P_3, etc., and T, which is one too many. However, since Equations (4) refer to any moment of the oscillation, one force may be chosen arbitrarily, for example, $P_1 = 1$.

An exact solution of these equations, which are of the second order, is not possible; but by using the first two equations and considering only P_2 and T as unknowns, a lower and an upper limit for T can be found, in substituting for P_3, P_4, etc., first the lowest possible and second the highest possible values. The lowest values are evidently zero; the highest values depend on the kind of deformation and can be expressed in terms of P_2, since from Equation (3) it follows that:

$$f_3 : \frac{P_3}{M_3} = f_2 : \frac{P_2}{M_2}$$

and f_3 will certainly not be greater than f_2. In a case of bending, as in Fig. 7 (c), f_3 must be less than three-fifths of f_2, and f_4 less than one-fifth of f_2.

The limits of T will be found to be near enough together for the purpose. Greater accuracy can be attained, of course, by systematically changing the values of P_3, P_4, etc.

If the deformation is only due to shear, and the mass uniformly distributed (frame office buildings), the lateral displacements, f, during an oscillation, are very nearly equal to those resulting from a system of loads, decreasing uniformly from a certain (arbitrary) value at the top, to zero at the base of the structure. This gives a considerable "short-cut," since now for the top section the lateral displacement can be computed at once, and T can be found immediately by applying Equation (3).

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PAPERS AND DISCUSSIONS

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THE STIFFNESS OF SUSPENSION BRIDGES

By S. Timoshenko,* Esq.

Synopsis

In this paper a method is developed by which the additional cable stress and the deflection curve produced in a suspension bridge either by live load or by temperature variation may be directly calculated. Due to the quick convergence of the series used, the deflections and the bending moments in the span truss can be calculated in much shorter time than by the usual method.

I.—Introduction

The theory of the stiffened suspension bridge originated with Rankine and was developed on the assumption that the suspended truss remains absolutely rigid under the action of live load; further development was made by Professor Melan,† who first took into consideration the deflection of the span truss. The application of this theory to the design of large American suspension bridges, such as the Manhattan Bridge, in New York, N. Y., and the Philadelphia-Camden Bridge, showed‡ that this more exact theory is of great practical importance; furthermore it resulted in a considerable economy of material.

In applying Melan's theory to determine the additional horizontal component of the cable stress due to any cause, such as live load or temperature change, the assumption has been made that not only dead load but also additional load may be considered as uniformly distributed along the span. In this manner an equation has been established from which the additional cable stress can be calculated by a "cut-and-try" method. The assumption

Note.—Written discussion on this paper will be closed in September, 1928.

* Ann Arbor, Mich.

† This theory is discussed in detail in "The Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, Vol. II, p. 276.

‡ *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 919.

of uniform load distribution is sufficiently accurate provided the span is fully loaded, or nearly so; but a considerable error may occur when only a small part of the structure is loaded.*

In this paper a method is developed by which (1) the actual distribution of live load can be taken into consideration; and (2) the additional cable stress produced either by such a load or by temperature variation may be directly calculated. In using this method the deflection curve of the span truss is presented in the form of a trigonometrical series, the terms of which can easily be calculated from a consideration of the energy of the system.

II.—Differential Equations for the Deflection Curve of the Span Truss

Consider first the case shown in Fig. 1.† It is assumed that the initial curve of the cable is parabolic and that the entire dead load is carried by the cable and causes no stress in the stiffening trusses. The truss is only stressed by live load and by changes in temperature from the normal.

Let,

H_w = horizontal component of cable stress due to dead load and mean temperature.

H = additional horizontal component of cable stress, due to any cause, such as live load or temperature change.

$\beta = \dfrac{H}{H_w}$.

η = deflection of truss and cable at any point, from the normal position, due to H plus any given live load (the effect of the stretch of the hangers is neglected).

w = dead load per unit length = load on cable, including its own weight.

p = live load per unit length over a part or all of the span truss.

EI = flexural rigidity of the span truss, assumed to be constant along the span.

q = additional load on the cable due to live load or temperature change.

f = the sag of the cable.

l = the span of the stiffened truss.

The differential equation of the initial curve of the cable is:

$$H_w \frac{d^2 y}{d x^2} = -w \qquad (a)$$

By integration the known parabolic deflection curve is:

$$y = \frac{4 f x (l - x)}{l^2} \qquad (1)$$

When an additional vertical load, q, is acting on the cable, the differential equation of the curve of the cable becomes:

$$(H_w + H) \frac{d^2}{d x^2} (y + \eta) = -(w + q) \qquad (b)$$

* "The Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, Vol. II, p. 312.

† The notations and initial assumptions are the same as those contained in "The Theory and Practice of Modern Framed Structures."

From Equations (a) and (b) the load, q, transmitted to the cable will be obtained:

$$q = -H\frac{d^2}{d\,x^2}(y+\eta) - H_w\frac{d^2\,\eta}{d\,x^2}$$

or,

$$q = \beta\,w - H_w(1+\beta)\frac{d^2\,\eta}{d\,x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

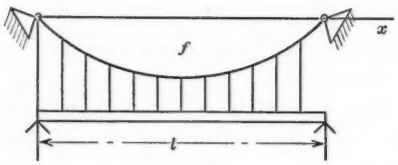

FIG. 1.

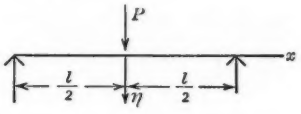

FIG. 2.

The load transmitted to the span truss will be:

$$p - q = p - \beta\,w + H_w(1+\beta)\frac{d^2\,\eta}{d\,x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Substituting this in the known differential equation of the deflection curve of a beam of uniform cross-section, the following equation for the deflection curve of the span truss will be obtained:

$$E\,I\frac{d^4\,\eta}{d\,x^4} = p - \beta\,w + H_w(1+\beta)\frac{d^2\,\eta}{d\,x^2} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

This is a linear differential equation with constant coefficients, which can easily be solved in any particular case provided the load, p, and the change, H, of the horizontal component of the cable stress are known.

III.—Particular Case

Assume that a concentrated load, P, is applied at the middle of the span truss (Fig. 2) and that the change, H, in the horizontal component of the cable stress can be neglected in comparison with H_w. Then, $p = 0$, $\beta = 0$, and Equation (4) becomes:

$$E\,I\frac{d^4\,\eta}{d\,x^4} - H_w\frac{d^2\,\eta}{d\,x^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The general solution of this equation is,

$$\eta = A\,e^{kx} + B\,e^{-kx} + C\,x + D \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which, A, B, C, and D are arbitrary constants, e is the Napierean base, and,

$$k = \sqrt{\frac{H_w}{E\,I}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

The arbitrary constants must now be determined from the conditions at the ends and at the middle of the span. At the end, $x = \frac{l}{2}$,

$$(\eta)_{x=\frac{l}{2}} = 0\,; \quad \left(\frac{d^2\,\eta}{d\,x^2}\right)_{x=\frac{l}{2}} = 0 \quad \ldots\ldots\ldots\ldots\ldots (c)$$

At the middle, the tangent must be horizontal and the shearing force equal to $-\frac{P}{2}$; therefore,

$$\left(\frac{d\,\eta}{d\,x}\right)_{x=0} = 0;\ E\,I\,\frac{d^3\,\eta}{d\,x^3} = \frac{P}{2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (d)$$

It is easy to show by differentiation that Equation (5) and the conditions, (*c*) and (*d*), will be satisfied by taking,

$$\eta = \frac{P}{2\,k\,H_w}\left(-\frac{\sinh k\left(\frac{l}{2}-x\right)}{\cosh\frac{k\,l}{2}} + k\left(\frac{l}{2}-x\right)\right) \qquad \ldots\ldots\ldots (8)$$

from which,

$$\eta_{\max.} = \frac{P}{2\,H_w}\left(\frac{l}{2} - \frac{1}{k}\tanh\frac{k\,l}{2}\right)$$

$$M_{\max.} = -\,E\,I\left(\frac{d^2\,\eta}{d\,x^2}\right)_{x=0} = \frac{P}{2\,k}\tanh\frac{k\,l}{2}$$

Take, for instance,*

$P = 100\,000$ lb.;
$E\,I = 29 \times 10^6 \times 43\,900 \times 144$ lb. per sq. in.;
$H_w = 10.48 \times 10^6$ lb.; and
$l = 1\,447.12$ in.

Then,

$$\frac{1}{k} = \sqrt{\frac{E\,I}{H_w}} = 349 \text{ ft.}$$

$$\eta_{\max.} = 22.0 \text{ in.};\ M_{\max.} = \frac{P}{2\,k}\tanh\frac{k\,l}{2} = 0.968\,\frac{P}{2\,k}$$

Comparing these results with the deflection,

$$\frac{P\,l^3}{48\,E\,I} = 59.5 \text{ in.}$$

and with the maximum bending moment,

$$M_{\max.} = \frac{P\,l}{4} = \frac{P \times 1\,447}{4} \text{ ft-lb.}$$

for a simply supported beam it can be concluded that due to the support provided by the cable the deflection of the span truss is reduced to about one-third and the maximum moment to about one-half that of an unsupported truss.

For a load, P, unsymmetrically applied or a distributed load, p, covering only a part of the span, the determination of the arbitrary constants of the general solution, Equation (6), becomes more complicated and a method based on the use of a trigonometrical series for the deflection curve (Section IV) is simpler for numerical calculations.

* These numerical data represent the middle span of the Manhattan Bridge.

IV.—Application of Trigonometrical Series

The deflection curve of a beam with simply supported ends (Fig. 3) always can be represented in the form of the trigonometrical series,

$$\eta = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{2 \pi x}{l} + a_3 \sin \frac{3 \pi x}{l} + \ldots\ldots\ldots (9)$$

in which a_1, a_2, a_3, . . ., are coefficients which can be calculated from a consideration of the energy of the system.

By using for the potential energy of bending according to the well-known equation,

$$U = \frac{E I}{2} \int_0^l \left(\frac{d^2 \eta}{d x^2} \right)^2 d x$$

substituting from Equation (9) for η; and taking into consideration that,

$$\int_0^l \sin \frac{m \pi x}{l} \sin \frac{n \pi x}{l} d x = 0$$

and,

$$\int_0^l \sin^2 \frac{m \pi x}{l} d x = \frac{l}{2}$$

the potential energy becomes,

$$U = \frac{E I \pi^4}{4 l^3} (a_1^2 + 2^4 a_2^2 + 3^4 a_3^2 + \ldots.) \ldots\ldots\ldots\ldots (10)$$

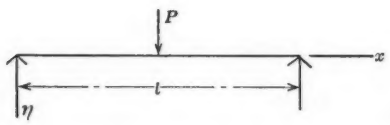

Fig. 3.

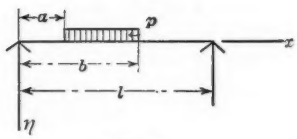

Fig. 4.

Assuming that a small increase, δa_n, is given to one of the coefficients, a_n, of the series in Equation (9), a small additional deflection, $\delta a_n \sin \frac{n \pi x}{l}$, will be superimposed on the deflection given by Equation (9); then, from the condition of equilibrium it can be concluded that the work done by loads acting on the beam during the displacement, $\delta a_n \sin \frac{n \pi x}{l}$, must be equal to the increase of the potential energy, Equation (10). In the case already considered the beam was under the action of a load, P, applied at the middle, and of a distributed load, $H_w \frac{d^2 \eta}{d x^2}$ (see Equation (5)), representing the action of the cable on the beam. The work done during the small displacement, $\delta a_n \sin \frac{n \pi x}{l}$, by the load, P, will be,

$$P \delta a_n \sin \frac{n \pi}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (e)$$

The work done by the distributed load, $H_w \dfrac{d^2\eta}{dx^2}$, will be,

$$\int_0^l H_w \frac{d^2\eta}{dx^2}\,\delta a_n \sin\frac{n\pi x}{l}\,dx$$

or, substituting for η from Equation (9), and considering that only the term containing the coefficient, a_n, gives an integral different from zero, the expression for this work becomes,

$$-H_w\,\delta a_n a_n \frac{n^2\pi^2}{2l} \qquad (f)$$

The increase of the potential energy, U, Equation (10), due to the small displacement, $a_n \sin\dfrac{n\pi x}{l}$, will be:

$$\frac{\delta U_n}{\delta a_n}\,\delta a_n = \frac{EI\pi^4 n^4}{2l^3}\,a_n\,\delta a_n \qquad (g)$$

Equating Equation (g) to the sum of Equations (f) and (e),

$$\frac{EI\pi^4 n^4}{2l^3}\,a_n = -H_w a_n \frac{n^2\pi^2}{2l} + P\sin\frac{n\pi}{2}$$

from which, when n is odd,

$$a_n = \frac{2Pl^3}{EI\pi^4}\,\frac{\sin\dfrac{n\pi}{2}}{n^4+n^2\alpha} \qquad (h)$$

and, when n is even,

$$a_n = 0$$

in which,

$$\alpha = \frac{H_w l^2}{EI\pi^2}* \qquad (11)$$

Substituting Equation (h) in Equation (9),

$$\eta = \frac{2Pl^3}{EI\pi^4}\left(\frac{\sin\dfrac{\pi x}{l}}{1+\alpha} - \frac{\sin\dfrac{3\pi x}{l}}{3^4+3^2\alpha} + \frac{\sin\dfrac{5\pi x}{l}}{5^4+5^2\alpha}\cdots\right)$$

The deflection at the middle $\left(x=\dfrac{l}{2}\right)$ will be,

$$\eta_{max.} = \frac{2Pl^3}{EI\pi^4}\left(\frac{1}{1+\alpha} + \frac{1}{81+9\alpha} + \frac{1}{625+25\alpha} + \cdots\right)$$

The series is quickly convergent and very convenient for numerical calculations.

Applying this to the previous numerical example,

$$\alpha = \frac{H_w l^2}{EI\pi^2} = 1.75$$

*The term, α, has a very simple meaning. It represents the ratio of the horizontal component, H_w, of the cable stress to the column load of the span truss.

and,

$$\eta_{\max.} = \frac{2\,P\,l^3}{E\,I\,\pi^4}\left(\frac{1}{2.75} + \frac{1}{96.9} + \frac{1}{668.7} + \ldots\ldots\right) = 0.376\,\frac{2\,P\,l^3}{E\,I\,\pi^4} = 22.0 \text{ in.}$$

Within the accuracy of the slide-rule this result coincides with the former solution.

This method can also be applied to the general case given by the differential Equation (4), in which the load acting on the truss is given by Equation (3). Taking, again, the series of Equation (9) for the deflection curve and assuming that the live load, p, is uniformly distributed between the limits, $x = a$ and $x = b$, as shown in Fig. 4, the work done by the live load on a small displacement, $\delta\,a_n \sin\frac{n\,\pi\,x}{l}$, will be,

$$p\,\delta\,a_n \int_a^b \sin\frac{n\,\pi\,x}{l}\,d\,x = \delta\,a_n\,\frac{p\,l}{n\,\pi}\left(\cos\frac{n\,\pi\,a}{l} - \cos\frac{n\,\pi\,b}{l}\right)$$

The work done by the reactions from the cable on the truss will be,

$$-\delta\,a_n\,\beta\,w\int_0^l \sin\frac{n\,\pi\,x}{l}\,d\,x + \delta\,a_n\,H_w\,(1+\beta)\int_0^l \frac{d^2\,\eta}{d\,x^2}\sin\frac{n\,\pi\,x}{l}\,d\,x$$
$$= -\delta\,a_n\,\frac{l}{n\,\pi}\,\beta\,w\,(1-\cos n\,\pi) - a_n\,\delta\,a_n\,H_w\,(1+\beta)\,\frac{n^2\,\pi^2}{2\,l}$$

The equation for determining a_n becomes,

$$\frac{p\,l}{n\,\pi}\left(\cos\frac{n\,\pi\,a}{l} - \cos\frac{n\,\pi\,b}{l}\right) - \frac{l}{n\,\pi}\,\beta\,w\,(1-\cos n\,\pi)$$
$$- a_n\,H_w\,(1+\beta)\,\frac{n^2\,\pi^2}{2\,l} = \frac{n^4\,E\,I\,\pi^4}{2\,l^3}\,a_n$$

from which,

$$a_n = \frac{\dfrac{p\,l}{n\,\pi}\left(\cos\dfrac{n\,\pi\,a}{l} - \cos\dfrac{n\,\pi\,b}{l}\right) - \dfrac{l}{n\,\pi}\,\beta\,w\,(1-\cos n\,\pi)}{\dfrac{n^4\,E\,I\,\pi^4}{2\,l^3} + H_w\,(1+\beta)\dfrac{n^2\,\pi^2}{2\,l}}$$

or, when n is odd,

$$a_n = \frac{2\,l^4}{E\,I\,\pi^5\,(n^5 + n^3\,\alpha\,(1+\beta))}\left(p\left(\cos\frac{n\,\pi\,a}{l} - \cos\frac{n\,\pi\,b}{l}\right) - 2\,\beta\,w\right)$$

and, when n is even,

$$a_n = \frac{2\,p\,l^4}{E\,I\,\pi^5\,(n^5 + n^3\,\alpha\,(1+\beta))}\left(\cos\frac{n\,\pi\,a}{l} - \cos\frac{n\,\pi\,b}{l}\right) \quad \Big\}\ .(12)$$

Substituting from Equations (12) in Equation (9), a quickly convergent series will be obtained from which the deflections can be calculated for any distribution of live load, provided that the additional horizontal component, H, of the cable stress is known.

Assuming, for instance, that the live load covers only one-quarter of the span to the left support of the truss, and substituting $a = 0$ and $b = \frac{l}{4}$ in

Equations (12), the deflection curve is obtained in the form of the following series:

$$\eta = \frac{2\,l^4}{E\,I\,\pi^5}\left(\frac{p\left(1-\cos\frac{\pi}{4}\right)-2\,\beta\,w}{1+\alpha\,(1+\beta)}\sin\frac{\pi\,x}{l}+\frac{p\sin\frac{2\,\pi\,x}{l}}{2^5+2^3\,\alpha\,(1+\beta)}\right.$$
$$\left.+\frac{p\left(1-\cos\frac{3\,\pi}{4}\right)-2\,\beta\,w}{3^5+3^3\,\alpha\,(1+\beta)}\sin\frac{3\,\pi\,x}{l}+\ldots\right)\ldots\ldots\ldots\ldots(13)$$

from which, the deflection can be calculated if $\beta = \frac{H}{H_w}$ is known.

For calculating the bending moments the second derivative of Equation (13) must be calculated and substituted in the known equation, $M = -E\,I\,\frac{d^2\,\eta}{d\,x^2}$.

If there is no live load and if the effects of temperature alone on the deflection are being considered, then from Equations (12), when n is odd,

$$a_n = -\frac{4\,\beta\,w\,l^4}{E\,I\,\pi^5\,(n^5+n^3\,\alpha\,(1+\beta))}$$

and, when n is even,

$$a_n = 0$$

The deflection curve will then be,

$$\eta = -\frac{4\,\beta\,w\,l^4}{E\,I\,\pi^5}\left(\frac{\sin\frac{\pi\,x}{l}}{1+\alpha\,(1+\beta)}+\frac{\sin\frac{3\,\pi\,x}{l}}{3^5+3^3\,\alpha\,(1+\beta)}\right.$$
$$\left.+\frac{\sin\frac{5\,\pi\,x}{l}}{5^5+5^3\,\alpha\,(1+\beta)}+\ldots\right)\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

The minus sign indicates that an increase in cable stress, ($\beta > 0$), produces a deflection of the truss in an upward direction.

V.—Determination of the Additional Horizontal Component of Cable Stress

To determine H, the additional cable stress due to live load or to change in temperature, the change in potential energy of the cable must be calculated. The initial tensile force in the cable for any section is,

$$H_w\,\frac{d\,s}{d\,x}$$

and the potential energy of tension in an element, $d\,s$, will be,

$$\frac{\left(H_w\,\frac{d\,s}{d\,x}\right)^2}{2\,A\,E}\,d\,s$$

in which, A is the area of the cross-section of the cable.

The increase of this energy, due to an additional horizontal component, H, will be,

$$\frac{(H_w + H)^2 \left(\frac{d\,s}{d\,x}\right)^2 d\,s}{2\,A\,E} - \frac{H_w{}^2 \left(\frac{d\,s}{d\,x}\right)^2 d\,s}{2\,A\,E} = \frac{H\left(H_w + \frac{1}{2}H\right)}{A\,E} \frac{d\,s^3}{d\,x^2}$$

so that the complete change in the energy of the cable will be:

$$\frac{H\left(H_w + \frac{1}{2}H\right)}{A\,E} \int_0^l \frac{d\,s^3}{d\,x^2} = \frac{H\left(H_w + \frac{1}{2}H\right)}{A\,E} L \quad \ldots\ldots\ldots (15)$$

in which,

$$L = \int_0^l \frac{d\,s^3}{d\,x^2}$$

Taking as the basis of calculation the parabolic curve given by Equation (1),

$$L = l\left[\frac{1}{4}\left(\frac{5}{2} + \frac{16\,f^2}{l^2}\right)\left(1 + \frac{16\,f^2}{l^2}\right)^{\frac{1}{2}} + \frac{3}{32}\log_e\left(\frac{4\,f}{l} + \left(1 + \frac{16\,f^2}{l^2}\right)^{\frac{1}{2}}\right)\right] \quad \ldots\ldots\ldots\ldots (16)$$

The increase in energy, Equation (15), must be equal to the work done by the load acting on the cable. The intensity of the initial load is w. The intensity of the final load is (Equation (2)),

$$w + q = w + \beta\,w - H_w\,(1 + \beta)\,\frac{d^2\,\eta}{d\,x^2}$$

Assuming that, for small variations in cable stress, the deflections are proportional to the load, an average value of the intensity of the load must be taken when calculating the work done. Hence,

$$T = \int_0^l \left(w + \frac{q}{2}\right)\eta\,d\,x = \int_0^l \left(w + \frac{w\,\beta}{2} - \frac{H_w}{2}(1 + \beta)\,\frac{d^2\,\eta}{d\,x^2}\right)\eta\,d\,x$$

Substituting from Equation (9) for η,

$$T = \left(w + \frac{w\,\beta}{2}\right)\frac{2\,l}{\pi}\left(a_1 + \frac{a_3}{3} + \frac{a_5}{5} + \ldots.\right) + \pi^2\,\frac{H_w}{4\,l}\,(1 + \beta)\,(a_1{}^2 + 2^2\,a_2{}^2 + 3^2\,a_3{}^2 + \ldots.) \quad \ldots\ldots\ldots (17)$$

Equating this to the increase in potential energy of the cable,

$$\frac{H_w{}^2\,\beta\left(1 + \frac{1}{2}\beta\right)}{A\,E}\,L = \left(w + \frac{w\,\beta}{2}\right)\frac{2\,l}{\pi}\left(a_1 + \frac{a_3}{3} + \frac{a_5}{5} + \ldots.\right) + \frac{\pi^2\,H_w}{4\,l}\,(1 + \beta)\,(a_1{}^2 + 2^2\,a_2{}^2 + 3^2\,a_3{}^2 + \ldots.)$$

or,

$$\frac{H_w L}{A E}\beta\left(1+\frac{1}{2}\beta\right)=\frac{16 f}{\pi l}\left(1+\frac{\beta}{2}\right)\left(a_1+\frac{a_3}{3}+\frac{a_5}{5}+\ldots.\right)$$
$$+\frac{\pi^2}{4 l}(1+\beta)\left(a_1{}^2+2^2 a_2{}^2+3^2 a_3{}^2+\ldots.\right)\ldots\ldots\ldots\ldots(18)$$

The coefficients, a_1, a_2, a_3, . . ., can be calculated in each particular case as already explained (Section IV), so that Equation (18) establishes a definite relation between β and the live load, p. The simplest way to solve this equation for any particular load distribution is to assume a certain value for β, that is, a certain increase in horizontal component of cable stress and to calculate, from Equation (18), the intensity of the live load, p, necessary to produce such an increase in cable stress. Then assuming, for small fluctuation in cable stress, that β is proportional to p, the additional cable stress for any value of p can easily be calculated.

In order to show the procedure take the numerical example already considered, and assume that, $f = 145.3$ ft.; $A = 275$ in. square = area of cross-section of cable; and $w = 5\,820$ lb. per lin. ft. = intensity of dead load. Then, using the previous data,

$$\alpha=\frac{H_w l^2}{E I \pi^2}=1.75$$

and,

$$L=\int_0^l \frac{d s^3}{d x^2}=1\,564 \text{ ft.}$$

Consider the case where the live load covers only one-quarter of the span to the left support of the truss and the deflection curve is represented by Equation (13). Assume that the intensity, p, of the live load is such that a definite increase in cable stress is produced. Take, for instance,

$$\beta=\frac{H}{H_w}=0.2$$

Then the series, Equation (13), becomes,

$$\eta=\frac{2 l^4}{E I \pi^5}\left(\frac{0.2929\,p-0.4\,w}{1+1.75\times 1.2}\sin\frac{\pi x}{l}+\frac{p\sin\frac{2\pi x}{l}}{2^5+2^3\times 1.75\times 1.2}\right.$$
$$+\frac{1.7071\,p-0.4\,w}{3^5+3^3\times 1.75\times 1.2}\sin\frac{3\pi x}{l}+\frac{2p\sin\frac{4\pi x}{l}}{4^5+4^3\times 1.75\times 1.2}$$
$$\left.+\frac{1.7071\,p-0.4\,w}{5^5+5^3\times 1.75\times 1.2}\sin\frac{5\pi x}{l}+\ldots.\right)\ldots\ldots\ldots\ldots(19)$$

and, hence,

$$a_1+\frac{1}{3}a_3+\frac{1}{5}a_5+\ldots=\frac{2 l^4}{E I \pi^5}(0.0965\,p-62.81)$$

and,

$$a_1{}^2+2^2 a_2{}^2+3^2 a_3{}^2+\ldots=\left(\frac{2 l^4}{E I \pi^5}\right)^2(0.01096\,p^2-11.89\,p+3\,920)$$

Substituting these values in Equation (18), the following quadratic equation for determining p, will be obtained,

$$5.45 = 1.827(0.0965\ p - 62.81) + 0.001797(0.01096\ p^2 - 11.89\ p + 3\,920)$$

or,

$$0.0000197\ p^2 + 0.1555\ p - 113.4 = 0$$

from which,

$$p = 671 \text{ lb. per lin. in.}$$

This is the intensity of the live load necessary to produce an increase, H, in the horizontal component equal to 0.2 H_w. The increase in cable stress for any other intensity of live load can easily be calculated. Assume, for instance, that a live load of 4 000 lb. per lin. ft. is covering the truss between the limits, $x = 0$ and $x = \frac{l}{4}$; then the corresponding increase in the horizontal component of the cable stress will be,

$$H = \frac{0.2\ H_w \times 4\,000}{12 \times 671} = 1\,040\,000 \text{ lb.}$$

In these calculations it has been assumed that β varies in the same proportion as the load, p; but as can be seen from the fundamental Equation (18), this assumption is not strictly correct. In order to show what errors may be involved in such an assumption additional calculations were made for the load covering only one-quarter of the span to the left support of the truss and in two different ways.* In the first case, after calculating p for $\beta = 0.05$, in the same manner as before, the values of p for other values of β were found on the assumption that there is a simple proportionality between β and p. In this manner the figures in the second line of the following tabulation were obtained. In the second case, the loads, p', were calculated from Equation (18) independently for each value of β.

Value of β =	0.05	0.1	0.2	0.4
Value of p =	169.4	338.8	677.6	1 355
Value of p' =	169.4	338.5	673.0	1 322

It is seen from this tabulation that there is a slight deviation from the proportionality between β and p, namely, β is increasing somewhat more rapidly than p. This means that with the increase in load, p, a large proportional part of the load will be taken by the cable. At the same time this deviation from porportionality is small and a conclusion can be made that in calculating p from Equation (18) a rough approximation for β will be sufficient for obtaining p with a satisfactory accuracy.

In the deviation of the general expression for the work done by the load acting on the cable (see Equation (17)), the average value, $w + \frac{q}{2}$, for the load was used. This is strictly correct only when the deflections are proportional to the loads. To find what errors are involved in this method of calcula-

* These calculations were made by Professor G. C. Priester. A more accurate value for α, namely, 174.6, was taken in this case and all calculations were made to four significant figures.

tion an additional investigation was made by dividing the process of loading into two steps, each step representing the loading by a load of one-half the final intensity. In calculating the cable stress for the first step, the previous method was used. For the second step the calculations of the cable stress were made by taking into account the changes in the cable stress and in the loads produced by the loadings of the first step. These calculations showed that the errors involved in this method of determining the work done by the loads acting on the cable are very small for the usual values of β and can be neglected.

In addition, it is interesting to note that the second term on the right side of Equation (18), representing the influence of deflection of the truss on the distribution of the load transmitted to the cable, is usually small. This fact explains why the ordinary theory of suspension bridges, which assumes that the load acting on the cable is uniformly distributed, yields satisfactory results.

VI.—The Case of Three Spans

The method just developed can be applied in the case of three spans, such as shown in Fig. 5. Assuming that the cable can move without friction at the top of the towers, the horizontal component of the cable stress will be constant along all three spans. For calculating the deflections of the middle span the equations already derived can be used.

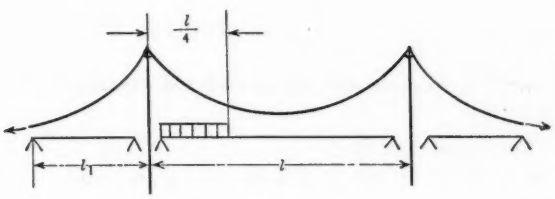

Fig. 5.

In considering the side spans let,

l_1 = length of side span;
w_1 = dead load of side span; and
EI_1 = flexural rigidity of side-span truss.

Taking the deflection curve of the side-span truss in the form of the series:

$$\eta_1 = b_1 \sin \frac{\pi x}{l_1} + b_2 \sin \frac{2 \pi x}{l_1} + b_3 \sin \frac{3 \pi x}{l_1} + \ldots\ldots\ldots\ldots (9a)$$

in which, x is measured from the left end of the span. The coefficients b_1, b_2, ..., can be calculated by using equations similar to Equations (12). Assuming, for instance, that there is no live load on the side spans (Fig. 5), Equation (14) becomes

$$\eta_1 = -\frac{4 \beta w_1 l_1^4}{E I_1 \pi^5} \left(\frac{\sin \frac{\pi x}{l}}{1 + \alpha_1 (1 + \beta)} + \frac{\sin \frac{3 \pi x}{l}}{3^5 + 3^3 \alpha_1 (1 + \beta)} + \ldots \right) \ldots (14a)$$

in which,

$$\alpha_1 = \frac{H_w\, l_1^2}{E\, I_1\, \pi^2}$$

For calculating the additional horizontal component, H, of the cable stress an equation analogous to Equation (18) can be derived. For this purpose, it is only necessary to put the complete length of the cable for L on the left side of Equation (18) and add to the right side the work done by loads acting on the side parts of the cable. This latter work will be:

$$T_1 = 2 \int_0^{l_1} \left(w_1 + \frac{w_1\, \beta}{2} - \frac{H_w}{2} (1 + \beta) \frac{d^2\, \eta_1}{d\, x^2} \right) \eta_1\, d\, x$$

Substituting the series, Equation (9a) for η_1,

$$T_1 = 2 \left(w_1 + \frac{w_1\, \beta}{2} \right) \frac{2\, l_1}{\pi} \left(b_1 + \frac{1}{3}\, b_3 + \frac{1}{5}\, b_5 + \ldots \right)$$

$$+ \frac{\pi\, 2\, H_w}{2\, l_1} (1 + \beta)\, (b_1^2 + 2^2\, b_2^2 + 3^2\, b_3^2 + \ldots.) \ldots\ldots\ldots (20)$$

The equation for determining H will be,

$$\frac{H_w^2\, L}{A\, E}\, \beta \left(1 + \frac{1}{2}\, \beta \right) = \left(w_1 + \frac{w_1\, \beta}{2} \right) \frac{4\, l_1}{\pi} \left(b_1 + \frac{1}{3}\, b_3 + \frac{1}{5}\, b_5 \right)$$

$$+ \frac{\pi^2\, H_w}{2\, l_1} (1 + \beta)\, (b_1^2 + 2^2\, b_2^2 + 3^2\, b_3^2 + \ldots) + \left(w + \frac{w\, \beta}{2} \right) \frac{2\, l}{\pi}$$

$$\left(a_1 + \frac{1}{3} a_3 + \frac{1}{5}\, a_5 + \ldots \right) + \frac{\pi^2\, H_w}{4\, l} (1 + \beta)\, (a_1^2 + 2^2\, a_2^2 + 3^2\, a_3^2 + \ldots). \quad (21)$$

For the numerical example already considered assume that,

$l_1 = 713.5$ ft.
$w_1 = 6\,130$ lb. per lin. ft.
$E\, I_1 = 29 \times 10^6 \times 50\,860 \times 144$ lb. per sq. in.; and
$L = 3\,452$ ft.

Then, assuming the loading shown in Fig. 5 the deflection curve of the middle span truss will be given by Equation (19) as before. The deflection curves for the side span trusses will be represented by Equation (14a), so that, substituting the numerical values, and taking $\beta = 0.2$,

$$\alpha_1 = \frac{H_w\, l_1^2}{E\, I_1\, \pi^2} = 0.3664$$

and,

$$\eta_1 = -\ 23.46 \sin \frac{\pi\, x}{l} - 0.132 \sin \frac{3\, \pi\, x}{l} + \ldots..$$

then,

$$b_1 + \frac{1}{3}\, b_3 + \ldots.. = -\ 23.5 \text{ in.}; \quad b_1^2 + 3^2\, b_3^2 + \ldots = 552 \text{ in. square.}$$

Substituting in Equation (21),

$$11.97 = 1.827\ (0.0965\ p - 62.81) + 0.001797\ (0.01096\ p^2 - 11.89\ p + 3\,920)$$
$$- 13.8 + 0.38$$

or,

$$0.0000197\ p^2 + 0.1555\ p - 133.3 = 0$$

from which,

$$p = 780 \text{ lb. per lin. in.}$$

If a load of the intensity of 4 000 lb. per ft. is acting on the part of the middle span shown in Fig. 5, the increase in the horizontal component will be,

$$H = \frac{0.2\ H_w \times 4\ 000}{12 \times 780} = 897\ 000 \text{ lb.} \pm$$

VII.—Effect of Change in Temperature

The same method can also be used in calculating the additional horizontal component, H, of the cable stress produced by a change in temperature. Let,

$\omega = 66 \times 10^{-7}$ = coefficient of linear expansion per degree Fahrenheit; and

$$L_t = \int_0^l \frac{d\ s}{d\ x}\, d\ s$$

Then the work in the cable due to temperature change is:*

$$\int_0^l \left(H_w + \frac{H}{2}\right) \frac{d\ s}{d\ x}\, \omega\ t\ d\ s = H_w \left(1 + \frac{\beta}{2}\right) \omega\ t\ L_t$$

Using for the change in potential energy of the cable, Equation (10), as before, the total work in the cable becomes,

$$\frac{H_w^2\ \beta \left(1 + \frac{\beta}{2}\right)}{A\ E} L + H_w \left(1 + \frac{\beta}{2}\right) \omega\ t\ L_t$$

This work must be equal to that done by the vertical load acting on the cable. For this later work Equations (17) and (20) will be used. The equation for determining H will be,

$$\frac{H_w^2\ \beta \left(1 + \frac{\beta}{2}\right)}{A\ E} L + H_w \left(1 + \frac{\beta}{2}\right) \omega\ t\ L_t = \left(w + \frac{w\ \beta}{2}\right) \frac{2\ l}{\pi}$$
$$\left(a_1 + \frac{1}{3}\ a_3 + \frac{1}{5}\ a_5 + \ldots\right) + \frac{\pi^2\ H_w}{4\ l}\ (1 + \beta)\ (a_1^2 + 2^2\ a_2^2 + 3^2\ a_3^2 + \ldots)$$
$$+ \left(w_1 + w_1 \frac{\beta}{2}\right) \frac{2\ l_1}{\pi} \left(b_1 + \frac{1}{3}\ b_3 + \frac{1}{5}\ b_5 + \ldots\right)$$
$$+ \frac{\pi^2\ H_w}{4\ l_1}\ (1 + \beta)\ (b_1^2 + 2^2\ b_2^2 + 3^2\ b_3^2 + \ldots\ldots) \ldots\ldots \quad \ldots(22)$$

Applying these equations, as before, to a bridge with three spans and assuming that $\beta = 0.2$ and $L_t = \int \frac{ds^2}{d\ x} = 3\ 324$ ft., and letting $p = 0$,

$$11.97 + 1.1\ \omega\ t \times 3\ 324.12 = -1.827 \times 62.81 + 0.001979 \times 3\ 920$$
$$- 13.8 + 0.38$$

* It is assumed that the fluctuations in the cable stress are small; and the average stress, $H_w + \frac{H}{2}$, is taken in calculations.

or,

$$1.1 \times 3\,324 \times 12 \times \omega\, t = -133.3$$

from which,

$$\omega\, t = -0.00304$$

Assuming that $\omega = 66 \times 10^{-7}$ in. per in. per degree Fahrenheit,

$$t = -461° \text{ Fahr.}$$

Now the additional horizontal component, H, for any variation in temperature can easily be calculated. Assuming, for instance, that the temperature increased 55° Fahr. above normal,

$$H = -\frac{H_w \times 0.2 \times 55}{461} = -250\,000 \text{ lb.}$$

Combining this effect of the increase in temperature with that already calculated, due to the load distributed as shown in Fig. 5, the complete change in the horizontal component of cable stress will be,

$$H = 897\,000 - 250\,000 = 647\,000 \text{ lb.}$$

which differs only about 2½% from the result given by Messrs. Johnson, Bryan, and Turneaure.* This agreement seems satisfactory considering that all the calculations were made with a slide-rule and that an approximate value for the coefficient of thermal expansion was used. A better approximation will be obtained by using the given value of $E\,I$ as a first trial and by calculating a more accurate value of β from Equation (22).

Conclusions

It has been shown that by using a trigonometric series for the deflection curve of the span truss a direct calculation of the change in the horizontal component of the cable stress becomes possible; and that the actual distribution of the live load can easily be taken into account in this calculation. Due to the quick convergence of the series representing the deflection curves, the calculation of the deflections and of the bending moments in the span truss can be made without difficulty and in a much shorter time than by the usual method.

* "The Theory and Practice of Framed Structures," Vol. II, p. 296.

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PAPERS AND DISCUSSIONS

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NOTES ON ARCHED GRAVITY DAMS

Discussion*

BY B. F. JAKOBSEN, M. AM. SOC. C. E.†

B. F. JAKOBSEN,‡ M. AM. SOC. C. E. (by letter).§—Mr. Rich‖ has checked Equation (7)¶ by a more elegant derivation and agrees that curvature should not be disregarded unless the dam is properly grouted under pressure.

Mr. Floris** presents a somewhat different method of arriving at the same result.

Mr. Jorgensen's discussion†† will be studied with considerable interest especially his Figs. 5‡‡ and 6§§.

Mr. Werner develops formulas for the interesting condition of a dam curved up stream instead of down stream.‖‖

Mr. Bauman insists that gravity dams are monolithic structures,¶¶ homogeneous and isotropic, and that arch action, therefore, must take place and may be determined in the usual manner. If shrinkage did not occur, there would be no reason for providing contraction joints and there would be no explanation for the large cracks which have appeared in the Mulholland Dam.*** This dam was not provided with contraction joints, but an effort was made to produce a monolithic structure. Unless the writer knows definitely that arch action must be present, he prefers to neglect it, because this is the safe

* Discussion on the paper by B. F. Jakobsen, M. Am. Soc. C. E., continued from January, 1928, *Proceedings*.
† Author's closure.
‡ Cons. Engr. (LaRue & Jakobsen), Los Angeles, Calif.
§ Received by the Secretary, January 24, 1928.
‖ *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 2060.
¶ *Loc. cit.*, August, 1927, Papers and Discussions, p. 1138.
** *Loc. cit.*, October, 1927, Papers and Discussions, p. 2061.
†† *Loc. cit.*, November, 1927, Papers and Discussions, p. 2355.
‡‡ *Loc. cit.*, p. 2356.
§§ *Loc. cit.*, p. 2358.
‖‖ *Loc. cit.*, p. 2360.
¶¶ *Loc. cit.*, January, 1928, Papers and Discussions, p. 219.
*** *Loc. cit.*, p. 246.

assumption—and a safe assumption regarding arch action is particularly requisite for gravity dams, such as the Exchequer Dam. Uplift was neglected at this dam, although it may exist in spite of the drainage pipes provided. Commenting on this subject, Alfred D. Flinn,* M. Am. Soc. C. E., Director of Engineering Foundation, writes,

"Some reduction of uplift may be gained also, but drainage passages cannot be depended on permanently because the leaching and depositing of salts from the masonry tend to seal the designed and accidental passages."

Likewise, in arch dam design the writer prefers to ignore the effect of the cantilevers,† and assume that the arches carry the entire load.

Mr. Bauman considers the deflection of only one cantilever, that at the crown, which is hardly sufficient,‡ and he appears to have applied Professor Cain's deflection formula for arches to the primary or original arches instead of to the secondary arches, which procedure may be equivalent to assuming that tension can be transmitted across contraction joints.§ In this manner Mr. Bauman estimates the arch action as much greater than it actually is.

Professor Cain shows‖ that when the up-stream face is sloped, as is customary, the effect of curving the dam is to decrease the compression, and this checks what the writer found for a vertical up-stream face. Professor Cain concurs in the writer's conclusion, that it is dangerous to neglect curvature in a curved gravity dam.

Mr. Hanna¶ quotes Wegmann to the effect that the Habra and Bouzey Dams were not arched in plan and that poor foundation was the main reason for their failure. The writer did not state that they were arched in plan, but that they failed from tension in the up-stream face and, as he knew nothing about the matter from personal observation, he referred to the source from which he quoted.**

Dr. Unwin has stated that both dams were built of somewhat inferior material and that in each the line of the resultant fell considerably outside the middle third, so that, on the ordinary theory, there was considerable tension in the up-stream face. Both dams, he has stated, failed about half way up the dam, along a line such as AB in Fig. 19, and he was agreed with the French engineers, that in both dams the initial fracture was a tension fracture, which occurred at the point where the tension was greatest, according to the law of the middle third.

Mr. Hanna takes the writer to task for using a radial-sided slice, 1 ft. wide, at the up-stream face and contends that this method cannot yield reliable results.†† He states further:‡‡

* *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 249.

† *Loc. cit.*, Vol. 90 (1927), p. 520.

‡ "Etude sur les barrages arqués," Extrait du *Bulletin de la Suisse Romande* (1922), by Alfred Stucky.

§ See the writer's discussion of the paper by Frederick Hall Fowler, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2783.

‖ *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 228.

¶ *Loc. cit.*, p. 232.

** *Loc. cit.*, August, 1927, Papers and Discussions, p. 1137; see, also, American Civil Engineers' Handbook," Fourth Edition (1920), Section 7, Article 8, p. 686.

†† *Loc. cit.*, January, 1928, Papers and Discussions, p. 233.

‡‡ *Loc. cit.*, p. 237.

"* * * they should not be allowed to displace the customary and reliable method of computing stresses in arched gravity dams by the parallel-sided slice theory."

A dam designed by the radial-sided slice method is safe when tested by the parallel-sided slice theory, but the reverse is not true. The writer showed this for a vertical up-stream face,* and Professor Cain has shown† that it remains true when the up-stream face is sloped. Therefore, in the absence of some conclusive proof that the parallel-sided slice theory is correct, the writer prefers the radial-sided slice theory, because it is the safer of the two assumptions.

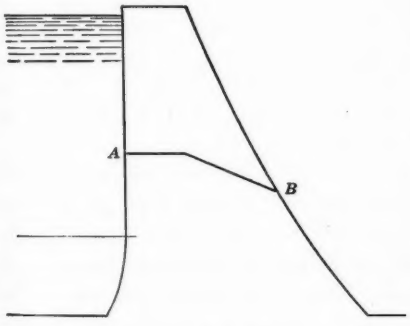

FIG. 19.

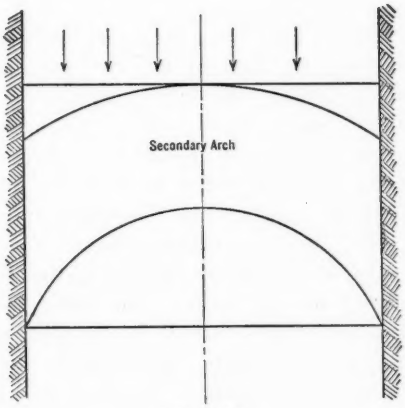

FIG. 20.

Mr. Hanna assumes a monolithic structure, that is, he denies that shrinkage occurs, and then reasons that a radial-sided slice cannot deflect without encountering arch resistance. That is absolutely correct, but he overlooks the essential fact that it is equally correct for a parallel-sided slice; that is, for a straight gravity dam, provided it is monolithic. A parallel-sided slice of a straight gravity dam is not independent of the remainder of the structure for two reasons:

1.—A set of secondary horizontal arches, free from tension, exist and will resist displacements of the cantilevers in a radial direction; and,

2.—The adjoining cantilevers are not generally of the same length or height as the one under consideration, and, therefore, the deflections at the same level of these three cantilevers are different, so that one re-acts on the others.

As an illustration, take a straight gravity dam, 200 ft. high, with 50 ft. between the vertical abutments.‡ Such a dam would act mainly as an arch dam, notwithstanding its straight up-stream face (see Fig. 20), provided it is monolithic, as assumed by Mr. Hanna.

* *Proceedings*, Am. Soc. C. E., August, 1927, Papers and Discussions, p. 1135.

† *Loc. cit.*, January, 1928, Papers and Discussions, p. 228.

‡ Résal, "Formes et Dimensions des grands barrages en maçonnerie," *Annales des Ponts et Chaussées*, II, 1919, p. 26.

Equation 55* gives the deflection of a cantilever due to bending and shear. Assume that the adjoining cantilever is longer by $d\,x$, so that, for this cantilever, $H + d\,x$ and $h + d\,x$ should be substituted for H and h in the equation in order to obtain its deflection at the same elevation, $H - h$ ft. from the crest (see Fig. 14).† The difference between the deflections may be obtained most conveniently by differentiating Equation (55), bearing in mind that the ratio, $\frac{H}{B}$, remains constant. This gives the additional deflection, $d\,\Delta_h$, due to the additional length, $d\,x$,

$$d\,\Delta_h = \left(\frac{62.5\,H^3}{E\,B^3}\,2\,h + \frac{46.9}{E\,B}\,2\,H\right) d\,x \ldots\ldots\ldots\ldots\ldots (76)$$

This means, even if arch action is not present, that when the abutment is steep, so that in a short horizontal distance a considerable change in height occurs, the longer cantilever tends to deflect more than the shorter one. Under this condition an overloading of the shorter cantilever may occur, which may result in uplift under the shorter cantilever. For this reason, also, it is well to insist, as does Professor Cain, that the base should always be thicker than the middle-third theorem prescribes.‡

Mr. Hanna develops formulas for the effect of curvature on stresses§ for a slice of any width instead of one having a width of only 1 ft. as was considered by the writer. In order to integrate his equations, however, Mr. Hanna assumes that H is constant, and his formulas, therefore, have a very limited application. When H is variable, shear stresses are present in the vertical radial planes, because Δ_h also varies and the calculations become quite complicated, even when arch action is ignored. Of course, this holds also for a straight gravity dam, provided it is monolithic.

Mr. Wiley's discussion‖ is of particular interest, because the Exchequer Dam was accepted by him as Consulting Engineer for the Merced Irrigation District. The gravity design was given preference over an arch dam, which required 60 000 cu. yd. less material. With reference to his Fig. 5, Mr. Jorgensen stated:¶

"The Exchequer Dam sustains a higher unbalanced water pressure than any other dam, and it possesses the slimmest cross-section thus far attempted for high dams."

When this gravity dam is properly analyzed, it is found to have considerable tension in the up-stream face.** The writer was informed by the engineer in charge that the concrete weighed 152 lb. per cu. ft., which checks well with his own experience for this locality. The value of 158 lb. per cu. ft. used in the design, according to Mr. Wiley,†† seems too high and has the appearance of being an assumption on the side of danger.

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 237.

† *Loc. cit.*, p. 234.

‡ *Loc. cit.*, p. 229.

§ *Loc. cit.*, p. 238, and Fig. 15.

‖ *Loc. cit.*, p. 254.

¶ *Loc. cit.*, November, 1927, Papers and Discussions, p. 2360.

** *Loc. cit.*, August, 1927, Papers and Discussions, p. 1141, and Professor Cain's discussion, *Loc. cit.*, January, 1928, Papers and Discussions, p. 228.

†† *Loc. cit.*, p. 255.

The expansion of the arch ring due to lateral deformation, is, according to Mr. Wiley, 0.000032 ft., while the least contraction in a 50-ft. joint, measured 22 ft. from the surface, is, 0.0324 in., or a unit shrinkage of 0.000054,* which is nearly 70% greater than the lateral deformation, as computed by Mr. Wiley. In addition, he states that there was still some setting temperature in the dam, when these measurements were taken.

In determining the lateral deformation, Mr. Wiley assumes Poisson's ratio as $\frac{1}{5}$ and that the modulus of elasticity is 2 000 000 lb. per sq. in. The writer believes that both these values are much too favorable. Turneaure and Maurer† give for Poisson's ratio, for 1:2:4 concrete, 60 days old, $\frac{1}{6.25}$ to $\frac{1}{10}$, and for 1:3:6 concrete, 60 days old, $\frac{1}{12.5}$; and this last value would probably be more nearly correct for the very lean concrete in the Exchequer Dam. A value of $\frac{1}{8}$ for Poisson's ratio and 4 000 000 lb. per sq. in. for the modulus of elasticity is more nearly correct and at any rate more conservative, for a concrete having a compressive strength of 2 000 lb. per sq. in., or probably more‡ and with these assumptions the lateral deformation near the toe of the base is 0.000010, or one-fifth of that required to annul the smallest measured shrinkage. For points at this elevation, but farther away from the toe, the lateral deformation becomes smaller, because the vertical compression is smaller, and for higher elevations the lateral deformation also becomes smaller. For these higher elevations the shrinkage is possibly greater, because the section is thinner and not so well protected against drying out and cooling, and it is the arch action in the upper parts of the dam and not at the base, that is of importance.

Mr. Wiley's figures show that the contractions decrease with increasing distance from the face, and he concludes that a little farther inside the dam, it may be assumed that no contraction takes place. There is another explanation which seems more reasonable, or at any rate more conservative, that is, that the contraction inside the dam is delayed due to the fact that the setting temperature remains longer in the interior. Mr. Wiley admits that when the measurements were taken, there was still some setting temperature in the dam. All authorities seem to agree that Poisson's ratio is exceedingly difficult to measure for concrete and, therefore, if it is to be seriously considered, a very conservative value should be used. Moreover, for the upper part of the dam, where arch action might be of considerable importance, the lateral deformation is at any rate quite negligible compared to the shrinkage.

Mr. Wiley's surmise that the writer may have obtained shear deflection 144 times too great,§ is incorrect.

* Mr. Wiley gives this as 0.000022, but has accepted the writer's figure as correct.

† "Concrete Construction," p. 25.

‡ Stanton Walker, Assoc. M. Am. Soc. C. E., "Modulus of Elasticity of Concrete," *Bulletin No. 5*, Structural Materials Research Laboratory, Lewis Inst., Chicago, Ill.

§ *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 257.

Mr. Wiley thinks that because very little leakage appears through shrinkage and contraction cracks, that shrinkage does not penetrate more than about 20 ft. below the surface.* Where water-stops are provided in the up-stream face, leakage cannot very well take place, but when the writer last visited the Mulholland Dam (which was not provided with contraction joints), there was some leakage through the vertical-radial cracks and the foreman stated that an attempt was being made to stop the leakage by introducing sawdust. It is quite true that two surfaces may be in contact and transmitting heavy stresses, as Mr. Wiley contends, and still there may be some leakage. However, it is equally true that two surfaces, as at a shrinkage crack, may not be in contact and yet very little leakage may be in evidence. This latter is the safe and conservative assumption. Why accept the unsafe assumption merely because it may sometimes happen to be correct? The writer has made some calculations to determine what the leakage would be through a shrinkage crack $\frac{1}{16}$ in. wide under such heads and length of paths as would occur in the Mulholland Dam and finds that the discharge would be exceedingly small. If account is taken of the fact that the channel formed through a shrinkage crack is not uniform, being obstructed by small particles, etc., and that the crack is not a plane, the computed leakage is practically negligible. Therefore, the fact that there is only a very small leakage does not justify the conclusion that there must be arch action in the dam.

In summing up, the writer feels that Mr. Wiley has failed to show that there probably will be arch action in the Exchequer Dam, after the setting temperature is gone; and much less has he proved that it must be present, as would be required before any claims can be made for arch action. It is undoubtedly true that arch action might prevent a complete failure, but as the writer sees it, this cannot be relied upon until after the cantilever has failed and uplift must then also be admitted.

Mr. Kramer's discussion† is very able and most fair. He admits that shrinkage will take place and then proceeds to show that the cantilever may possibly deflect sufficiently to bring about arch action, when certain reasonably advantageous assumptions are made. Mr. Kramer considers a block 50 ft. wide and assumes that the shrinkage is that corresponding to a temperature drop of 20° Fahr.‡ This corresponds to a unit shrinkage of 0.000110 which should cover both temperature and setting shrinkage and the writer believes this may not be quite enough.§ Mr. Kramer assumes the modulus of elasticity to be 2 000 000 lb. per sq. in., and states that this is a rather low value, which will give high values for the deflections. Stanton Walker, Assoc. M. Am. Soc. C. E., gives the following formula‖ for the relation

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 258.

† *Loc. cit.*, p. 242.

‡ *Loc. cit.*, p. 245.

§ See, Mr. Wiley's discussion, *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 256, bearing in mind that the measured contractions cannot be considered as final. See, also, "Stresses in Multiple-Arch Dams," *Transactions*, Am. Soc. C. E., Vol LXXXVII (1924), p. 310.

‖ *Bulletin No. 5*, Structural Materials Research Laboratory, Lewis Inst., Chicago, Ill., p. 65.

between the initial tangent modulus of elasticity, E_i, and the compressive strength, S:

$$E_i = 40\,000\ S^{0.6} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (77)$$

Equation (77) gives the following values:

Compressive strength, in pounds per square inch.	Modulus of elasticity, in pounds per square inch.
1 000	2 520 000
2 000	3 820 000
3 000	4 800 000
4 000	5 900 000

These values represent the grand average of a great many tests. At Exchequer Dam there was no control over the quantity of water used in mixing, but an effort was made to secure the proper mix, and by the time the dam was ready for loading, it is fair to assume that the average strength was probably 2 000 lb. per sq. in., or more, considering that most of the concrete would be at least one year old. This would give a modulus of elasticity of about 4 000 000 lb. per sq. in., or twice that assumed by Mr. Kramer, which would reduce the deflections of the cantilever proportionately.

The writer will elaborate on Mr. Kramer's analysis and carry the calculations a step farther. When the cantilever is bent, it is able to compensate for shrinkage and the amount of this is given by the difference in length between $A\,B$ and $A'\,B'$ in Fig. 21, in which, $A\,B$ is the length of a short element of the crest before the deflection, y, takes place. That difference is,

$$2\,\phi\,R_u - (R_u - y)\,2\,\phi = 2\,\phi\,y$$

If $A\,B$ is 1 ft., $2\,\phi = \dfrac{1}{R_u}$ and,

$$e = \frac{y}{R_u} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (78)$$

in which, e is the unit shrinkage that a deflection, y, can compensate for, when the up-stream radius is R_u. The compensation, therefore, depends directly on the deflection. Mr. Kramer determines* the crest deflection at the crown, $y_{max.} = 0.1154$ ft.; and with $R_u = 674.7$ ft., he obtains $e_{max.} = 0.000171$, which is 1.55 times greater than the assumed shrinkage.

However, the maximum deflection is applicable to only one single point in the structure and is not a fair measure of the structure's ability to compensate for shrinkage. The writer calculated the bending and shear deflections for a gravity dam 200 ft. high, of triangular section and with the water standing level with the crest,† and found the deflection line to be approximately straight. Half way up the dam, therefore, the deflection is only one-half of what it is at the crest (in reality, it is somewhat less than that), while the shrinkage is not materially less, although it may take a longer time for the thicker sections to reach their final shrinkage. At the lower

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 245.

† *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 101, Fig. 32.

elevations, especially near the toe, some compensation will be derived from the lateral deformation.

Even at the crest the maximum deflection is not a fair measure, since the crest deflection at the abutment is zero. The crest deflection due to bending and shear, as developed for the 200-ft. dam previously referred to, is,

$$y = \frac{0.434}{E} (a^3 + 0.72\, a)\, h^2 \text{ ft} \ldots\ldots\ldots\ldots\ldots\ldots (79)$$

in which, E = modulus of elasticity in pound per square inch, h = the height of the cantilever, and $a = \frac{\text{height}}{\text{thickness}} = 1.59$ for the Exchequer Dam. Assuming $E = 2\,000\,000$ as does Mr. Kramer, the crest deflection is,

$$y = 1.12 \times 10^{-6} \times h^2 \text{ ft} \ldots\ldots\ldots\ldots\ldots\ldots (80)$$

This checks Mr. Hanna's Equation (55). For $h = 320$ ft., Equation (80) gives $y_{\max.} = 0.1146$ ft., which checks Mr. Kramer's value of 0.1154 ft.

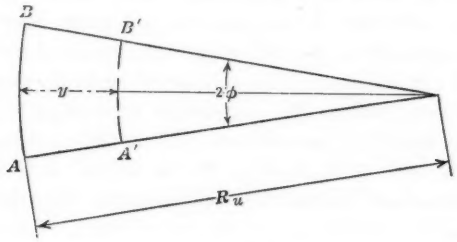

FIG. 21.

In order to have definite ideas, assume that the developed profile of the site, taken along the center line of the crest, is a straight line and let h be the height corresponding to an angle, θ, measured from the crown, so that,

$$h = c\,(\theta_1 - \theta) \text{ ft} \ldots\ldots\ldots\ldots\ldots\ldots (81)$$

in which, θ_1 is one-half the central angle. At the crown, $\theta = 0$ and $h_{\max.} = c\,\theta_1$, which determines the constant, c. If $h = h_{\max.}$ throughout (then, of course, Equation (81) would not apply), $y_{\max.}$ would be the average deflection. Since, however, h is variable in accordance with Equation (81), the average deflection for the crest is,

$$y_{\text{ave.}} = \frac{1}{\theta_1}\int_0^{\theta_1} y\, d\,\theta = \frac{1.12 \times 10^{-6}}{\theta_1} \int_0^{\theta_1} c^2\,(\theta_1 - \theta)^2\, d\,\theta$$

or,

$$y_{\text{ave.}} = 1.12 \times 10^{-6}\, \frac{c^2\, \theta_1^2}{3} = \frac{1}{3}\, y_{\max.} \ldots\ldots\ldots\ldots (82)$$

Equation (81) represents an unusual profile. One that is more nearly representative is shown in Fig. 22 and is determined by:

$$h = c \left[1 + \cos\left(\pi \frac{\theta}{\theta_1}\right)\right] \ldots\ldots\ldots\ldots\ldots\ldots (83)$$

For the crown section, $\theta = 0$ and $h_{max.} = 2c$. Proceeding exactly as before, the deflection at any point is given by Equation (80) when the value of h from Equation (83) is substituted, and the average deflection, which determines, at least approximately, the average unit shrinkage that the deflections can compensate for, is,

$$y_{ave.} = 0.375\ y_{max.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (84)$$

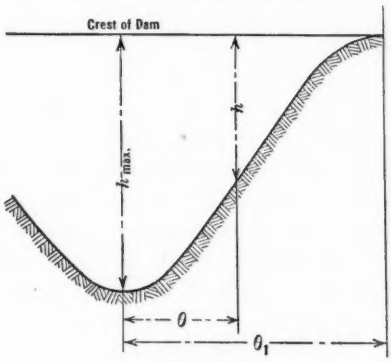

FIG. 22.

In other words, instead of using the maximum crest deflection, $y_{max.}$ only about 0.333 $y_{max.}$ should be used and that will compensate for a shrinkage of,

$$e = \frac{0.333 \times 0.1154}{674.7} = 0.00006$$

The shrinkage assumed by Mr. Kramer is 0.00011. The conclusion is, that even with Mr. Kramer's rather favorable values for shrinkage and modulus of elasticity, the deflection at the crest is sufficient to compensate for only about one-half the assumed shrinkage. At elevations below the crest, the deficiency is even greater.

From these considerations may be concluded that an arched gravity dam not provided with contraction joints is less likely to crack near the crown and more likely to crack nearer the abutments. This checks well with the location of the cracks in the Mulholland Dam. Likewise, dams provided with contraction joints would be expected to show larger openings toward the abutments than near the crown.

The writer agrees with Mr. Kramer that no allowance should be made for arch action, unless it be shown that it must be present under the most unfavorable conditions which may be reasonably expected, and that the burden of proof should be on the designer.

As has been shown it appears questionable whether the Exchequer Dam, after it has cooled, can develop arch action before the cantilevers fail and introduce uplift; so many indeterminate factors enter into the calculations as to make these uncertain. To the factors already mentioned, may be added the deflections due to the yielding of the foundation, which are by no means negligible* and which will influence both the arch and the can-

* "Stresses in Thick Arches of Dams," *Transactions*, Am. Soc. C. E., Vol. 90 (1927). p. 520, and Dr. Vogt's discussion, p. 554.

tilever deflections, the influence of swelling of the up-stream face* due to water-soaking, the rapidity with which the reservoir is filled, temperature during construction, method of construction, when the dam was pressure-grouted, etc. The writer also agrees with Mr. Kramer that when properly calculated much higher stresses can be used in design than has been customary in the past. The allowable stress should be dependent on the actual strength of the concrete as determined by test. This will encourage the production of good concrete.

Mr. Creager believes the Exchequer Dam would probably hold as an arch dam irrespective of its strength as a gravity dam.† This is to be hoped, because as a gravity section, it has considerable tension in the up-stream face with only the water load, and if an earthquake shock, traveling up the river bed, should occur while the reservoir was full, the gravity action could not be depended upon for any material assistance. If, in the last analysis, the Exchequer Dam derives its stability, or its factor of safety, from arch action or mainly from arch action, would it not have been wiser to have taken account of this in the design? Mr. Bauman gives the central angles for the Exchequer Dam‡ and Mr. Jorgensen gives the central angles for the proposed arch dam.§ A comparison shows that the central angles are much larger and the radii much smaller for the arch design than for the gravity design, and the conclusion is inescapable that from the point of view of arch action, the Exchequer Dam is hopelessly inferior in spite of its much greater cost.

Referring to Dr. Unwin's statement, it seems to the writer quite likely that the Bouzey and Habra Dams carried a portion of the load on the secondary arches during the first years of their existence. As shrinkage and cooling became effective, the arch action was greatly reduced, tension made its appearance in the up-stream face and eventually led to failure.‖

The writer agrees with Mr. Godfrey¶ and Mr. Kramer that uplift merits serious consideration and that drainage systems are not always dependable.

Mr. Turner's discussion** takes him somewhat far afield. The writer stated expressly in his paper that he was aware of the limitations involved and gave his reasons for accepting the usual assumptions.†† He had no intention of formulating an elastic theory. Mr. Turner seems to be under the impression that there is a principal distinction to be made between straight and curved gravity dams. This question has already been discussed herein. The twisting stresses to which Mr. Turner refers, have been discussed by others, but as far as the writer knows, consideration of them has not gone beyond the conversational stage, and what engineers need are figures by which to

* "Stresses in Thick Arches of Dams," *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 517, and Fig. 26.

† *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 259.

‡ *Loc. cit.*, p. 222, Table 1, Column (4); for the crest, the central angle is about 80 degrees.

§ *Loc. cit.*, November, 1927, Papers and Discussions, p. 2358, and Fig. 6.

‖ See Mr. Kramer's discussion concerning the Don Pedro Dam, *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, pp. 247–248.

¶ *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 260.

** *Loc. cit.*, p. 261.

†† *Loc. cit.*, August, 1927, Papers and Discussions, p. 1142.

design, based on conservative assumptions. A complete solution of the stresses in a gravity dam, straight or arched, is so complicated that it may not be attempted for some time to come. A good and conservative approximation, however, will satisfy most engineers. As study and experiments go on, knowledge of the stress distribution will be perfected.

General Remarks.—There is one factor, which has not been brought into the discussion, but which cannot be ignored, when calculating deflections, which occur under prolonged loading. This is the so-called time effect. The modulus of elasticity is defined as the ratio between stress and strain, when both are measured immediately after the load is applied. Tests show that when the stress is maintained for some time, the deformation continues to increase and that it approaches its maximum value asymptotically. This continued yielding under constant stress is known as the time effect. Due to this characteristic of concrete, the stress multiplied by the modulus of elasticity, as ordinarily measured, gives only the deformation, which occurs at once, when the load is applied and this deformation is much smaller than the actual deformation due to the load acting over a considerable period of time.

F. R. McMillan, M. Am. Soc. C. E., has tested* a reinforced concrete beam in bending over a considerable period of time. The concrete was of good quality and was 6 weeks old when the test was started. Immediately after the load had been applied, the deformation of the concrete was 0.0021 in. on an 8-in. gauge length. This was measured in the center of the span and on the compression face of the beam. After nearly 60 days under the same load, the deformation was 0.0039 in., an increase of nearly 100 per cent. At this time an additional load was applied, increasing the deformation to 0.0047 in.; 60 days later this had increased to 0.0063 in. and after an additional 60 days to 0.0072 in. Expressing the last three figures as percentages, they are 100, 134, and 153. Another beam† had a deformation of 0.0027 in. in an 8-in. gauge length, immediately after the load had been applied; 20 days later, it was 0.0064 in., and after another 20 days, it was 0.0069 in. About a year after the test was started the deformation was 0.0101 in. and this is 3.74 times the deformation which took place at once and from which the modulus of elasticity would be derived. In all these tests the shrinkage was measured separately and allowed for, so that the change in deformation given, is that due to time effect only.

Mr. Charles M. Montgomery reports‡ a compression test on an 8 by 16-in. concrete cylinder, which was about 4 years old at the time of test. It had been stored in damp sand for more than 3 years and the remainder of the time it had been exposed to air. The mix was approximately 1:2:4 and the crushing strength was 3 550 lb. per sq. in. A load equivalent to a stress of 300 lb. per sq. in. was applied on March 22 and the deformation was 0.0014 in., measured immediately after the load was applied. After 24 hours the load

* "Shrinkage and Time Effect in Reinforced Concrete," Univ. of Minnesota, Studies in Engineering, No. 3, March, 1915, Fig. 8.

† *Loc. cit.*, p. 26, Fig. 11-*a*.

‡ *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1712.

was removed and immediately replaced and the deformation was 0.0024 in. After another 24 hours, it was 0.0028 in. The results of the test are shown in Fig. 23. After about 12 to 14 days, the concrete seems to have taken its final deformation and this is about three times as great as the one first measured. The concrete appears to have been of a good grade and the stress was less than 10% of the ultimate strength. No allowance was made for shrinkage in this case, but it seems very unlikely that this could have effected the deformations more than a few per cent.

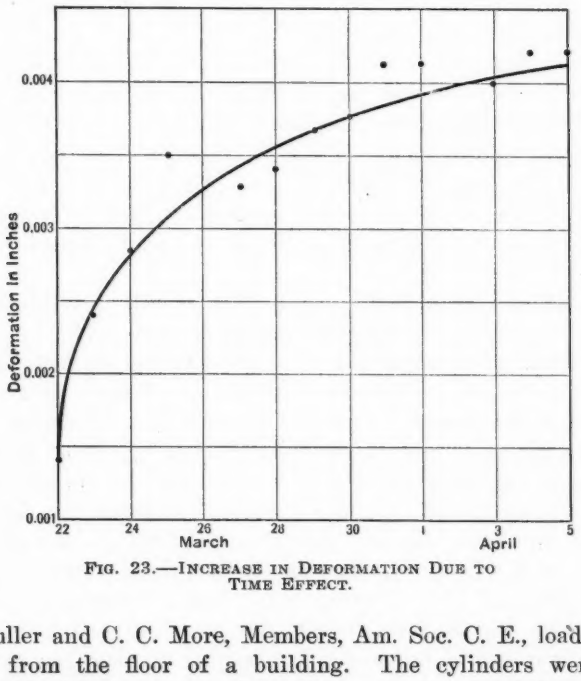

FIG. 23.—INCREASE IN DEFORMATION DUE TO TIME EFFECT.

A. H. Fuller and C. C. More, Members, Am. Soc. C. E., loaded three cylinders* cut from the floor of a building. The cylinders were from 101 to 137 days old. With a stress of 650 lb. per sq. in., the deformation of one of the cylinders, 14 days after load had been applied, was 15 dial units, and 54 days later it was 42 or 2.8 times the first measurement. The deformation was still increasing, but at a slower rate. Another of the cylinders was tested under a stress of 1 150 lb. per sq. in.; 14 days after test started, the deformation was 33 dial units and 54 days later, it was 78 or 2.36 times the first measurement. The first reading taken on this cylinder was about a week after applying the load, when a deformation of 22 dial units was found; 250 days after starting the test, the deformation was 120 or nearly six times as large as the reading after one week of test.

These tests will suffice to show the very great influence of the time factor; it appears that part of the deformation is plastic and part elastic and that time is also a factor in the recovery. The rapidity of filling of the res-

* "Time Tests of Concrete," *Proceedings*, Am. Concrete Inst., v. 12, p. 302, 1916; quoted from *Bulletin No. 24*, by W. K. Hatt, M. Am. Soc. C. E., Purdue Eng. Station, November, 1925, p. 41.

ervoir will determine to what extent the time effect may be counted on to compensate for shrinkage.

These tests on small samples bring up the question as to whether concrete in mass will obey the same law. In discussing the Lake Spaulding Dam, F. A. Noetzli, M. Am. Soc. C. E., computed the deflection at crest* as 0.36 in., based on a modulus of elasticity of 2 000 000 lb. per sq. in. and he states that the measured deflection is 3.44 in. and, further, that:

"This * * * shows, that the cantilever deflected, as measured, about ten times as much as the laws of elasticity really would permit."

Mr. Noetzli did not take account of the swelling of the up-stream face, the yielding of the foundation, and the shear deflection, but these omissions are not sufficient to account for the very great discrepancy, although they may more than double the calculated deflection. If, in addition, the time effect is sufficient to increase the deformation from three to four times, then there is a very fair agreement between calculated and measured deflections. The tests previously referred to indicate that such an increase is quite possible.

Mr. Noetzli concluded that the cantilever must have cracked at the base, since otherwise he could not account for the fact that the measured deflection was ten times larger than his computed deflection. Mr. Jorgensen, however, stated† that if the cantilever was cracked to any depth, leakage should occur through the 6-in. drain pipes placed in the up-stream third of the dam, but that virtually no leakage had occurred. He also stated that the zero point for deflection was probably located before the chemical heat had been dissipated and while the cantilever was bent up stream by the swelling of the arches due to heat; but this effect, combined with the effect of swelling, yielding of the foundation, and deflection due to shear, is not sufficient to account for the very great discrepancy and it seems to the writer that it must be admitted that time effect was responsible for a considerable portion of the additional deflection.

On the other hand, the leakage through the Mulholland Dam, already referred to, indicates that the total deformation is not sufficient to compensate completely for the shrinkage effect.

It must be evident that the problem is a complicated one and engineers designing dams should be careful to make only such assumptions as are known to be conservative. It is also evident, that the relation between stresses and deformations in a large concrete structure, is not a simple linear one, as it is for steel, and that much care is necessary in order to draw correct conclusions regarding stress distribution from deformations in concrete.

It was the writer's intention to show that arched gravity dams should be calculated as such and not as straight gravity dams and after reviewing the discussions, he believes that, hereafter, they will be so treated. He intended to limit the investigation to gravity action, but nearly every engineer who took part in the discussion brought in the question of arch action. That is

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 54.

† *Loc. cit.*, p. 70.

interesting, because it indicates a shifting of preference from gravity to arch dams. If arch action must be depended upon to provide the real factor of safety, this should be given due consideration in the design and that will mean large central angles and small radii; that is, arch dams. Gravity action must exist in any dam to some extent; yet engineers designing arch dams do not have to bring in gravity action to show the existence of a factor of safety, while, as this discussion has shown, the reverse is not true.

The writer feels amply rewarded for the time he has devoted to this subject by the interest shown by other engineers and in closing he wishes to thank those who have participated in the discussion.

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PAPERS AND DISCUSSIONS

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PRECISE WEIR MEASUREMENTS

Discussion*

By Messrs. R. E. Ballester, and Gardner S. Williams.

R. E. Ballester,† Esq. (by letter).‡—The authors deserve great credit for performing numerous and precise experiments and for the complete care and clearness with which their studies have been described.

The tentative formula:§

$$Q = 3.33\, L \left[\left(h + \frac{V\, a^2}{2\, g} \right)^{\frac{3}{2}} + \left(\frac{V\, b^2}{2\, g} \times h \right) \right]$$

shows that it is almost impossible to frame a general formula that takes into account all the factors that influence the flow of water over a weir, because "if the velocity conditions are unknown it is necessary to make measurements to find the distribution."

With this new series of observations it may be possible to find an expression or formula composed of two quantities, one expressing the characteristics of the weir itself, as to roughness, height, etc., and the other giving the effects of velocity conditions up stream from the weir, instead of including them in the constant, 3.33, as in the Francis formula.

With this in mind, the writer has determined the formula for discharge over a broad-crested weir with a cylindrical up-stream face (Fig. 62(*d*)), and he wishes to present the results of his observations as to the influence of the velocity of approach, and the determination of Bazin's coefficient, α.

The weir is located in the main canal of the Rio Negro Irrigation Works, Argentina. Its dimensions are shown in Fig. 62(*b*), (*c*), and (*d*). The channel is lined with concrete for a distance of 200 m. (656 ft.) up stream from the weir. The scale that measures the height of the nappe over the

* Discussion of the paper by Ernest W. Schoder, M. Am. Soc. C. E., and the late Kenneth B. Turner, Esq., continued from April, 1928, *Proceedings*.

† Chf. Engr., Rio Negro Irrig. Works, Fuerte General Roca, Argentine Republic.

‡ Received by the Secretary, January 4, 1928.

§ *Proceedings*, Am. Soc. C. E., September, 1927, Papers and Discussions, p. 1399.

crest is placed in a stilling-well connected with the canal by means of a pipe. This pipe is 3 in. in diameter, reduced at the entrance to 1 in., and placed 4.60 m. (15.09 ft.) up stream from the weir.

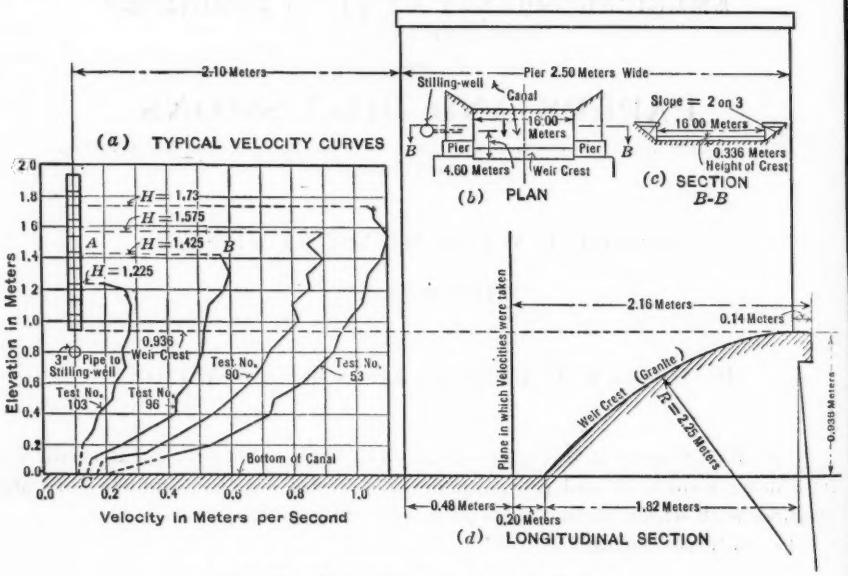

FIG. 62.—SECTIONS OF CANAL AND WEIR.

Discharge Measurements.—When the Irrigation Service was started, the discharge table was computed by means of the weir formula. The coefficient of discharge was determined from a weir with a crest similar in form to that described in the "Experiments of United States Deep Waterways Board at Cornell University, 1899".*

In attempting to control the distribution of water, some discrepancies were found, so it was decided to make discharge measurements by means of a current meter. The measurements were made by reading velocities all in the same vertical plane. At each point in the plane selected for observation, readings were taken one above the other from 0.10 m. (0.33 ft.) above the bottom of the canal (see Fig. 62).

The current meter used was an Ott Model IV. It was not possible to measure the velocity at the bottom and at the surface and these points were estimated on the curve (see Fig. 62(*a*)). The current meter was rated several times during the measurements, and no appreciable change of the constant occurred.

At first, two vertical planes for measuring velocities were used for each test, one at the vertical face at the crest and the other 2.16 m. up stream (see Fig. 62). The measurements at the crest were not precise, because the velocities were two to three times greater than they were up stream and there were more changes or oscillations of the water surface.

* *Water Supply Paper No. 200*, U. S. Geological Survey, Series 19, p. 90.

The Calculation of Discharge.—For each test the height at the crest was determined by a reading at the stilling-well and the velocity was observed in the vertical plane at intervals of 0.10 m. (0.33 ft.).

The velocity curves (Fig. 62(*a*)) were then plotted and the area that represents the discharge per unit of length of weir (for example, see Area *A B C D*, Fig. 62(*a*)), was calculated by the Simpson formula.

When a set of forty-three measurements was completed under heads that ranged between 0.282 m. and 0.815 m. (0.926 ft. and 2.67 ft.) above the crest, the data were plotted (see Fig. 63) in terms of head and discharge per unit length of weir. A curve was traced which showed a visual average of the results. After this, ten sets of readings that conformed most nearly to the curve were selected for a more detailed analysis and for the determination of the coefficient of discharge.

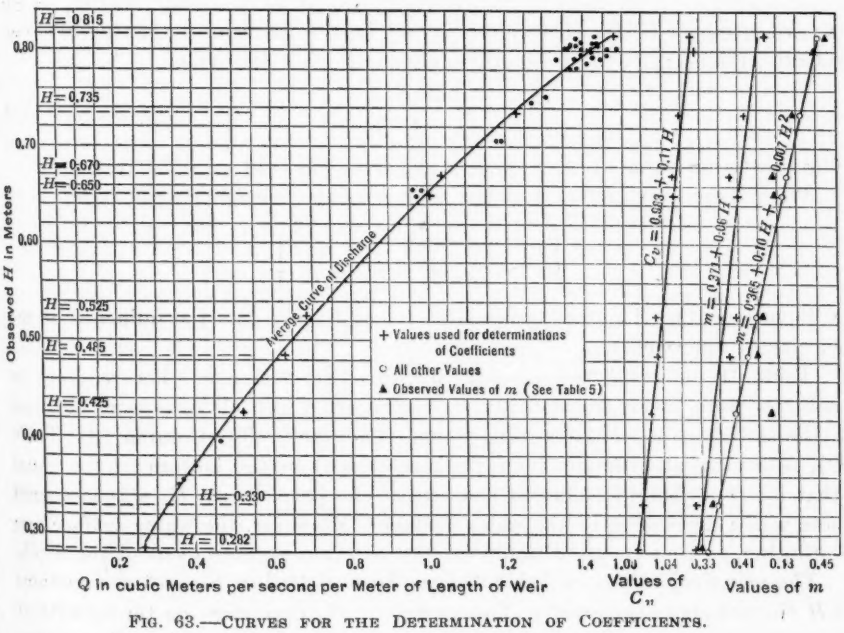
H = 0.815
H = 0.735
H = 0.670
H = 0.650
H = 0.525
H = 0.485
H = 0.425
H = 0.330
H = 0.282
Observed H in Meters
Q in cubic Meters per second per Meter of Length of Weir
Average Curve of Discharge
+ Values used for determinations of Coefficients
○ All other Values
▲ Observed Values of m (See Table 5)
Values of C_v
Values of m

FIG. 63.—CURVES FOR THE DETERMINATION OF COEFFICIENTS.

This procedure, of course, does not make complete use of all the measurements taken, but it was judged sufficient because the precision of measurements that is required on an irrigation system of ample supply of water is not extreme and greater precision was not consistent with that obtained by the current meter measurements.

No extreme care was taken in measuring the height of water above the crest by means of floats or other devices, but methods were used throughout that were considered adequate for irrigation purposes.

The Coefficient of Discharge.—The general formula used in determining the value, m, was:

$$Q = m\,L\,H\,\sqrt{2\,g\,H}\left[\left(1+\frac{\alpha\,h}{H}\right)^{\frac{3}{2}}-\left(\frac{\alpha\,h}{H}\right)^{\frac{3}{2}}\right] \quad \ldots\ldots\ldots\ldots (1)$$

in which,

Q = discharge, in cubic meters per second.
m = coefficient of discharge.
L = length of weir, in meters.
H = height of water above the crest, in meters.
g = acceleration of gravity (= 9.81 m. per sec.^{2}).
h = velocity head $\left(= \frac{u^2}{2g}\right)$.
α = coefficient to correct the influence of the different velocities in the vertical plane.
U = mean velocity in the canal at 2.16 m. (7.08 ft.) up stream from the vertical face of the crest.

If all water filaments had the same velocity, U, the kinetic energy, would be expressed as $U^3 H$. Considering each water filament as having its own velocity, the corresponding kinetic energy is $\Sigma u^3 \Delta H$. Then,

$$\alpha = \frac{\Sigma u^3 \Delta H}{U^3 H} \quad \ldots\ldots (2)$$

The maximum value of ΔH used in the tests was 0.10 m. (0.33 ft.).

With these measurements it has been possible to calculate the term,

$$\left[\left(1 + \frac{\alpha h}{H}\right)^{\frac{3}{2}} - \left(\frac{\alpha h}{H}\right)^{\frac{3}{2}}\right]$$

of Equation (1). This has been called C_v, and represents the influence of the velocity of approach.

Table 61 shows all the velocities measured in each run. The readings in each test were taken one directly above the other and all tests were confined to the same vertical cross-section plane. The depth, H_1 (Column (3), Table 62), is somewhat different than the elevation from the bottom of the canal (that is: H + height of crest) because, at the place where H_1 was measured there was a depression of the water surface. Moreover, the water surface was oscillating, so that it was difficult to take an exact measurement of the depth.

The principal values in Table 62 have been plotted in Fig. 63, as functions of H (height above the crest). The coefficient, C_v, dependent on the velocity of approach, may be represented by a linear function of H with sufficient precision, within the range of measurements. The equation of the curve is,

$$C_v = 0.983 + 0.11\, H \quad \ldots\ldots (3)$$

Columns (15), (16), and (17), Table 62, show that there is sufficient agreement between computed and observed values for practical purposes.

The coefficient of discharge, m, may be also represented by a linear function of H. (See Fig. 63.) This does not mean that it must be necessarily so because the number of observations is not great enough to prove it. The equation of the m-curve in Fig. 63 is:

$$m = 0.371 + 0.06\, H \quad \ldots\ldots (4)$$

within the range of measurements given.

TABLE 61.—Velocity Determinations at Ten Different Heights of the Nappe Over a Weir with a Cylindrical Up-Stream Face.

Test No. 103*.		Test No. 104.		Test No. 102.		Test No. 96*.		Test No. 101.		Test No. 92.		Test No. 90*.		Test No. 88.		Test No. 52.		Test No. 53*.	
Elevation, in meters.	Velocity, in meters per second.	Elevation, in meters.	Velocity, in meters per second.	Elevation, in meters.	Velocity, in meters per second.	Elevation, in meters.	Velocity, in meters per second.	Elevation, in meters.	Velocity, in meters per second.	Elevation, in meters.	Velocity, in meters per second.	Elevation, in meters.	Velocity, in meters per second.	Elevation, in meters.	Velocity, in meters per second.	Elevation, in meters.	Velocity, in meters per second.	Elevation, in meters.	Velocity, in meters per second.
0.00	0.100	0.00	0.100	0.00	0.150	0.00	0.185	0.00	0.175	0.00	0.185	0.00	0.170	0.00	0.180	0.00	0.220	0.00	0.22
0.20	0.112	0.10	0.117	0.15	0.212	0.10	0.143	0.10	0.194	0.10	0.194	0.10	0.189	0.10	0.192	0.10	0.31	0.10	0.35
0.30	0.163	0.20	0.165	0.30	0.246	0.20	0.264	0.20	0.269	0.20	0.375	0.15	0.322	0.20	0.447	0.20	0.57	0.20	0.53
0.40	0.186	0.30	0.255	0.30	0.290	0.30	0.330	0.30	0.330	0.30	0.458	0.20	0.380	0.30	0.580	0.30	0.80	0.30	0.62
0.50	0.215	0.40	0.224	0.40	0.330	0.40	0.414	0.40	0.385	0.40	0.535	0.30	0.460	0.40	0.635	0.40	0.68	0.40	0.70
0.60	0.234	0.50	0.248	0.50	0.356	0.50	0.419	0.50	0.404	0.50	0.568	0.40	0.539	0.50	0.690	0.50	0.70	0.50	0.73
0.70	0.255	0.60	0.285	0.60	0.380	0.60	0.468	0.60	0.450	0.60	0.640	0.50	0.584	0.60	0.753	0.60	0.80	0.60	0.83
0.80	0.247	0.70	0.293	0.70	0.411	0.70	0.499	0.70	0.508	0.70	0.712	0.60	0.637	0.70	0.775	0.70	0.89	0.70	0.86
0.90	0.273	0.80	0.306	0.80	0.448	0.80	0.506	0.80	0.542	0.80	0.788	0.70	0.700	0.80	0.909	0.80	0.94	0.80	0.93
1.00	0.278	0.90	0.322	0.90	0.442	0.90	0.510	0.90	0.567	0.90	0.786	0.80	0.721	0.90	0.852	0.90	0.91	0.90	0.94
1.10	0.282	1.00	0.310	1.00	0.476	1.00	0.522	1.00	0.577	1.00	0.824	0.90	0.765	1.00	0.859	1.00	0.95	1.00	0.97
1.20	0.259	1.10	0.342	1.10	0.489	1.10	0.539	1.10	0.603	1.10	0.765	1.00	0.811	1.10	0.883	1.10	1.02	1.10	1.02
1.225	0.200	1.20	0.368	1.20	0.499	1.20	0.583	1.20	0.597	1.20	0.844	1.10	0.797	1.20	0.941	1.20	1.01	1.20	1.03
.....	.....	1.25	0.350	1.30	0.503	1.30	0.605	1.30	0.617	1.30	0.770	1.20	0.852	1.30	0.909	1.30	1.06	1.30	1.05
.....	.....	1.275	0.300	1.35	0.499	1.40	0.583	1.40	0.617	1.40	0.835	1.30	0.841	1.40	0.915	1.40	1.05	1.40	1.09
.....	.....	.....	.....	1.38	0.420	1.425	0.570	1.45	0.594	1.50	0.767	1.40	0.899	1.50	0.976	1.50	1.08	1.50	1.10
.....	.....	.....	.....	.....	.....	.....	.....	1.48	0.585	1.55	0.855	1.50	0.888	1.60	0.915	1.60	1.10	1.60	1.07
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.575	0.840	1.55	0.896	1.625	0.933	1.65	1.07	1.65	1.06
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.575	0.880	1.65	0.900	1.70	1.04	1.675	1.07
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.73	1.04

*See Fig. 62 (a).

TABLE 62.—CALCULATION OF COEFFICIENTS, m AND C_v.

(1) Test No.	(2) Observed head above weir, H, in metres.	(3) Distance above bottom of channel, H_1, in metres.	(4) Discharge, per meter of weir, Q, in cubic meters per second.	(5) Mean velocity of approach, $U = \frac{Q}{H_1}$, in meters per second.	(6) $U^3 H_1 = \frac{Q^3}{H_1{}^2}$.	(7) $\Sigma\,(u^3 \Delta H_1)$.	(8) $a = \frac{\Sigma\,(u^3 \Delta H_1)}{U^3 H_1}$.	(9) $h = \frac{U^2}{2g}$, in metres.	(10) $\frac{a\,h}{H}$.	(11) $\left(\frac{a\,h}{H}\right)^{\frac{3}{2}}$.	(12) $1 + \frac{a\,h}{H}$.	(13) $\left(1 + \frac{a\,h}{H}\right)^{\frac{3}{2}}$.	(14) C_v = Column (13) — Column (11).	(15) C_v, computed.	(16) Difference, Column (15) — Column (14).	(17) Percentage of difference.	(18) $H\sqrt{2g\,H}$.	(19) $m = \frac{Q}{C_v\,H\sqrt{2g\,H}}$.	(20) m, computed.	(21) Difference, Column (20) — Column (19).	(22) Percentage of difference.	(23) C_v = Column (14) × Column (19).	(24) m_1, computed.	(25) $\mu = \frac{3}{2} \times$ Column (24).	(26) Variation from Francis' ($\mu = 0.623$).	(27) Percentage of difference, $\frac{\text{Column (26)}}{0.623} \times 100$.
108	0.282	1.225	0.2600	0.212	0.0117	0.0146	1.248	0.0023	0.010	0.001	1.010	1.015	1.014	1.014	....	....	0.663	0.387	0.388	+0.001	0.3	0.392	0.393	0.589	−0.034	5.5
104	0.330	1.275	0.3317	0.26	0.0224	0.0281	1.250	0.0034	0.013	0.001	1.013	1.020	1.019	1.019	.....	.....	0.836	0.388	0.391	+0.003	0.8	0.395	0.399	0.598	−0.025	4.0
103	0.425	1.360	0.5288	0.389	0.0754	0.0915	1.212	0.0074	0.021	0.003	1.021	1.032	1.029	1.030	+0.001	0.1	1.227	0.415	0.396	−0.019	4.6	0.427	0.408	0.612	−0.011	1.8
96	0.485	1.425	0.6278	0.440	0.1213	0.1550	1.277	0.0099	0.026	0.004	1.026	1.039	1.038	1.039	.....	.....	1.495	0.405	0.400	−0.005	1.2	0.419	0.415	0.622	−0.001	0.2
109	0.525	1.460	0.6606	0.467	0.1505	0.1770	1.176	0.0110	0.024	0.003	1.024	1.036	1.038	1.041	+0.003	0.1	1.685	0.409	0.402	−0.007	1.7	0.422	0.419	0.628	+0.005	0.8
92	0.650	1.575	1.0011	0.635	0.4044	0.5110	1.263	0.0205	0.040	0.008	1.041	1.063	1.054	1.057	+0.003	0.3	2.429	0.406	0.411	+0.005	1.2	0.428	0.435	0.652	+0.029	4.6
90	0.670	1.575	1.0402	0.660	0.4540	0.5670	1.249	0.0222	0.041	0.008	1.041	1.063	1.054	1.060	+0.006	0.4	2.791	0.411	0.415	+0.001	0.2	0.439	0.442	0.663	+0.040	6.4
88	0.735	1.650	1.2247	0.742	0.6746	0.8126	1.204	0.0280	0.046	0.010	1.046	1.070	1.060	1.064	+0.004	0.4	3.170	0.419	0.419	.....	.....	0.449	0.449	0.673	+0.050	8.0
52	0.800	1.700	1.4229	0.837	0.9967	1.2308	1.234	0.0357	0.055	0.013	1.055	1.084	1.071	1.071	+0.002	0.2	3.258	0.423	0.420	−0.003	0.7	0.453	0.451	0.676	+0.053	8.5
53	0.815	1.720	1.4740	0.852	1.071	1.2978	1.212	0.0370	0.055	0.013	1.055	1.084	1.071	1.073												

The results are shown in Columns (20), (21), and (22) of Table 62. Except for $H = 0.425$, in which the percentage difference is 4.6, the variation does not exceed 1.7 per cent. It seemed a good approximation for measurements of discharge by means of a current meter, especially in view of the intended use of the weir, and, therefore, the corresponding values of C_v and m were adopted.

By multiplying these two coefficients a general coefficient, m_1, may be obtained. There is no object in doing this, however, because it obscures the separate effects of velocities of approach and the characteristics of the weir itself. This value will be (see Fig. 63):

$$m_1 = 0.365 + 0.10\,H + 0.007\,H^2 \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The variations of this formula with Francis' simple weir formula:

$$Q = \frac{2}{3}\,\mu\,L\,h\,\sqrt{2\,g\,h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

are given in Columns (25), (26), and (27) of Table 62, in which $\mu = 0.623$.

GARDNER S. WILLIAMS,* M. AM. SOC. C. E. (by letter).†—This paper goes a long way toward settling many of the questions that have agitated experimenters dealing with the measurement of water in open channels. In the writer's judgment it is entitled to a place beside the classic experiments of Francis and Fteley and Stearns in this country and those of Bazin abroad, and in precision of measurement it exceeds them all.

To evaluate its position in the series properly it may be well to recall that quantitative knowledge of the flow of water over weirs, before the publication of this paper, was dependent on 93 experiments of Francis made at the Lower Locks at Lowell, Mass., in 1852; 41 experiments of Fteley and Stearns, made on the Sudbury Aqueduct, near Boston, Mass., in 1877 and 1878; and 153 experiments of Henri Bazin made at Dijon, France, in 1886. The work of Rehboch, published in 1911, however interesting, was with weirs of such small dimensions as to be hardly entitled to a place with the others, and 86 of Bazin's determinations might be logically rejected on the same ground. However, accepting all those volumetric determinations, they make a total of 287 to be compared with 1 276 made by the authors, and to these may be added the 1 162 additional volumetric measurements made by their collaborators.

The writer, as a result of his own experiments and the study of those of others, came to the conclusion long ago that the then accepted discharges of sharp-edged weirs were likely to be questioned and modified for causes then unknown. In preparing for publication the results of his experiments on weirs with irregular crests he expressed the coefficients for them in terms of the discharge of a sharp-edged weir under similar conditions, instead of presenting a coefficient to be introduced directly in the Francis or some other formula.‡ While he is not entirely sure that there may not be more in the method of reading head than the authors seem to indicate, the writer believes

* Cons. Engr., Ann Arbor, Mich.

† Received by the Secretary, March 31, 1928.

‡ "Hydraulic Tables," by Gardner S. Williams and Allen Hazen, Members, Am. Soc. C. E., Wiley & Sons, New York; and American Civil Engineers' Pocket Book, edited by the late Mansfield Merriman, M. Am. Soc. C. E., Wiley & Sons, New York, N. Y.

this paper has carried the subject of weir measurement many steps farther toward definite evaluation than anything else done in the past thirty years.

The importance of the protection of a sharp-edged weir from the effects of corrosion was apparently appreciated by Francis whose custom it was to dry the weir crest and coat it with grease at the conclusion of the day's work. Other experimenters do not seem to have been so particular. The determination of the magnitude of the corrosion effects has never been attempted before and without the very ideal measuring apparatus used by the authors, it could not now have been successfully evaluated.

Perhaps the most interesting and valuable conclusion of the authors is that approving the use of weirs under heads greater than the height of the weir.* This will often make the application of the weir as a measuring device much less difficult.

The authors are to be commended for the completeness of the presentation of the data upon which the paper is based, and by which its utility to future investigators will be greatly enhanced.

* *Proceedings*, Am. Soc. C. E., September, 1927, Papers and Discussions, p. 1465.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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HISTORIC REVIEW OF THE DEVELOPMENT OF SANITARY ENGINEERING IN THE UNITED STATES DURING THE PAST ONE HUNDRED AND FIFTY YEARS

A SYMPOSIUM

Discussion*

BY MESSRS. JAMES E. BROOKS, W. G. STROMQUIST, S. H. MCCRORY,† AND J. A. LEPRINCE†

JAMES E. BROOKS,‡ ESQ.—The human mind is generally so burdened with the things of the present that it forgets the things of the past; especially so if the things of the past have not been personal experiences. People think very little about the terrors and horrors of smallpox and yellow fever epidemics. In this part of the world they are things of the past, and few communities have experienced them. However, the victory over yellow fever came so much later than that over smallpox that some of those who took part in the victory over yellow fever at Havana are still living.

Mr. LePrince, who is a civil engineer, was with the late General Gorgas at Havana, and, subsequently, at Panama. He is a pioneer in the fight against disease-carrying mosquitoes, and he is still carrying on the fight.

Mr. LePrince has not painted the picture of yellow fever black enough. Of the epidemic of 1878, Gorgas says: "In this epidemic over 13 000 people in the Mississippi valley alone lost their lives, and the loss of wealth is estimated at considerably more than a hundred million dollars." Yellow fever has been epidemic as far north as Quebec, Canada, and Swansea, in Wales.

* Discussion of the Symposium on Historic Review of the Development of Sanitary Engineering in the United States During the Past One Hundred and Fifty Years, continued from April, 1928, *Proceedings*.

† Authors' closures.

‡ Cons. Engr., Essex County Mosquito Extermination Comm., Newark, N. J.

In the fight against disease-carrying mosquitoes the physician, the biologist, and the engineer should work together, hand in hand. The late Dr. Henry R. Carter was not only a physician but a graduate civil engineer. In 1898, while he was a surgeon in the U. S. Marine Hospital Service, Dr. Carter wrote a paper on his observations upon the extrinsic incubation of yellow fever, made at Orwood and Taylor, Miss. It has been said that,

"Measured by the results produced, this was one of the most important papers ever written." * * * It was one of the great steps in establishing the true method of the transmission of yellow fever."

The life habits of the different species of mosquitoes vary greatly. While in the larva and pupa stage they all live in water, but some develop in salt water and some in fresh water; some require clean water, while others will thrive in water so polluted that fish die in it within a few minutes. Some pass the winter in the adult or winged stage, some as eggs, and a few in the larval form.

It is fortunate that warfare against mosquitoes began with a species that is readily brought under control. The successful fight against the yellow fever carrying mosquito at Havana and Panama encouraged people to fight other species without a realization of the greater difficulties to be encountered. The mosquito which carries yellow fever is called *Stegomyia fasciata* by some and *Aëdes ægypti* by others. Malaria is carried from person to person by mosquitoes of the genus *Anopheles*.

Mr. LePrince speaks* of the rapidity with which yellow fever was brought under control, but he makes no such statement about malaria. The war against *Anopheles* will be long and expensive, but the victory will be worth the price.

In New Jersey the fight began at South Orange in 1901, and it has been going on with increasing intensity ever since. It was started to get rid of an intolerable nuisance, and was directed against mosquitoes in general without regard to species.

To-day (1928) the nine counties along the eastern border, from Bergen to Cape May, and Passaic County, have mosquito commissions acting under a law passed in 1912, and their total annual appropriations amount to more than $300 000. In only one or two of these ten counties does the control work cover the entire county. Essex County, with an area of about 125 sq. miles, and 500 000 population, spends $75 000 per year for mosquito control. This is approximately 15 cents per person. There are still eleven counties without mosquito commissions, and of all the salt marsh in the State a little more than one-half has been drained. The work is being advanced each year. Most of the undrained salt marsh is in the southern portion of the State.

Methods of Mosquito Control.—All mosquitoes, in their larval stage, live in water. At present, the attack is directed against the larva; and obviously, the elimination of standing water is the most effective method. This may be accomplished either by drainage or filling.

* *Proceedings*, Am. Soc. C. E., September, 1927, Papers and Discussions, p. 1638.

On the salt marsh, filling is expensive and is proceeding slowly, while drainage has been accomplished rapidly. Three methods are used: Open ditches connected with tide-water; dikes with automatic tide-gates; and pumping.

On the upland, the practice of draining and filling is followed wherever it is feasible. Where water cannot be removed it may be stocked with some top-feeding fish, or treated with some chemical called a larvæcide, or the surface of the water may be covered with a film of oil. In warm weather the larvæcide or oil film must be renewed at intervals of from 8 to 10 days. As an example of routine work, in Essex County about 10 500 sewer catch-basins are treated with oil once every 10 days from the middle of June to the end of September. A cheap, safe, effective, and lasting larvæcide has not been discovered.

Results.—The relief obtained through mosquito control work in New Jersey has been very great, and has more than justified the expenditures made, both from the standpoint of increased personal health and comfort, and the increase in taxable ratables.

Failures.—A female mosquito of the genus *Aëdes* will fly about laying eggs, one at a time, on the surface of the ground where some day they may be covered with water. This may be near a pool of water or in a depression in some field or woods. Months, or even years, may pass, when a heavy rain during the warmer half of the year changes the dry ground to a pool of water, and in a few minutes the eggs have hatched. If the pool of water disappears before the larvæ have time to mature they die. The water may run off; it may seep into the ground; it may be evaporated; or it may remain a week or so and permit a brood of mosquitoes to reach the adult or winged stage.

Occasionally the time comes when rains follow each other at such short intervals, and the evaporation is so slight, that the hundreds and thousands of large and small pools and pockets of water (too numerous to be drained or covered with oil within the short time allowable) will turn out vast numbers of adult mosquitoes.

During the year 1926 the control work in New Jersey during May, June, and July, was more effective than ever before, but during August and September weather conditions were so unfavorable that, in spite of all that the mosquito commissions could do with the means and methods at hand, a large brood of *Aëdes* mosquitoes reached maturity.

Additional drainage and the greater use of machinery for digging and cleaning ditches, and a way of increasing the working force temporarily, following rainy periods, are needed to increase the efficiency of mosquito control in New Jersey.

W. G. STROMQUIST,* ASSOC. M. AM. SOC. C. E. (by letter).†—Because of the many factors involved in his review of the control of disease-bearing

* San. Engr. and Director, Bureau of Rural Sanitation, Jefferson County Board of Health, Birmingham, Ala.

† Received by the Secretary, March 12, 1928.

mosquitoes, Mr. LePrince said very little about the relation of artificial lakes to malaria. This is a subject of growing importance which should be given more attention by engineers than has been done in the past.

In former years, when water power projects were on a small scale only, there were many relatively small constant-level ponds which were frequently the cause of severe outbreaks of malaria; or, as they were often termed, "serious epidemicals". More recent developments of hydro-electric power are on a large scale, with immense impounding reservoirs. The danger of malaria is greatly increased. This is particularly true in the Southern States, where there is a combination of prevalent malaria with a rapid and extensive increase in hydro-electric projects.

Fortunately, the Engineering Profession is beginning to understand the possible relation of these projects to malaria outbreaks of serious importance that result in damage suits against the power company, or against the municipality developing a surface water supply. However, in too many cases the necessary precautions, under experienced supervision, are not taken until they are required by health authorities or threatened damage suits. Preventive measures are generally less expensive than corrective measures.

The production of *Anopheles* (malaria-carrying mosquitoes) is most prolific in bights or indentations of constant-level reservoirs where floatage protects the mosquito larvæ against its natural enemies. Aquatic or semi-aquatic growths in shallow parts of reservoirs also favor mosquito production, as does any protection against, or lack of, severe wave action.

Some of the conditions inimical to the development of *Anopholes* are: Distinct wave action against a clean shore; the presence of *Gambusia* (top-water minnows) which eat mosquito larvæ if they are not protected by floatage, vegetation, etc.; and pronounced fluctuations of water level at 10-day intervals during the mosquito season (from 2 ft. on steep slopes to 4 ft. on flat slopes).

Having recognized the danger of building reservoirs without proper precautions against malaria, the State Boards of Health in several States have established regulations governing reservoir construction. The rules imposed are adjusted to local conditions, but, in general, they require:

(1) Clearing the reservoir site of all trees, brush, and undergrowth that would extend above the surface at low-water level, in order to prevent the collection and anchorage of floatage.
(2) Clearing the shore line at a specified distance beyond high-water level to prevent the collection of floatage along the shore.
(3) Establishing hatcheries for *Gambusia* prior to impounding the water so that, as the reservoir is filled, the branches and indentations will be well stocked with these enemies of the mosquito.
(4) Other supplementary control measures, such as removing floatage, clearing shallow parts of vegetation, oiling, etc., under trained supervision, for the first few years after the reservoir is built and until the proper biological balance is obtained in it.

With clearing charges of from $25 to $40 per acre, the necessary work of preparing a reservoir site may cost several hundred thousand dollars. In an effort to develop less expensive methods of control, the U. S. Public Health Service

has, since 1926, conducted some very interesting experiments to determine the feasibility of controlling the production of *Anopholes* in large reservoirs by means of Paris green distributed from airplanes. The development of this new control measure may in time eliminate from many projects the need of costly clearing operations.

It is advisable for any engineer connected with an impounded water project, be it a water supply or hydro-electric development, where there is potential danger of malaria, to consult with the Engineering Division of the State Board of Health, or with the U. S. Public Health Service, before he goes far with his plans, so that serious errors will not be made.*

It is worthy of note that, of the three men who were most prominent in originating and developing malaria control—the late General W. C. Gorgas, U. S. A., Dr. H. R. Carter, and Mr. LePrince—the latter two were graduate Civil Engineers. The control of malaria in cantonments and extra-cantonment zones, with the remarkable results mentioned by Mr. LePrince,† was directed mainly by engineers.

S. H. McCrory,‡ M. Am. Soc. C. E. (by letter).§—The writer is grateful to those who have taken part in the discussion. The development of drainage laws has been a slow process. Frequently, as Mr. Morgan suggests,‖ laws have been passed to meet the immediate purpose. At times, emphasis perhaps has been placed on legal and engineering technicalities, and the agricultural side of the problem has been neglected. The tendency in recent laws has been to make the organization of the districts easier. It is probable that recent laws have been too liberal in this respect, and that in future laws the organization of districts will be made more difficult. In considering new drainage projects there is an increasing appreciation of the importance of examining all possible uses to which the land can be put, and selecting the use for which it is best adapted.

Under-drainage is needed over a large area in the United States. Its advantages are well described by Mr. Jones.¶ The mole drain described by Mr. Shaw** had been almost abandoned when changed economic conditions made it necessary to find a cheaper method of under-drainage than the use of tile, and the mole drain came into use again. By substituting the tractor for the capstan and thus speeding up the work it is possible to construct drains 2 rods apart at the rate of 30 acres per day, at an approximate cost of about $2 per acre. Where the soil is of such texture that these drains can be used, they afford fairly adequate drainage at low cost.

Mr. Hidinger emphasizes†† the importance of proper balance in designing drainage systems with modern excavating machinery. Drains can be cut to

* Possible errors that engineers may make in the construction of impounding reservoirs have been outlined in the *Proceedings* of the National Malaria Committee, 1927.

† *Proceedings*, Am. Soc. C. E., September, 1927, Papers and Discussions, p. 1639.

‡ Chf., Div. of Agricultural Eng., Bureau of Public Roads, U. S. Dept. of Agriculture, Washington, D. C.

§ Received by the Secretary, April 9, 1928.

‖ *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2762.

¶ *Loc. cit.*, February, 1928, Papers and Discussions, p. 611.

** *Loc. cit.*, December, 1927, Papers and Discussions, p. 2764.

†† *Loc. cit.*, January, 1928, Papers and Discussions, p. 307.

specified size, and slopes can be adjusted to the material through which the drain passes. The control of silt brought down by small hill streams by settling basins, as suggested by Mr. Towl,* can be used to advantage on many streams. Studies now in progress under the writer's direction indicate that such basins should usually be proportioned so as to have, within the basin, from 1% to 2% of the area of the water-sheds.

The permanence of irrigated agriculture depends to a large extent on the establishment and maintenance of satisfactory drainage. As Mr. Hart states,† these drains must often be constructed to prevent the loss of large areas of productive land which, under irrigation, are becoming water-logged and alkaline.

J. A. LePrince,‡ Esq. (by letter).§—The California State Board of Health has issued a *Bulletin*‖ of value to those who are interested in prevalence or elimination of malaria. To a considerable extent this prevalence of malaria is due to irrigation, mining, and other operations of an engineering nature. Irrigation with inadequate drainage, seepage waters, and irrigation canals overgrown with vegetation demand attention and treatment by engineers. In the engineering schools of South Carolina, the students are taught to appreciate the importance of this phase of their work.

With regard to the mosquito control activities of New Jersey, all "mosquitoölogists" agree that it would be financially advantageous for all States with mosquito problems of economic importance to follow the lead and methods taken by New Jersey. Within the past year (1927) some unjustifiable criticism relative to anti-mosquito work in New Jersey and on Cape Cod, Massachusetts, was published in a prominent American engineering journal. The writer investigated the situation on Cape Cod and found that the total appropriation made for mosquito control was $1 000. Yet some one expected mosquito freedom over all the Cape Cod territory for that sum and stated that the enormous sums spent, had been wasted. Such news items in engineering journals are at least misleading.

Mr. Brooks states¶ that the annual appropriations now made by the counties of New Jersey exceed $300 000 and it is probable that less than $500 000 are being appropriated by the counties of all other States taken together for mosquito control.

If such a sum were available for a few years exclusively for the control of malaria, this disease would soon disappear from the country, assuming that the elimination work was directed by competent specialists applying the newer control methods that have been devised and are being developed by the U. S. Public Health Service. These new methods are now being used in a number of countries of both hemispheres.

In connection with the elimination of pestiferous mosquitoes, the time will soon come when the public will expect the Engineering Profession to avoid

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 309.
† *Loc. cit.*, p. 307.
‡ Senior San. Engr., U. S. Public Health Service, Memphis, Tenn.
§ Received by the Secretary, April 2, 1928.
‖ *Bulletin No. 44*, State Board of Health, Sacramento, Calif.
¶ See p. 1502.

building mosquito-breeding places in thickly settled communities. It means that storm-water catch-basins must be designed so that they will be dry except at time of rain.

Mr. Brooks has shown unusual foresight in the general plan of ditching of the meadows of New Jersey. To have used additional smaller ditches to care for the unusually rainy seasons would have increased first costs as well as maintenance costs and, since emergency control measures are now being investigated, it is probable that sanitary engineers will be able to use the new and satisfactory supplementary measures in the years when the rains are spaced so as to be particularly favorable to mosquito life.

In regard to the statement* that it is fortunate that warfare against mosquitoes began with a species (*Aedes aegypti*) that can be readily brought under control, it should be remembered that when the first campaign started it was not known that such was the case, and the public press of Havana claimed the efforts of the sanitary engineers were useless because all mosquitoes destroyed would be replaced by "others carried in from the yellow fever districts by favorable winds."

This pioneer work done in Cuba in 1901 had little effect on the reduction of the yellow fever mosquito in the Southern States, as is evidenced by the rapid spread of dengue fever in 1922. In that year there were more cases of dengue than have ever existed at any time in American history.

The relation of impounded water to malaria prevalence is now being studied by malariolögists of other countries and they are sending to America for information, because the State health regulations in regard to the subject in this country have been published, and are based on scientific investigations conducted by the U. S. Public Health Service.

* See p. 1502.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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PROBLEMS IN CONCRETE DAM CONSTRUCTION ON THE PACIFIC COAST

Discussion*

By Fred A. Noetzli, M. Am. Soc. C. E.

Fred A. Noetzli,† M. Am. Soc. C. E. (by letter).‡—The problems encountered in dam construction are many and varied. No two dams are exactly alike, each site or type of dam involving new problems in one way or another. Usually, it is hard-earned experience gained in handling similar work that points the way for the proper planning and most efficient execution of the work. Lack of experience in dam construction often leads contractors to under-estimate rather than over-estimate the difficulties and, therefore, the cost.

In some recent cases the estimate of the highest bidder was almost double that of the lowest bidder, which is hard to explain for relatively simple structures like concrete dams, especially when the contract is to be let on the basis of unit prices. Perhaps the low and "successful" bidder has the right explanation for this discrepancy after he is through with the job.

The author's organization has built more dams than probably any other construction firm on the Pacific Coast. His paper deserves most careful consideration by engineers as well as by contractors.

In dam construction, probably more than in any other type of building work, a close co-operation between the constructors and the supervising engineers is essential to the economic and structural success of the work. The engineering plans and specifications should be as complete as possible. Besides structural features, they should also include as much information as practicable on the peculiarities of the rock formation, especially as to what extent

* Discussion of the paper by Arthur S. Bent, Affiliate, Am. Soc. C. E., continued from March, 1928, *Proceedings*.

† Cons. Hydr. Engr., Los Angeles, Calif.

‡ Received by the Secretary, February 29, 1928.

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‡ Received by the Secretary, February 29, 1928.

blasting will be permitted; and on the stream flow and methods of river diversion, size of temporary flood openings in the dam, and the methods and anticipated time of closing these openings.

On the other hand the contractor should not fail to make due allowance in his bid for many items which, in estimating, are often overlooked or simply included in the bid prices per cubic yard of rock excavation or concrete. The proper cleaning of the surface of the bed-rock and the joints between successive layers of concrete are especially important items. Many dams suffer from unsightly leaks, usually in some joints. Lack of adequate engineering supervision and inspection is not always to be blamed entirely for this evil.

The author cites* some instances in which the sound bed-rock was found at a much greater depth below the stream bed than was anticipated. Such conditions may call for a practically complete re-design of the dam, especially in case of an arch dam of variable radius. For instance, during the construction of several dams of this type with which the writer is familiar, the excavation was carried somewhat farther into the side abutments than had been assumed in the preparation of the original designs. It became necessary, therefore, to change the arch radii at practically all elevations of the dam. Prompt co-operation between the designers and the contractors was necessary to take care of these and other incidental changes, in order not unduly to delay the construction program of the contractors.

Mr. Bent stresses the desirability of a railroad for major projects.† Where the concrete aggregate is to be brought from the outside over considerable distances, a railroad is no doubt advisable. However, where cement, steel, and contractor's equipment are the main items involving transportation, the use of trucks has been found to be more economical. For the Coolidge Dam, now under construction on the Gila River, in Arizona, the contractors have installed an aerial tramway between the gravel-pits in the river bed below the dam site and the mixing plant on the side hill above the dam. All other materials are hauled by truck. The nearest railroad station is San Carlos, located less than 10 miles from the dam site. This job is completely motorized, and is conspicuous by the absence of horses and mules.

The writer has been identified with the design and construction of about twelve dams that have been built since 1918, or are now in the course of construction. In more than one instance he has seen contractors lose large sums of money, quite unnecessarily, on account of the violation of one or the other of the fundamental principles that are so ably presented by the author.

* *Proceedings*, Am. Soc. C. E., September, 1927, Papers and Discussions, p. 1722.
† *Loc cit.*, p. 1723.

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PAPERS AND DISCUSSIONS

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NEW THEORY FOR THE CENTRIFUGAL PUMP

Discussion*

BY MESSRS. R. T. LIDDICOAT, LLOYD H. DONNELL, AND S. TIMOSHENKO.

R. T. LIDDICOAT,† ESQ. (by letter).‡—The writer has studied the problem and his conclusions are squarely in line with Professor Sherzer's arguments.

It seems that some of those interested in centrifugal pumps do not realize what findings Professor Sherzer has presented. If he is correct in his work, he gets an amount of energy h ft-lb. per lb. of water pumped at a given revolution per minute, whereas by the present theory he should get $2h$; but this is not all. He assumes that he pumps water without losses as do the proponents of the present theory. Both theories cannot be right. To determine the maximum energy a given pump can give to the water at a given revolution per minute the following assumptions are made:

1.—No losses occur.
2.—The prime mover maintains a constant revolution per minute.
3.—A balanced condition has been reached; that is, output of water and input are equal.
4.—The casing rotates with the impeller.

The writer proposes to prove that for these conditions, the energy per pound of water discharged by the pump is $\frac{V^2}{2g}$, in which, V is the tangential velocity of water as it leaves the prime mover. This tangential velocity will be the velocity of a point on the rim of the impeller, if and only if, the impeller completely fills the casing. Professor Sherzer has shown that if the impeller has less diameter than the casing, the value, V, must be corrected to give the real tangential velocity of the water. To avoid any confusion regarding conditions after the particles are thrown off by the prime mover, the writer proposes to build a casing around the impeller, so as to know definitely what water is and is not rotating. To be more specific, the energy given to the water by the prime mover is the important factor to be considered.

* Discussion of the paper by A. F. Sherzer, Assoc. M. Am. Soc. C. E., continued from April, 1928, *Proceedings*.

† Asst. Prof., Dept. of Eng. Mechanics, Univ. of Michigan, Ann Arbor, Mich.

‡ Received by the Secretary, February 29, 1928.

What is done with the energy after it is developed is of no interest at present. The writer is only trying to prove that the most the prime mover can possibly give the water is $\frac{V^2}{2\,g}$ and not $\frac{V^2}{g}$.

In building the centrifugal pump, the purpose is to construct an apparatus with which to whirl water as efficiently as possible. Considering, first, the shut-off conditions when no water is pumped, suppose the water whirled is contained in a cylindrical box of a radius, r. Suppose that in this box there are radial vanes, quite close together. For convenience, put the box on a vertical axis as shown in Fig. 32(a).

Fill the box at atmospheric pressure and then seal it completely. Now the prime mover brings the water to a stated speed and then keeps it moving uniformly. The pressure developed according to well-known laws will be atmospheric at the center and will attain a maximum at the outside. Taking the center as an origin and plotting pressure against radial distance, a parabolic curve is obtained as shown in Fig. 32(b).

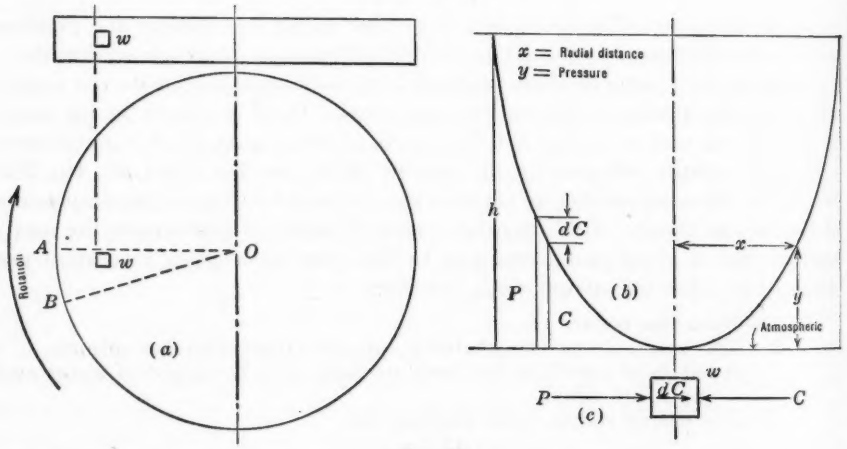

FIG. 32.

A careful study of conditions in the closed box will throw considerable light on the centrifugal pump. Consider the particle, w, of Fig. 32(a) after the cylinder has attained its constant speed. In Fig. 32(c), the particle, w, is shown as a free body with the radial forces acting on it.

The radial forces acting will be: P, caused by the water pressure back of the particle, w; C, caused by inertia due to centrifugal force, of the column of water between w and the axis of rotation; and dC, caused by the inertia due to the centripetal acceleration of the particle, w.

P will always be greater than C by an amount just sufficient to keep particle, w, traveling in a circular path. This is the important factor in the writer's argument. Particle w must be forced to travel in a circle; it will not do so otherwise. However, it travels in a circle only as long as the difference between P and C is dC (Fig. 32(c)). This will hold absolutely true as long as P and C follow the parabolic pressure distribution shown in Fig. 32(b).

Now, suppose that, by some means, it were possible to give w a slight radial velocity, V_r, toward the center of the cylindrical tank. Particle w, now having all radial forces balanced, then moves toward the center of rotation, O. Having more tangential velocity than it needs for the new position, it is slowed up tangentially by pressing on Vane AO (Fig. 32(a)). At the same time the particle taking the position formerly occupied by w must be speeded up tangentially. Therefore, it presses against Vane BO so that the prime mover is not affected. The radial forces are still perfectly balanced on the particle, w, although P, C, and dC have all changed in value; w continues on its way to the center at this constant velocity. To get a clear idea of this behavior, consider the actual water surface to take the form of the paraboloid of revolution (Fig. 32(b)) and then, putting in the guide-vanes, consider the motion of the particle, w, in the form of a wood block having the density of water and acting under similar initial radial velocity conditions at the water surface.

Conclusions to Be Drawn from Shut-Off Condition.—Neglecting losses and assuming perfect guide-vanes, a particle of water is free to move radially. Given a radial velocity, V_r, that velocity will be maintained as long as the parabolic pressure distribution, originally assumed, exists.

Now, suppose water is admitted at the center of the pump with whatever radial velocity, V_r, is needed to supply the demand. Then a series of holes is opened all around the periphery of the cylindrical tank. The prime mover maintains the constant speed necessary to make input equal to output. As long as the pump remains full and slip is neglected (which will be the best condition attainable), the pressure distribution within the cylindrical box will have the parabolic distribution. In any position, the forces acting upon the particle, w (P, C, and dC), are balanced just as they were in the box at shut-off.

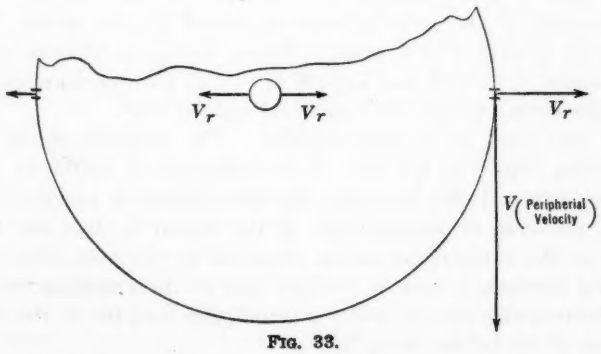

FIG. 33.

P is just enough greater than C to cause w to travel in a circular path. Particle w now has come from the center and has a radial velocity toward A, that remains a constant as long as the particle is in the pump. It is important to note the fact that, as long as Particle w is in the cylindrical box, it must travel in a circle. Since it travels in a circle only as long as P is greater than C, it follows that P is necessary to keep the particle in the box and after

it leaves, of course, P, C, and dC all vanish together. Therefore, the writer believes that, after a careful unbiased study, it will be agreed the particle, w, entered with a radial velocity, V_r, and left with identically the same radial velocity.

Based on this discussion, the energy considerations are quite simple. The velocity of the jet issuing from the cylindrical box (Fig. 32(a)) will be as shown in Fig. 33.

The total energy contained in the water is obviously $\frac{V_r^2}{2g} + \frac{V^2}{2g}$, but $\frac{V_r^2}{2g}$ was given to the water before it entered and must be deducted from the output to give that furnished by the prime mover.

Some will say that the pressure head at the point of discharge has been neglected. This pressure was needed to keep the particles in the box in their circular paths.

Another point of view on energy is that particles entering with radial velocity, V_r, and no tangential velocity, are given a tangential velocity, V, by the prime mover, or the input is $\frac{V^2}{2g}$.

The purpose of allowing the casing to rotate is simply to place definite limits on the subject-matter. The energy given to the water by the prime mover is the important factor.

LLOYD H. DONNELL,* ESQ. (by letter).†—The author brings up the interesting question as to whether, in a rotating body of liquid, there is both kinetic energy due to its motion and potential energy due to its pressure, or whether these are merely two successive steps in the transformation of the same energy from one form to another.

At first, the latter view does not seem unreasonable. It is natural to feel that the pressure is due solely to the motion and that this pressure would instantly disappear if the velocity were destroyed by any means, by friction or by changing it into useful work. Hence, trying to change the velocity energy into useful work and then expecting to still have pressure energy available seems like trying to "eat the cake and have it too".

However, this rests on a misconception. The pressure of any part of a body of rotating liquid is not due to its velocity; it would be possible to destroy its velocity and still have the pressure remaining exactly the same as before. The pressure of any particle of the liquid is due, not to its own velocity, but to the centrifugal forces produced by the velocities of the particles of liquid between it and the center, just as the crushing pressure on a brick at the bottom of a pile of bricks is principally due, not to its own weight, but the weight of the bricks above it.

A centrifugal pump can be considered roughly as a casing containing a body of liquid kept rotating by a motor-driven paddle-wheel or rotor, water being drawn off as desired from the outer layers, which are under pressure, and allowed to flow in at the center. The water might be imagined to be divided along the radius into thin layers. The outermost layer flows out through the discharge pipe and the next layer moves up to take its place, etc.

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† Received by the Secretary, February 29, 1928.

The point to be noted is that the tangential velocity of each successsive outermost layer could be completely destroyed just before it was discharged, without affecting its pressure (as this pressure is due to the centrifugal force on the inner layers and only negligibly to the centrifugal force on the outermost layer itself, if the layers are assumed to be thin enough). The velocity could be destroyed by friction with the casing wall (as must actually be the case, to a large extent, at very low discharge rates), converting its energy into heat; or, theoretically, it could be changed into useful work with a set of miniature water-wheels. Thus, water would be discharging under the full pressure head and at the same time the kinetic energy of all the water discharged (no matter how thin each successive layer was assumed) would be converted into heat or some other separate form of energy.

Hence, there is, in the casing, both pressure energy (of which full use is made) and kinetic energy, which must have been paid for at the drive shaft, but which there is evidently some difficulty in utilizing. Some of it can be used as velocity in the discharge pipe, especially at high discharge rates when this velocity is comparable to the tangential velocity in the casing (hence it is possible to get high energy efficiencies at full discharge). However, especially at low discharge, much of the velocity energy must be changed into pressure energy, or else it will probably be wasted in eddy currents.

It is possible to change velocity energy into pressure energy efficiently, by providing an expanding closed path in which the velocity gradually decreases as the pressure increases, as in a Venturi meter. It is hard to see how a spiral casing provides any such thing, because the expanding passage formed is entirely open all along one side and the pressure kept more or less constant all along this side. However, a closed expanding passage can be provided, either outside the casing with the small end forming the discharge opening of the casing, or in the form of a number of small passages arranged around the rotor inside the casing. To be effective, however, the velocity inside such a passage at the small end (which depends on its area and the discharge rate) must be the same as that of the water just outside the passage (which depends largely on the speed of rotation of the pump). With a fixed pump speed this can be the case only at one particular rate of discharge. Such a passage will be ineffective or even harmful at other rates of discharge—unless its size can be varied by some mechanical means.

S. TIMOSHENKO,* ESQ. (by letter).†—This paper contains in several places a severe criticism of the usual theory for centrifugal pumps. The accepted theory is based on the assumption of an "ideal fluid" and on the neglect of various kinds of losses of energy; hence it is quite natural to expect considerable discrepancy, between the theoretical and experimental results. Although this discrepancy is very large the writer does not agree with Professor Sherzer that the causes are fundamental errors in the present theory, such as an erroneous addition of the "pressure head" to the "velocity head"‡ or a neglect of the pressure in the casing which prevents the free radial acceleration.§

* Dept. of Eng. Mechanics, Univ. of Michigan, Ann Arbor, Mich.
† Received by the Secretary, March 3, 1928.
‡ *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, pp. 1777–1778.
§ *Loc. cit.*, p. 1781.

The technical part of the paper contains very interesting experimental results and applications of these results in design. However, the writer wishes to discuss only that part in which the usual theory is criticized. By use of two simple examples, it may be shown that the "velocity head" and the "pressure head" are not "part and parcel of each other," but are separate and distinct quantities, both of which can be utilized under certain conditions. In the usual theory, the pressure in the casing does not affect the amount of work done on the water in the radial grooves of the runner. Consider, first, a fluid rotating in a circular cylindrical container with a constant angular velocity, w (Fig. 34 (a)). It is well known that the surface of the rotating fluid will be a paraboloid of revolution. At the center, a, the velocity of the fluid and the pressure are both zero. At the point, c, the pressure is also zero, but the velocity is $v = wz_1$ and kinetic energy of 1 lb. of the fluid at this point will be $\frac{v^2}{2g}$. At the point, b, the kinetic energy and the so-called "pressure energy" are both equal to $\frac{v^2}{2g}$.

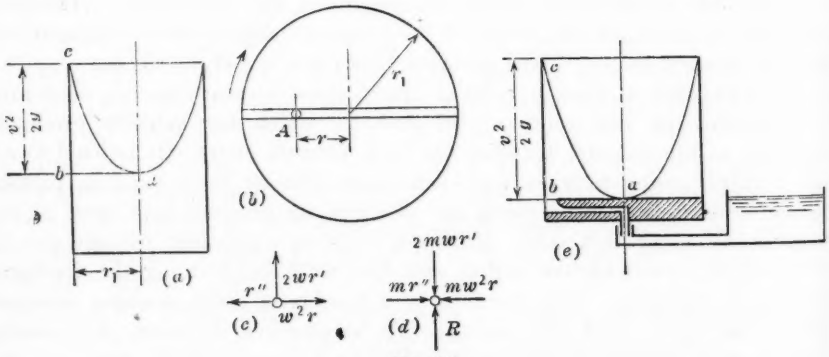

FIG. 34.

Consider, as a second elementary problem, the motion of a ball in a smooth groove of a rotating disk. For simplicity, assume that the groove is radial (Fig. 34(b)), that there is no friction, and that the ball can move freely along the groove. In order to investigate the motion of the ball, apply the principle of d'Alembert. On this basis equations for bodies in motion can be obtained in the same manner as equations for bodies in equilibrium, it being necessary only to add the inertia forces to the external forces. The inertia forces will be obtained by multiplying the components of acceleration with $-l$ and with the mass of the moving ball. In the present case, the complete acceleration of the ball will consist of three parts: (a) The acceleration of the ball in its relative motion along the radial groove, which will be represented by the second derivative, r'', of the radial distance, r, of the ball with respect to time; (b) the acceleration of A the instantaneous point of contact between the ball and the disk. In the case of uniform rotation, this is given by the magnitude of the centrifugal acceleration, w^2r; (c) Corioli's acceleration, $2wr'$, which is perpendicular to the relative velocity, r', in the direction of motion. These

three components of acceleration and the corresponding inertia forces are shown in Fig. 34(c) and (d). In addition to inertia forces, the external forces acting on the ball should be considered. Neglecting the weight of the ball, only one force remains—the reaction, R, of the groove, which produces the work on the ball. The differential equations of motion, (Fig. 34(d)), will be:

$$mr'' - mw^2r = 0 \quad \ldots\ldots(52)$$

$$R = 2mwr' \quad \ldots\ldots(53)$$

Equation (52) determines the motion of the ball along the groove. It may be written in the form:

$$r = r_0 \cosh w\,t + \frac{r_0'}{w} \sin w\,t \quad \ldots\ldots(54)$$

in which, r_0 denotes the initial displacement, and r_0' the initial relative velocity of the ball. Consider only the particular case, $r_0' = r_0 w$; that is, the ball at the initial instant has its radial and circumferential velocities equal. Under such an assumption:

$$r = r_0\,(\cosh wt + \sinh wt) \quad \ldots\ldots(55)$$

$$r' = r_0 w\,(\cosh wt + \sinh wt) = rw \quad \ldots\ldots(56)$$

The radial component of the velocity remains equal to the tangential component during the motion of the ball in the groove, and the ball will leave the disk with the total velocity equal to,

$$\left(\sqrt{(r')^2 + w^2 r^2}\right)_{r=r_1} = \sqrt{2}\, w\, r_1 = \sqrt{2}\, v$$

in which, v denotes the circumferential velocity of the disk. The kinetic energy of the ball is equal to,

$$E = m v^2 \quad \ldots\ldots(57)$$

Neglecting the initial value of this energy, and assuming that the ball has been introduced at the center of the disk, the kinetic energy, Equation (57), must be equal to the work done by the reaction, R, which is the only force acting on the ball. In calculating this work, Equation (53) will be used. The reaction, R, is always perpendicular to the radius, and its moment with respect to the center of the disk will be $2\,m\,w\,r'\,r$. The work done by this moment per element of time, $d\,t$, will be $2\,m\,w^2\,r\,d\,r$, and the complete work produced on the ball during its motion from $r = 0$, to $r = r_1$ will be:

$$W = \int_0^{r_1} 2\,m\,w^2\,r\,d\,r = m\,v^2 \quad \ldots\ldots(58)$$

which agrees with Equation (57). Imagine now that the ball is no longer free to move along the groove, and that a radial force, F, directed always toward the center of the disk, opposes the free motion of the ball in the groove. It is easy to see that in such a case Equation (54) remains unchanged and Equation (52) must be replaced by:

$$m\,r'' - m\,w^2\,r + F = 0 \quad \ldots\ldots(59)$$

The relative velocity of the ball will depend now on the magnitude of the force, F. When this force is very small and can be neglected, Equation (52) is sufficient. When the value of F approaches $m\,w^2\,r$ (the centrifugal force), the relative velocity, r', approaches zero, the total velocity of the ball leaving

the groove approaches v, and the kinetic energy of the ball will be $\frac{m\,v^2}{2}$; that is, only one-half the kinetic energy obtained for the case of the free relative motion of the ball in Equation (57). Then, considering the work done on the ball during this very slow motion along the groove, it is easy to see, from Equation (53), that the reaction, R, of the groove diminishes in the same proportion as the relative velocity, r', but the time of action of this force increases in the same proportion and, finally (Equation (58)), the work done on the ball does not depend on the relative velocity, r', and is always equal to $m\,v^2$. In this case, however, only one-half this work is transformed into the kinetic energy of the ball and the other half is spent on work against the pressure, F.

In order to apply this discussion to the study of pumps, unite the two problems represented in Fig. 34(a) and (b), into that shown in Fig. 34(c). The circular cylinder rotates about its vertical axis with a constant angular velocity, w, such that the parabolic surface of the liquid is tangent to the bottom of the container at the center, a. The bottom of the container consists of a circular disk with a smooth radial channel, $a\,b$, and provision is made for bringing the liquid to the center, a, of the disk with zero pressure and zero velocity. When the opening, b, is closed the same conditions occur as in the case shown in Fig. 34(a), and if friction is neglected, rotation with constant angular velocity will proceed without the introduction of any work from the outside. Imagine, now, that there is an opening at b such that the liquid is moving slowly along the channel, $a\,b$, and that the depth of the container is a little less than $\frac{v^2}{2\,g}$. Then, for each pound of fluid brought (at zero pressure and zero velocity) to the point, a, 1 lb. of liquid will be delivered at the point, c, where the velocity is equal to v and the pressure is equal to zero. From the discussion of Fig. 34(b), it is clear that the work done on each pound of liquid during its motion along the channel, $a\,b$, is equal to $\frac{1}{g}\,v^2$. This work will be spent partly in increasing the kinetic energy of the fluid and partly in overcoming the pressure.

Considering now the extreme case of a very slow motion in the channel it is found, as before, that one-half the work will be transformed into kinetic energy, $\frac{v^2}{2\,g}$, and the other half into the so-called "pressure energy." In this manner there occurs at the point, b, a "velocity head" equal to $\frac{v^2}{2\,g}$ and a "pressure head" equal to $\frac{v^2}{2\,g}$. Both these parts have a definite physical meaning, namely, for each pound of liquid introduced at the point, a, there will be obtained a pound of liquid at c, at which point it has the kinetic energy, $\frac{v^2}{2\,g}$ and the potential energy, $\frac{v^2}{2\,g}$. The sum of these two forms of energy is equal to the

work, $\frac{v^2}{g}$, done during motion of the liquid along the channel, $a\ b$, which work is exactly the same as in the case of the free radial acceleration, which occurs in pumps without casings and discharging in the air.

By a special arrangement, the kinetic energy of liquid at the point, c (Fig. 34(c)), can be utilized and if there are no friction losses or other sources of dissipation of energy, the liquid will be raised to the level of $\frac{v^2}{g}$ given by the usual theory.

Thus, it is evident that there are no fundamental errors in the usual theory. It will give correct results provided the assumptions on which it is derived are fulfilled. In actual pumps, if such a level as $\frac{v^2}{g}$ is not attained, it must be attributed to various kinds of losses in energy which are not taken into consideration by this theory, and to other deviations from the assumptions made. It is assumed, for instance, that in its relative motion the fluid has a velocity uniformly distributed over the cross-section of the groove and that this velocity has a radial direction. More detailed investigations* show that the distribution of the velocity over the cross-section is not uniform. Velocity is smaller at the side of the groove at which pressure on the liquid is produced and larger on the opposite side. The same investigations show also that in a radial groove the relative velocity of the fluid near the periphery is no longer in a radial direction, and that it has a component in a circumferential direction. These deviations from the assumptions made in the usual theory must affect the final result, and may yield some explanation of discrepancies between theoretical and experimental values.

* See, for instance, W. Kacharski, "Stroemungen einer reibungs freien Fluessigkeit bei Rotation fester Körper," 1918, and W. Müller, "Allgemeine Sätze über die Strömung in Kreiselzädern und Turbinen," *Zeitschrift für angewandete Mathematik und Mechanik*, Bd. 7, 1927, s. 347.

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WATER-POWER APPRAISALS

Discussion*

By Messrs. J. H. Lance and William H. Cushman†

J. H. Lance,‡ M. Am. Soc. C. E. (by letter).§—The author has apparently overlooked the fact that appraisals or valuations are differently made for different purposes. He does not state the particular purpose which he had in mind, in writing this paper. If he had in view the valuation for purchase or sale, condemnation, capitalization or taxation, the writer is heartily in accord with the statement‖ that the only exact method of valuation is through capitalization of earnings.

If, however, the object is rate-making, then the method of real estate sales will be found to be the best guide. It has the sanction of most public service commissions. Manifestly, the capitalization of earnings which are based on rates, cannot be used for computing values from which rates are to be determined.

The author has rejected one method of making a valuation, which he calls "The Steam Substitution Method" while accepting "The Capitalized Net Earnings Method." As a matter of fact, in practically all sections of the United States, the use of the latter method embodies that of the former. In other words, in order to determine the earnings, the cost of the output must be compared with that of similar power in the vicinity. In most cases this similar power will be produced at a steam-generated station.

There is no fixed value of a unit of power. The value of any power site to a purchaser depends on the amount of power which can be developed, the cost of its development, the nearness of the site to a load center, and the value of a unit of power at that load center. It depends also on the form of the load curve, and on the position on that load curve which will be occupied by the block of power that may be produced at the water power site. Within reasonable limits, then, due to reasonable variations in basic assumptions, the earning capacity of a site may be definitely established. The capitalization of these earnings at a fair rate of return represents the upper limit of the values or the value to a purchaser.

The lower limit of value is probably that of wild mountain land in the vicinity. Somewhere between these two limits will be found the selling price

* Discussion of the paper by William H. Cushman, M. Am. Soc. C. E., continued from March, 1928, *Proceedings*.

† Author's closure.

‡ Cons. Engr. (J. H. & W. Lance), Wilkes-Barre, Pa.

§ Received by the Secretary, March 5, 1928.

‖ *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1844.

of the power site, the amount of which is agreed upon by bargaining, and is dependent largely on whether or not the seller is informed.

There seems to be considerable misapprehension as to the meaning of the word, "value". The selling price may represent what the Courts term the "fair market value." It does not represent the value to the seller, which it exceeds, and it is necessarily less than the value to the buyer. If this were not so, there would be no sale, for there would be no advantage to either party in the sale. Glass beads in Holland or England, perhaps, have had one value to the seller and another to the buyer. Those same beads at some trading post in the Far North have had one value to the trader and another to the Indian trapper, who exchanged for them furs worth many times the value of the beads to the trader, and perhaps thousands of times their original cost. Thus, the seller of anything would always rather have the money price than the article itself; and the buyer would rather have the article than the money price. The article actually has a different value to each, of which value the money price approximates the mean.

The author states* that "a method of valuation that has no standing in Court is unavailable for an appraisal engineer." It must be admitted, though, that to date, the consideration of previous real estate sales has had the favor of the Courts, as against the capitalization of earning capacity. This may be due to a not unnatural reluctance on the part of the Courts to delve into computations which may appear to be abstruse; or, on the other hand, it may be a heritage from the days when every piece of land was just so much land, to be valued at the same acreage rate as "similar lands in the neighborhood," regardless of its special value for special purposes. Previous sales of lands in the vicinity can, by no stretch of the imagination, be used as a basis for the valuation of a water power property. They may be so used in the case of city real estate; and even then must be carefully weighted by reason of the purpose for which the property is to be used, the traffic, and many other considerations. The purchaser of a water power site does not think in terms of past sales. He says, "What can I make out of it?" Then he fixes its value to him on that basis, rather than on the basis of what some one else has paid for a water power property.

That there is a trend of thought in the Courts toward the allowance of evidence of special values, where such values exist, is clear from a perusal of some recent Court decisions. However, the scientific demonstration of the amounts of such values has been barred, as being too long a stride from the path of precedent. Instead, witnesses may give their opinions of the value of the property as a whole, but they may not, on direct examination, enter into an analysis with the intention of demonstrating such values. In the recent decision of the Pennsylvania Supreme Court, in the case of McSorley *vs.* Avalon Borough School District, the syllabus was to the effect that:

"In determining the damages to which an owner is entitled for the taking of his property under the right of eminent domain, it is not proper to appraise separately the various things which give to the property its value, and add them together for the purpose of fixing the damages: the only permissible evidence is as to the value of the property in its entirety, but the witnesses may testify as to those things which tend to give it value, including

* *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1844.

buildings or anything else on or under it, which a proposed purchaser would probably consider in determining whether or not he would buy it at an enhanced price."

The exclusion from the evidence of computations of value results, of course, in giving the witnesses far greater latitude. A study of any condemnation case indicates that the wide variation in valuations by the different witnesses can be explained only through this cutting loose from facts and figures; and, possibly, through tying up to the imagination instead. In the case referred to, it is interesting to read from the decision:

"No other evidence is as unsatisfactory as that necessarily received in this class of cases. Courts and juries are given by the witnesses, matters of opinion only, the accuracy of which is far more difficult to test than are disputed questions of fact."

Can it not be possible that the condition just complained of by the learned Court is one of its own making, through the principle which it enunciated in the same case, just quoted, and which has been followed in numerous other cases?

The writer ventures to think that most of the Court's troubles in this respect would disappear if it would not only abandon this position, but would insist that a witness demonstrate to the satisfaction of the Court, the values to which he testifies, and that he give facts and figures, rather than "judgment".

The writer feels that thanks are due Mr. Cushman for opening so interesting a subject; and believes that the Society can do much to classify the present beclouded position by going on record as outlining a standard form of procedure for engineering witnesses in giving testimony in condemnation cases.

William H. Cushman,* M. Am. Soc. C. E. (by letter).†—In the preparation of this paper no hard and fast rule for valuation was attempted. In the "example" quoted,‡ the appraisal was made to show the productive value of a certain water-power property to a prospective purchaser. Other appraisals might have been quoted that would have shown values for rate-making, tax, or investment purposes. It is in no wise inconsistent that different valuations should be placed on the same property for differing uses.

In some instances appraisals have been made that simply developed the "annual net earnings" of a property, based on current rates for power. When the purpose for which the valuation is made, is not disclosed by the client, this would seem to be the best practice. The function of the engineer would seem to end at this point, as the client, having been advised of the earning capacity of the developed property, can then decide on his own system of financing or can adapt the rate of capitalization to the situation in hand. In other words, calculating the net earnings is engineering; capitalization enters into the realm of finance.

In several of the discussions, stress has been very properly laid on the basic importance of a correct determination of the several factors used. Cost of production, market value of power, and quantity of available power, must be studied in detail for each individual situation, because the local conditions are subject to wide variations.

* Hydr. Engr., Watertown, N. Y.

† Received by the Secretary, March 13, 1928.

‡ *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1848.

Mr. Huber,* after expressing a concurrent view as to differing valuations according to the purpose of valuation, states that the method under discussion is "wholly untenable as a basis for rate-making" on the ground that it is "reasoning in a circle". The public service regulation of the rates of a certain company are based on contemporary rates for similar situations. They are rarely, if ever, subject to "over-night" changes. With a rate so established in a certain district, a water power property located therein may be valued by the "Capitalized Net Earnings" method based on that rate, provided a market exists for this additional supply. There would seem to be no "reasoning in a circle" in such case.

Mr. McCurrach† criticizes the sale of the power at the point of production. In the "example" referred to, it so happened that the power was sold in this way and that such procedure is common practice in that vicinity. It goes without saying that if reaching the market required a distribution system, transformers, etc., that the cost and operating charges for these items would be included in the valuation calculations.

In mentioning the market rate of $27 per h.p.-year for power in the Pacific Northwest as against the $42 used in the "example", the contrast in rates between different localities is all that is shown. In the vicinity of the plant described, many paper mills are purchasing "primary" and "second-class" power at $42 and $20, respectively. The "super-power groups" exchange power at the rates of 1 cent and ½ cent per kw-hr. for day-shift and night-shift power, respectively. This power is deliverable also at the point of production.

The concluding paragraph of Mr. McCurrach's discussion, stating his experience that minor variations of basic data are "of major importance in the capitalized net earnings theory", is very much to the point and emphasizes the importance of correct determination of the factors used.

Mr. Justin's discussion‡ is effective in representing the attitude of large public service corporations engaged in power distribution work. Quoting from the concluding paragraph of his discussion: "The capitalized difference between the annual costs at the two plants [steam as against hydraulic power] may be taken as the ultimate value of the undeveloped water-power site to the power company". This may be a tenable theory as to the value of an addition to a group of power properties owned by a distributing company, but the thought occurs that the difference in value between two horses does not represent the value of either horse. The last four words, "to the power company", qualifies his statement.

Mr. Wood§ mentions several references to data on stream flow, and states that the method used for "analyzing stream flow and estimating power available should not pass unchallenged". There is nothing new in the use of "mean monthly averages" to determine the flow available for power development. It represents standard practice among a great many engineers. More recently, weekly averages have come into use and are possibly better suited to determine the limit of profitable plant installation.

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 325.
† *Loc. cit.*, p. 326.
‡ *Loc. cit.*, February, 1928, Papers and Discussions, p. 625.
§ *Loc. cit.*, p. 627.

"The proof of the pudding" is what counts. In the writer's experience, mentioned in the last paragraph of his paper,* several instances have occurred where the annual horse-power available for sale, was calculated according to the method under discussion. In these instances plant records, of actual power produced and sold, became available and the manner in which the actual practice checked up the calculations was surprising and, to the writer, convincing.

A typical instance occurred in the appraisal of a chain of seven water-power plants, operated by a public service corporation. Five years after the appraisal the production records of this company came to light. Taking the three years next preceding the date of appraisal (no earlier data being available) the average annual quantity of power produced and sold, checked the appraisal figures within less than 2% (1.76%). Again, a recent appraisal showed a calculated annual production of available power aggregating 61 000 000 kw-hr. per annum. This quantity being called in question, plant production records were consulted. Only the records since development were available, that is, 1926 and 1927; these years showed 65 000 000 and 59 000 000 kw-hr., respectively. There have been other instances of like character. A study of ten or twelve cases has developed no instance where the actual production record varied more than 6% from the calculated quantity and the average production always exceeded the calculated quantity.

The experiences quoted have led to a belief that a study of a typical year of stream flow, based on a reasonable cycle of gaugings, will show by analysis, considering the flow on the lowest day and up to the flow limit of profitable development, that the annual product will check the values found by using the mean monthly averages, within reasonable limits. The writer has recently reviewed a 9-year gauging record of a river, examining each day's flow and eliminating all water in excess of the "logical limit of development". The annual amount of available power proved to be 0.00112% less than that found by using mean monthly averages. In another case, a 17-year period of gaugings was similarly treated and the resulting quantity of gross horse power was reduced to below the "mean monthly" figures by slightly less than 3 per cent.

Mr. Wood's point† on the use of "primary" and "second class" as definitions is well taken, but so many grades of power seem to be indicated by terms in current use that here, again, the writer offers nothing new. Engineering literature teems with power classification terms, namely, primary, guaranteed, firm, continuous, secondary, periodical, surplus, and dump. In a recent suit, power was referred to as "day-shift" and "night-shift" power and, in this instance, it avoided misunderstandings.

Without attempting to reconcile the writer's theory of appraisal with the methods referred to by Mr. Wood, it is quite probable that a judicious selection of the rate of capitalization applied to the former method, would approximate values obtained by the methods that he quotes.

Mr. Davis‡ takes up the matter of valuation by the Real Estate Sales Method and the attitude of the Courts toward it. The rule under which condemnation proceedings are conducted is expressed in the phrase, "the value

* *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1850.
† *Loc. cit.*, February, 1928, Papers and Discussions, p. 628.
‡ *Loc. cit.*, p. 633.

before and after taking as between a willing buyer and a willing seller". This involves bargaining, and when a bargain is not tendered there usually is no sale. A property, valued on its full earning capacity, cannot be considered a "bargain" in the ordinary sense; hence most sales reflect less value than actually exists. Stocks and bonds are ordinarily sold on the basis of their full earning capacity; all the "bargain" presented lies in the hope that their value will be increased by future increased earning capacity. Certainly, water power has as promising a future as most commodities. A record of sales becomes a list of properties sold for less than they were worth, and Court findings, based on sales, operate to the disadvantage of the vendor.

The statement* is made:

"When a water power, * * *, is an integral part of a manufacturing operation, it should not ordinarily be separated from the other component parts of the operation and assigned a separate value. * * * The value of the part remaining might be practically destroyed in case the industry could not operate at a profit * * *."

Carrying this theory to its logical conclusion, consider an industry which, with its own water power, cannot operate except at a loss. The industry would become a liability instead of an asset. The manufacturing portion of the industry becomes scrap; but, what of the water power? As a part of the liability, does it become worth less than nothing? It would be equivalent to stating that the coal stored in the mill yard was valueless because the mill could not operate at a profit. Many decadent businesses "carry on" on the strength of owning water power, when the power, if sold, would produce a larger annual return than the business. Water power, located in the center of a good market, has a definite value easily to be determined. It is a commodity of commerce. Its misapplication to a losing industry in no way alters its sale value.

Mr. Sherman† attributes to the paper the attempt to lay down a hard and fast rule, whereas it simply specifies a line of procedure. The factors used must, of necessity, be determined from experience and the study of all the surrounding circumstances, such as would be used to formulate any mature judgment. As his discussion develops to the point where "pitfalls" and "capitalized errors" are treated, no question can arise as to the correctness of his viewpoint.

In discussing‡ "Load Factor", he expresses the usual experience, with which the writer agrees. In the "example" used, the load factor for "supplying the wholesale market" was known to be closely variable around 75 per cent. In instances where the factor cannot be so readily determined, extreme care must be used to avoid serious errors. The capitalization rates quoted by Mr. Sherman, on different characters of risks§, are in line with customary practice and coincide with the views of the writer. There is much other valuable material in his discussion.

Mr. Lance so fully expresses the views of the writer and puts his case so well, that the only exception taken to his reply is in regard to "rate-making".

In general, the discussions submitted have been fair and have presented new angles and viewpoints on a most interesting subject. The writer is grateful to the engineers who have given their views, based on experience.

* *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, p. 634.
† *Loc. cit.*, March, 1928, Papers and Discussions, p. 837.
‡ *Loc. cit.*, p. 838.
§ *Loc. cit.*, p. 841.

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PAPERS AND DISCUSSIONS

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SIDE SPILLWAYS FOR REGULATING DIVERSION CANALS

Discussion*

BY MESSRS. A. P. FLOCKART AND GEORGE M. BACON.

A. P. FLOCKART,† ESQ. (by letter).‡—The structures described by Mr. Nimmo were designed and erected prior to the publication of information on hydraulic jump problems§ by Julian Hinds, M. Am. Soc. C. E. It was intended that at full supply 450 cu. ft. per sec. should flow 5 ft. deep at 10 ft. per sec. through both wood and concrete flumes, no allowance being made for possible difference in hydraulic properties of the sections, nor for the obstruction caused by the gate stanchion. In operation, 410 cu. ft. per sec. ran 5 ft. deep in the wooden flume; it drew down through the transition to a depth of 3 ft. 6 in. just beyond the gate, and then gradually rose to normal depth some distance down the concrete flume. Experiments were then undertaken to determine what was causing the apparent throttling, with the result that the gate stanchion was ruled out, as also was the slight curve preceding the gate.

About 1920 the method of analysis by drawing the minimum energy line was introduced. This showed a hump at the end of the wooden flume section and also showed that the water levels down stream were not high enough to drown out this possible control, which, therefore, operated and caused a disturbance down stream. The throttling was thus caused solely by a change of shape of cross-section which has been avoided in the design for permanent works.

In Mr. Nimmo's proposed design, provision has been made for all the flow conditions that can be foreseen, and it is hoped that Equation (9)‡ will prove to be accurate. There is a great need, however, for some simpler

* Discussion of the paper by W. H. R. Nimmo, Assoc. M. Am. Soc. C. E., continued from February, 1928, *Proceedings*.

† Hydr. Engr., Hydro-Elec. Dept., Hobart, Tasmania.

‡ Received by the Secretary, April 3, 1928.

§ *Engineering News-Record*, 1920.

‖ *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1874.

expression which will indicate the effect of altering the various factors without having to make a complete calculation. It would certainly be of interest to know the effect of sloping the floor of the pool from Station 150 to the outlet with the object of reducing the yardage of concrete.

The statement* that the water surface becomes vertical when passing through the critical point is obviously not correct when, for instance, water passes over a broad-crested weir. An examination of Equation (11)† shows that, at the critical point, not only does the value of $\frac{\Delta d}{\Delta x}$ take the values of plus and minus infinity, but that the equation itself becomes discontinuous. This would be illustrated if the equation could be integrated and a more general expression for the relation between d and x written.

It is possible, however, to integrate the expression graphically by plotting $\frac{\Delta x}{\Delta d}$ on d as abscissa, in which case the areas under the curve represent the lengths of conduit between successive values of d, and so permit the surface curve to be drawn. This does away with the labor and uncertainty of "arithmetical integration", and permits both back-water curves and Equation (11) to be studied. The method is not applicable however, to Equation (9).

GEORGE M. BACON,‡ M. AM. SOC. C. E. (by letter).§—This paper should command the interest of all engineers who have to deal with problems involving the distribution of water. All such engineers are constantly on the watch for, and seeking after, devices which shall be perfectly automatic in their action. For engineers in general, the big problem is to provide for the situation when all is not going exactly as planned.

Past observations have long ago given the writer a definite suspicion that large volumes of flowing water fail to behave in accordance with any set of formulas. It would be interesting to see what results would be obtained from analyzing this problem on the basis of a thin sheet of water the lower surface of which was that bounded by horizontal lines extending across the flume from the lip of the spillway. This thin slab would naturally have the velocity of the water immediately below its lower bounding surface and the assumptions made as to one would, in a measure, apply to the other. This slab could be considered as being acted upon by two forces, one representing the momentum of the water along the direction of the flume and the other a force represented by the slope, at right angles to the flume, which the upper surface of the slab must take in discharging water over the edge of the flume.

In Equation (1),‖ how is the momentum that has just been mentioned taken care of? Independent of this case it is easy to see that this momentum might be sufficiently great to allow very little over-flow. A factor certainly should be taken into account covering the loss of momentum as the flow in the flume loses its velocity.

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* *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1875.
† *Loc. cit.*, p. 1874.
‡ State Engr., Salt Lake City, Utah.
§ Received by the Secretary, April 9, 1928.
‖ *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1873.

It is not entirely clear just how static pressures on the end areas can be compared, with no particular reference to the velocity involved or the force of gravity keeping up or modifying the velocity.

It would be extremely interesting to have the results of the experiment which the author states* is fully described in his thesis presented to the University of Melbourne. The results would be extremely valuable if they indicated some definite relation between over-flow, capacity of flume, and length of over-flow as calculated from the formula and that found from actual experiment. It is obvious that if no particular limit is fixed for the length of the spillway all surplus water will be ultimately lost especially if reasonable provision can be had for a considerable length of flume in which the flow of water has a chance to become quite uniform and regular.

The results of experiments of this kind might have a distinct bearing on what could be considered the reverse of the present problem, namely, that of a spillway channel of a reservoir which was fed by the ordinary over-flow on a long crest, the channel being designed so as to carry away from the edge of this over-flow the water as fast as it is fed into the channel. This problem would not occur very often in actual practice as conditions usually admit of making such a channel sufficiently deep and steep so that the water in it will not become crowded and thus build up a back-water resistance.

* *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1875.

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PAPERS AND DISCUSSIONS

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A GRAPHIC METHOD FOR DETERMINING THE STRESSES IN CIRCULAR ARCHES UNDER NORMAL LOADS BY THE CAIN FORMULAS

Discussion*

By Fred A. Noetzli, M. Am. Soc. C. E.

Fred A. Noetzli,† M. Am. Soc. C. E. (by letter).‡—This paper is of much value to designers of arch and multiple-arch dams. The labor of calculating the stresses in the elementary arch slices of such structures is reduced to the very minimum by the use of the curves presented by Mr. Fowler.

In order to get an idea of the degree of accuracy that may be obtained with these curves, the stresses in one of the arch slices of the Stevenson Creek Test Dam were calculated and compared with the stresses as determined from the tests.§ Table 5 gives the data for the elementary arch slice at a height of 30 ft. above the base of the dam. For this elementary arch, $\frac{t}{r} = \frac{2}{99} = 0.0202$. The central angle is $2\phi = 43° 30'$. The head of water for full reservoir is 30 ft., but according to the results of the tests only about 84% of the water pressure at this elevation was carried by the horizontal elementary arch, the remainder being supported by the vertical cantilever.‖ The full water pressure being $30 \times 62.5 = 1\,875$ lb. per sq. ft., the net load on the arch element as determined from the test was, therefore, approximately, $0.84 \times 1\,875 = 1\,570$ lb. per sq. ft. Consequently, the "multiplying factor" to be used in connection with Figs. 10 to 13,¶ inclusive, is $\frac{1\,570}{62.5 \times 10} = 2.52$, corresponding to a net

* Discussion of the paper by Frederick Hall Fowler, M. Am. Soc. C. E., continued from March, 1928, *Proceedings*.

† Cons. Hydr. Engr., Los Angeles, Calif.

‡ Received by the Secretary, March 12, 1928.

§ See *Bulletin No. 5*, Arch Dam Investigation, Engineering Foundation; also, *Engineering News-Record*, July 14, 1927, p. 66.

‖ See Fig. 5, *Bulletin No. 5*, Arch Dam Investigation, Engineering Foundation.

¶ *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, pp. 1908–1915.

head of water of 25.2 ft. The stresses in the dam as determined by means of the curves (for "fixed" arches) are given in Column (4) of Table 5. The stresses as found from the tests in 1926 are given in Column (5), and by way of comparison there are shown in Column (6) the values of the stresses as determined in the design of the dam made by the writer in 1925 for the Committee on Arch Dam Investigation.

TABLE 5.

Point.	Stress read from Figs. 10 to 13, inclusive, in pounds per square inch.	Multiplying factor.	Stress in Dams, in Pounds per Square Inch.				
			Fowler.	Test, 1926.	Design, 1925.	Design, 1925, adjusted for comparison (fixed ends).	Design, 1925 (hinged ends).
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Crown:							
Extrados...	310	2.52	780		665	782	692
Intrados...	95	2.52	240	260†	207	243	390
Abutments:							
Extrados...	−2	2.52	−5		−11	−13	516‡
Intrados...	400*	2.52	1 008	940†	879	1 032	516‡

* Exterpolated from curves.

† From "Tests on Stevenson Creek Dam: Deflections and Stresses," *Engineering News-Record*, July 14, 1927, Fig. 5.

‡ Average values.

The agreement between the various values is reasonably close considering the fact that in the design of the dam the net water pressure supported by the arch at the 30-ft. elevation was found to be 1 335 lb. per sq. ft., while the tests gave the somewhat larger figure of 1 570 lb. per sq. ft. In order to afford a comparison on the proper basis between the values of the stresses as determined by the use of the author's Figs. 10 to 13 and those computed in the designed, these latter values must therefore be multiplied by $\frac{1\,570}{1\,335} = 1.175$. The corresponding design stresses would then be as listed in Table 5, Column (7). Thus, if the same water pressure as determined in the tests, is used as a basis for the computations, there is found a reasonably close agreement between the theoretical values of the stresses and those calculated from the strains measured in the dam.

By the use of Figs. 7, 8, and 9* and the "multiplying factor" of 2.52, the stresses of this arch (for hinged ends) are as given in Column (8), Table 5.

A comparison of the values of the stresses found in the tests (Column (5)) with those computed from the curves for "fixed" (Column (7)) and "hinged" (Column (8)) arches, respectively, would indicate that undoubtedly there is a much better agreement between the test values and the theoretical stresses

* *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, pp. 1904–1906.

for a "fixed" arch than for "hinged" end conditions. Inasmuch as the arches of the Stevenson Creek Test Dam are probably thinner than those encountered in most service dams of the single-arch type, it may be concluded that the curves for fixed arches, including shear, are the ones that should be used in all practical cases. The thicker an arch dam the more nearly correct will probably be the assumption of "fixity" at the abutments.

The stresses in the arches of the Stevenson Creek Dam as determined in the design made in 1925 prior to construction were computed by treating rib-shortening as separate from axial arch compression.* In many cases, there is a certain advantage in this method in that it gives a clearer conception of the influence of the rib-shortening pull and of the corresponding stresses. The rib-shortening pull may be assumed as acting in the center of gravity of the arch. If the arch is not of uniform thickness from abutment to abutment, the rib-shortening pull acts in the center of gravity of the elastic weights.

The author's Figs. 10 to 13 do not permit of the determination of the temperature stresses. These stresses can be readily computed by means of the curves and formulas given by Wegmann.*

Temperature stresses in arch dams are of considerable importance as are those due to the shrinkage of the concrete. The stresses resulting from a uniform swelling of the arches due to water-soaking may be computed in a manner similar to those due to temperature. Water-soaking tends to expand the concrete and the effect on the stresses in an arch dam is beneficial rather than harmful.

The author's Figs. 10 to 13 will diminish the labor of computing the arch stresses from water pressure to such an extent that the analysis of an arch ring, either of a single arch dam or of a multiple-arch dam, can be accomplished in a few minutes.

* "The Design and Construction of Dams," by Edward Wegmann, M. Am. Soc. C. E., Eighth Edition, 1927, p. 448.

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PAPERS AND DISCUSSIONS

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SURVEYING AND MAPPING IN THE UNITED STATES

Discussion*

By C. H. Birdseye, M. Am. Soc. C. E.†

C. H. Birdseye,‡ M. Am. Soc. C. E. (by letter).§—This paper was presented at the first meeting of the Surveying and Mapping Division with the idea of giving the members a brief résumé of the history of surveying and mapping. The writer did not intend it to be a comprehensive presentation of the subject, and he is not surprised that the discussion of the paper uncovered several incomplete and immature statements.

Major Hall questions‖ the writer's division of the history of mapping into two major periods, "the first for military purposes and the second for economic or engineering uses." One would need to be well equipped to debate this subject with Major Hall, who is one of the most profound students of military engineering in the United States Army, and the writer is quite willing to admit that the division of mapping activities into these two periods is inadequate.

Mr. Pendleton is correct in his assumption¶ that the writer had in mind the use of aerial photography as practised by the Federal mapping agencies at the time the paper was written. It should be noted, however, that he specifically referred to densely timbered areas in connection with the statement,** "but the topographer must traverse these streams to determine their fall * * *." At the time the paper was written the writer was only vaguely familiar with the stereoscopic methods used by the firm of engineers with which Mr. Pendleton is associated. Later, opportunity to investigate these

* Discussion on the paper by C. H. Birdseye, M. Am. Soc. C. E., continued from March, 1928, *Proceedings*.

† Author's closure.

‡ Chf. Topographic Engr., U. S. Geological Survey, Washington, D. C.

§ Received by the Secretary, March 9, 1928.

‖ *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 342.

¶ *Loc. cit.*, p. 340.

** *Loc. cit.*, October, 1927, Papers and Discussions, p. 1926.

methods carefully has convinced him that the stereoscopic use of arerial photographs will provide elevations along rivers even in timbered country, well within the allowable error of small scale mapping. However, the writer has been unable to find any solution, by aerial photography, of the problem of locating section corners in the timbered areas of the Western Public Land States.

The U. S. Forest Service is a large user of topographic maps, and one of the principal requirements of that organization is the correct location of section lines and corners with relation to streams and other physical features. The principal point which the writer tried to bring out is that aerial photography cannot be expected to replace ground surveys entirely, and in densely timbered areas of the Western Public Land States it does not promise any great reduction in the cost of small scale mapping. The writer is still convinced of the soundness of this view, but does wish to emphasize his statement that, in most cases, the use of aerial photography will yield better maps.

Major Lee* stresses the value of the standard topographic map for military purposes, and emphasizes the importance, from the standpoint of National defense, of completing the mapping program authorized by the Temple Act.

The writer is exceedingly interested in Major Lee's testimony that studies of flood control in the Mississippi River Valley are seriously handicapped by the lack of adequate topographic maps, particularly in view of recent statements of some members of the Corps of Engineers that such maps are not essential to the solution of the flood problems. The area of the alluvial valley of the Mississippi River is about 30 000 sq. miles, of which about 20 000 sq. miles was flooded in 1927. Only 1 400 sq. miles, or about 7% of the alluvial valley, is topographically mapped, and it is inconceivable that any engineer engaged on flood control or flood relief problems does not need all the knowledge he can secure of the topography of the area subject to floods.

* *Proceedings*, Am. Soc. C. E., March, 1928, Papers and Discussions, p. 845.

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PAPERS AND DISCUSSIONS

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THE DELAWARE RIVER FROM PHILADELPHIA TO THE SEA

Discussion*

By Messrs. William Gerig and F. C. Boggs.†

William Gerig,‡ M. Am. Soc. C. E. (by letter).§—Practical as well as theoretical methods of improving the navigable channel of the Delaware River are set forth in this excellent paper. It is undoubtedly true that most of the shoaling is due to the building up of the middle grounds where the ebb and flood flows do not take the same channel, and to the gradual deepening of the entire river bed by material sliding into the dredged channel.

The author shows that a general system of contraction works is not economically justified and contends that a few additional dikes, properly placed for guiding the ebb and flow into one channel, will considerably reduce the maintenance. The writer believes that such dikes, although expensive to build, are justified, and can be placed so as to do a minimum amount of damage to adjoining property.

The material coming into the channel from the sides must be removed by dredging because, without contraction works, the velocity of the current is not sufficient to scour the bottom at this great depth. The average maximum velocity in a vertical section is about 2.4 knots per hour, or approximately 4.0 ft. per sec., which is insufficient to scour even the soft material at a depth of 30 or 35 ft. According to Kennedy's silt theory, the critical velocity, that is, the mean velocity for a channel 35 ft. deep, just sufficient to prevent silting or scouring, is approximately twice the maximum velocity found in the Delaware Channel. This low velocity accounts for the failure of the agitation method that was tried.

* Discussion of the paper by F. C. Boggs, M. Am. Soc. C. E., continued from January, 1928, *Proceedings*.

† Author's closure.

‡ Senior Engr., Office of Chf. of Engrs., U. S. Army, Washington, D. C.

§ Received by the Secretary, February 28, 1928.

For the upper river, the present method of dredging, that is, using self-propelled hopper dredges to clear the channel, and delivering the dredgings to a pipe-line dredge where they are pumped out of the river, is considered economical. On the lower river, however, where the haul does not exceed 30 to 50 miles, it is believed that it will be found more economical to dispose of the dredgings in deep water in the Bay Section, or outside the Capes.

The writer does not believe that placing the dredged material along either or both sides of the channel will be effective, because it will immediately come back into the channel. This is done in some rivers where the material encountered is usually sand, a material much heavier than the mud found in the Delaware River. The writer has observed that the material in the disposal areas of the Delaware takes a very flat slope, 1 ft. in 100 ft., or even less, after passing through the dredge. It would, therefore, be a difficult material to retain on the already moving sides of the channel.

For delivering material at long distances, the sea-going hopper dredge of proper design has proved more economical than the dipper or bucket dredge with scows. This is largely due to its facility for handling, its speed, its capacity and its non-interference with navigation. Also, each cut made by a sea-going hopper dredge improves the channel by the amount removed, while other types of dredges do not benefit the channel until dredging is practically completed. On the Delaware River, however, the channel deteriorates rapidly behind the latter types of dredges.

In economical work with hopper dredges, it is, of course, important that the economic loading time be carefully determined. The method for ascertaining this is well understood by hopper-dredge operators and has become standardized on Government-owned hopper dredges. If this loading time is not carefully obtained, the cost may be greatly increased.

F. C. Boggs,* M. Am. Soc. C. E. (by letter).†—The writer is heartily in accord with Professor Spofford‡ in his views as to the possible value of models for an indication of the probable effects of harbor and river improvement works. He trusts that one or more laboratories will be established in the near future, where these and other problems of hydraulics may be studied. In this connection attention is invited to the recommendations along these lines, made by Maj. Gen. Edgar Jadwin, Chief of Engineers, U. S. Army, M. Am. Soc. C. E., in his report on flood protection works for the Mississippi River.

Professor Spofford and Mr. Sherman§ raised the question as to cost keeping methods. General Taylor‖ has quite fully covered this phase of the subject. Professor Spofford also pertinently suggests¶ that if costs for work done by the Government plant are so low, it might be advisable to use this plant in Philadelphia Harbor instead of having dredging work there done by contract. The self-propelled hopper dredge, used by the Government on the

* Col., Corps of Engrs., U. S. Army; Dist. Engr., Philadelphia, Pa.

† Received by the Secretary, March 12, 1928.

‡ *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2789.

§ *Loc. cit.*, p. 2791.

‖ *Loc. cit.*, January, 1928, Papers and Discussions, p. 347.

¶ *Loc. cit.*, December, 1927, Papers and Discussions, p. 2790.

Delaware River, would not be satisfactory for work in the restricted limits of the harbor. The numerous passenger and car ferries, small boats, and towed barges, together with larger boats at anchor or entering or leaving their slips, would make the operation of a self-propelled dredge unsatisfactory and dangerous. The width of the harbor, about 1 800 ft., does not offer the opportunity of alternate routes as does the much wider Hudson River at New York. Moreover, the many under-water cable lines which can be readily handled by clam-shell or dipper dredges, would be subject to serious damage from the suction drag-head of a self-propelled dredge. While these difficulties are not insurmountable, they would add materially to the cost, which would be still further increased on account of the long run necessary to reach a disposal area. These dredges, when loaded, draw from 18 to 20 ft.

Mr. Sherman comments* on the statements made by the writer in regard to the municipally owned piers. In justice to those who have control of the policy of handling these piers, it should be stated that the Pennsylvania Courts have decided that, except by Special Act of Legislature, private piers as well as piers leased from the City of Philadelphia, must be open to all comers. As one jurist expressed it, "the owner of a wharf is under the same obligation as an inn-keeper to receive those who call, provided he has room, and they tender a reasonable compensation." Despite the legal aspects, the writer is still of the opinion that one or more municipally owned and operated piers would be advantageous. He wishes to admit an error in his statement that city piers are leased to ship brokers.

Professor Spofford asks† why inland ports, such as Philadelphia, should be developed by the Federal Government. This presents a matter which cannot be fully covered in a brief paper. The writer believes that a consideration of the comparative costs of transportation by rail from Boston, New York, or Baltimore, to the 5 000 000 or more people in the Philadelphia area, with costs of water deliveries through the Port of Philadelphia, will give a convincing answer to this inquiry.

Several of the discussors mention the increased height of the mean low-water plane in recent years, in the river above Artificial Island. The writer agrees that this is undoubtedly due to the constriction of the river between Artificial Island and Reedy Island dike. Whether this constriction, with the resultant raising of the low-water plane, has been harmful is open to question.

The writer cannot agree with Mr. Warner‡ in the theory that all the maintenance dredging required is due to silt originating in the upper river and its tributaries and deposited from suspension in the lower river. While undoubtedly some of the material removed in maintenance work is of this origin, the writer believes that the main cause of the formation of shoals in the channel is the gradual movement of the soft material forming the river bed. Under natural conditions he believes that the bed of the stream at any cross-section would maintain, with little change, a semi-permanent profile. If this is modified by dredging a channel or by placing over-burden on any

* *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2792.
† *Loc. cit.*, p. 2791.
‡ *Loc. cit.*, January, 1928, Papers and Discussions, p. 349.

part of the profile, the equilibrium of forces is disturbed. Cross-currents produced by the ebb and flood tides, which it may be noted attain approximately the same maximum strengths, facilitate this movement down the sloping bed of the river into the channel.

The writer again desires to state that he is not opposed to dikes constructed for the purpose of regulating flow. He is opposed to dikes in the tidal section of the Delaware River, constructed for the purpose of inducing strong currents, since such structures, due to practical equality of ebb and flood currents, would tend to induce scour during both these stages of the tide. The result would be that about the same amount of material would be carried up stream on the flood as would be carried down stream on the ebb. In his opinion, dikes to eliminate undue widths of the stream, or to guide currents in lines parallel to the channel, would be advantageous. Such dikes should be constructed only after careful and extensive current studies at each locality where shoaling occurs.

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TRAFFIC CONTROL BY ELECTRIC SIGNAL LIGHTS

Discussion*

BY M. O. ELDRIDGE, ASSOC. M. AM. SOC. C. E.†

M. O. ELDRIDGE,‡ ASSOC. M. AM. SOC. C. E. (by letter).§—It is gratifying to note the unanimity of opinion regarding the writer's suggestion that the amber signal, following the red, should be eliminated.

Since the suggestion was first made and the idea was given a trial in Washington, the City of Detroit, where 415 intersections are now controlled by traffic signal lights, has eliminated the amber light following the red. The reasons advanced for the change by the traffic authorities in Detroit are that it has a tendency to hold stopped traffic until after the intersection is cleared of moving traffic and that it prevents accidents.

The only real difficulty that has been experienced with traffic lights in Washington has been that some obstreperous motorists will start on the amber following the red, and it is believed that this will continue until the so-called "split amber" cycle is adopted for all master and individual timers.

Some cities have tried out the idea of overlapping the last 3, 4, or 5-sec. portion of the red signal with the amber light, as a warning that a change is imminent, but it is understood that even in these cities, the obstreperous drivers continue to start as soon as the amber light appears. The only solution, therefore, appears to be the elimination of the amber light after the red.

The writer agrees with Mr. Carmalt‖ in his contention that a 5-sec. interval does not give the pedestrian time to cross a wide street. It should be remembered in this connection, however, that the amber following the green is not intended as a "Go" or crossing signal for the pedestrian, but

* Discussion of the paper by M. O. Eldridge, Assoc. M. Am. Soc. C. E., continued from April, 1928, *Proceedings*.
† Author's closure.
‡ Director of Traffic, Washington, D. C.
§ Received by the Secretary, April 2, 1928.
‖ *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2799.

merely as a signal indicating that a change is imminent and that he should step lively and clear the intersection.

With a 35 to 40-sec. "Go" period, every pedestrian who desires to cross the street with safety and dignity has ample time to do so, but if he steps into the street after the amber light flashes, he must take the consequences and resort to undignified haste in reaching the opposite sidewalk.

Pennsylvania Avenue, in Washington, is 100 ft. wide from curb to curb. It has double car tracks in the center, and on each side of the car tracks there are safety zones, or refuges at all intersections, for pedestrians. If Pennsylvania Avenue had traffic lights, a 5-sec. interval would ordinarily provide the pedestrian with sufficient time to enable him to reach the safety zone or the curb, as the case may be.

While a longer amber period than 5 sec. might be desirable at some intersections, it is a fact that after two years of experiment this time interval appears to be satisfactory to pedestrians at most intersections.

Mr. McIntyre calls attention* to the criticism and opposition in Washington during what he refers to as the "cut-and-try process". The writer disagrees with Mr. McIntyre's conclusions as to the cause of this criticism.

When the lights were first installed in Washington they were synchronized; that is, all the lights were green or red at the same time. The total time cycle was about 120 sec. This arrangement, which is similar to the Fifth Avenue System, in New York, had the effect of holding traffic on cross streets about 60 sec. and of permitting traffic on 16th Street to move about 60 sec. This caused drivers of standing vehicles to become impatient while waiting for a green light, and caused the drivers of moving vehicles to go as far and as fast as they could before the lights changed. Rushing the lights and breaking the speed limit became the common pastime of many motorists, thus resulting in criticism and complaint.

It was then decided to shorten the time cycle and to stagger the lights in pairs of two or more, depending on the length of the blocks, so that motorists could drive from one end of the street to the other at a uniform speed of about 20 miles per hour without stopping. This arrangement produced what is known as the wave or co-ordinated system, and it is believed that Washington was the first city in the United States to develop such a system. The plan has now been adopted, in one form or another, in various cities throughout the country with general satisfaction.

At present, the total time cycle on all traffic lights in Washington is 80 sec., divided as follows: 35 sec. for green or "Go"; 5 sec. for amber, which means, when following the green "clear the intersection"; 35 sec. for red, or "Stop"; and 5 sec. for amber following the red, which means "get ready to go". In the new installation of lights now being planned for Washington, some change will be made in the amber light following the red.

The lights on 16th Street have now been installed about two years. During the first year of operation the records show that there were 2.9 accidents per intersection on that street, while on 15th Street there were 5.4 accidents

* *Proceedings*, Am. Soc. C. E., April, 1928, Papers and Discussions, p. 1217.

per intersection and on 17th Street there was an average of 6.4 accidents per intersection. This reduction has been brought about in spite of the fact that 15th Street carries only 44% and 17th Street only 59% as much traffic as 16th Street.

Only one fatality has occurred on 16th Street since the lights were installed, and this accident was caused by a person walking across the street between intersections and through traffic that was moving on proper signal. During the same period four persons have been killed in traffic on 17th Street and one on 15th Street. These streets are parallel to 16th Street and do not have traffic lights except at two intersections on each street.

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BOUNDARY SURVEYS

Discussion*

By Messrs. Alexander Blair and Clarence T. Johnston.†

Alexander Blair,‡ M. Am. Soc. C. E. (by letter).§—The excellent contribution by Professor Johnston opens, in a most interesting manner, the discussion of matters of vital importance to all civil engineers, whose practice includes land surveying.

If political boundaries are to be located to best advantage, it is essential that any commission authorized to define such dividing lines shall include a properly qualified civil engineer. Engineers are better organized to-day than ever before, due to various causes. One effect of such organization should be to stimulate local chapters of engineers to take action, leading to the appointment, on boundary commissions, of members who fitly represent their profession.

It is strongly urged that in defining political boundaries, not only should lines be run out by actual survey, but they should be well monumented, and it is especially desirable that the survey be related to true meridian. Wherever possible, it should tie into State or Federal control lines.

In regard to the use of true meridian, it is a common experience of engineers to find owners who are developing large tracts of land, unwilling to bear the expense of having observations for true meridian made. Where original monuments on tracts of this nature have been obliterated (or were never used), the subsequent owners have borne many times the cost of such observations as would originally have been made, and much entanglement has resulted from lost lines. This is an educational matter in which engineers should help by their practice and recommendations.

* Discussion of the paper by C. T. Johnston, M. Am. Soc. C. E., continued from February, 1928, *Proceedings*.

† Author's closure.

‡ Civ. Engr. and Surv., Westfield, N. J.

§ Received by the Secretary, January 30, 1928.

CLARENCE T. JOHNSTON,* M. AM. SOC. C. E. (by letter).†—Much has been written relative to the construction and use of instruments. The most difficult problem facing the surveyor relates to accuracy. The man in the field often works under directions outlined by those who know but little about surveying. His employers often believe that they know what they need and they generally emphasize the virtues of economy when they outline the budget for field work. The surveyor has no professional protection and he fully understands the handicap under which he suffers because of small appropriations for his work. If he could refer his employers to specifications for surveys, approved by some competent authority, he would be in a position where he could place responsibility for poor work where it belongs. As matters stand, the surveyor represents science, but the members of his profession either lack the ability or the courage to define the most critical and important phase of his activities.

Surveying should be placed among the applied sciences. This cannot be done until surveyors establish limits of error and carry on field work within such limits. Because only a few surveyors are able to analyze critically the data obtained in the field, many of them seem to have reached the conclusion that this is either undesirable or unattainable. To the layman, a survey may be anything from a good guess to mysterious results expressed in whole numbers and decimals. The layman seldom realizes that a good guess may be more reliable than figures which seem to represent the result of careful measurements. Refined equipment, careful work in the field, and clean, complete, and accurate notes point to reliable results. They are not necessarily a guaranty of reliability.

Experts in nearly every field of activity have discovered that the output, whatever it may be, must be regulated by critical tests if it is to be of uniformly high quality. Many manufacturers, a few years ago, considered accurate measurements and scientific tests of materials as an unnecessary refinement in manufacturing processes. The physician of forty years ago had but a few scientific facts to guide his judgment. As scientific information has become available, he has thrown away the traditional materia medica and replaced it by something that is supported by experimental data.

There are a few highly trained men who make surveys that have lasting value. The large majority of men engaged in surveying operations are unable to adhere to standards they would accept gladly if they were conscious of support of any kind. No specifications have been prepared relating to the maximum errors which may be accepted in connection with boundary surveys. Many textbooks discuss "ordinary" and "accurate" surveys without designating limits of accuracy for either. Since the purpose of boundary surveying is to obtain results which accurately represent linear dimensions on the ground and to set monuments at the intersection of important lines, the surveyor should not only know the limits of accuracy to be applied, but when he finishes his work, he should be able to tell how far it runs above or below the specified maximum error.

* Prof. of Geodesy and Surveying, Univ. of Michigan, Ann Arbor, Mich.
† Received by the Secretary, March 23, 1928.

Municipal surveys are now in progress in which limits of error do not exceed 1 in 50 000. Branches of Government service doing triangulation work have fixed the limits at 1 in 500 000. It would seem that the latter limits would be sufficiently refined for the best system of extensive control. Errors of 1 in 500 000 approximate $\frac{1}{8}$ in. per mile. An error of 1 in 50 000 is about $1\frac{1}{4}$ in. per mile. In these two branches of accurate surveying, the limits have been established and engineers can leave it to the experts to defend the standards they have accepted. The large volume of surveying performed in this country lies outside these two fields. The allowable error resulting from surveys of public lands, under the specifications issued to successful bidders on this kind of work, was 1 in 80. The writer doubts whether such work should be dignified by a classification which would relate it closely to surveying.

It is evident that the geographical position and direction of a political boundary are of more importance than its length. Such boundaries are defined by statute. No one is particularly interested in the length of the southern boundary of Michigan. If any attempt were to be made by an interested State to shift this line north or south, this country might witness hostility such as occurred in the great boundary dispute between Ohio and Michigan a hundred years ago. There is a great difference in fundamental purpose between a political boundary survey and primary triangulation work. The triangulation net is spread over a large area for the purpose of determining, with accuracy, the relative position of points on the surface of the earth. If it is required to know these positions within the nearest foot it is necessary to accept a limiting error of approximately 1 in 500 000, because the length of sides of triangles may run from 20 to 100 miles. In an equally accurate municipal survey, the network consists of smaller triangles and it is possible to locate points to the nearest foot by adhering to a limiting error of 1 in 50 000. The sides of triangles run from 2 to 6 or 8 miles in length for such surveys. Such a system would provide monuments which would support local control by accurate traverse so that horizontal position, expressed by co-ordinates referred to a common center and elevation above sea level, would be available at convenient places throughout the municipal area. These stations would be so distributed that accurate azimuths would be at once available from any one of them.

It is apparent that such systems of control would result in great economies, and surveyors would have no excuse for using approximate azimuths or for not making frequent checks on linear measurements. Evidently, surveys connected with such a system of control should locate property corners, street intersections, etc., within 0.1 ft. If the average length of lines measured is 500 ft., the precision of these local surveys would be 1 in 5 000. Such accuracy would be attainable without great expense after the control had been provided. Under average conditions now facing the surveyor, control is wanting, and only a few people in the country seem to appreciate its value. Control is not an expensive frill; it is essential to good work, and it leads to great economies. Under existing conditions the surveyor who is most competent

seldom receives enough financial support to enable him to check his own work properly, or to conduct it so that he is sure that he is obtaining results that are uniformly reliable.

If specifications for boundary surveys were to be prepared by a group of men of National standing, those who employ surveyors would receive protection that cannot be obtained under a system which is notorious because of an almost complete absence of standards. If a surveyor could show some plain specifications for boundary surveys to a prospective employer, he would protect himself as well as those who are to pay for the work. The specifications would be stimulating to all laymen who come in contact with them. If any boundary survey should then become a matter of litigation, the surveyor who had adhered to the specifications would have a decided advantage. In time, American Courts would begin to appreciate some of the important scientific elements of a good boundary survey.

This phase of boundary surveying is presented in the hope that it may be analyzed further until a substantial agreement is reached among those who are competent to assume an unselfish and qualified leadership. Specifications governing boundary surveys would be the logical outgrowth of such a study.

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BAFFLE-PIER EXPERIMENTS ON MODELS OF PIT RIVER DAMS

Discussion*

By Messrs. Ralph L. Parshall, H. W. Dennis, and I. A. Winter.

Ralph L. Parshall,† Assoc. M. Am. Soc. C. E. (by letter).‡—The control of scour below dams resulting from the impact of the water passing over the spillway, presents a problem of great importance in the design of such structures, for the reason that the erosion, unless prevented, comprises a potent element of failure. The authors' paper on the practical method of the dissipation of energy is a valuable contribution to the Engineering Profession.

In April, 1917, the writer designed and built a concrete spillway for a small reservoir near Windsor, Colo. (Fig. 45). This spillway, which is essentially a chute, has a width of 50 ft. at the crest, narrowing to 8 ft. in a distance of 75 ft., with a uniform grade or drop of 8.5 ft. in that distance. The bottom width is increased to 24 ft. in the next 10 ft. of length. The axis of the crest of the spillway forms an angle of 80° with the north side of the channel which is straight, while the south side curves in plan from a width of 50 ft. to 8 ft., as stated. The side slopes are 1 on 1. The depth of this converging section is 2 ft. The side walls at the lower end are 4 ft. high.

A vertical drop of 4 ft. occurs in the grade at the lower end of this chute. Across the channel at the drop is a vertical concrete baffle-wall, 4 ft. high and 8 in. thick, joined to the 1 on 1 side slope of this basin and set 1.5 ft. above the floor line. This vertical wall is supported by three equally spaced buttress walls on the lower face of this transverse baffle-wall; 5 ft. down stream is a transverse parallel wall, 1.5 ft. high, on the floor of the basin. A hole is left at the floor line of this low wall for drainage purposes. Most of these features are shown in Fig. 45.

* Discussion of the paper by I. C. Steele and R. A. Monroe, Members, Am. Soc. C. E., continued from April, 1928, *Proceedings*.

† Irrig. Engr., U. S. Dept. of Agriculture, Fort Collins, Colo.

‡ Received by the Secretary, March 5, 1928.

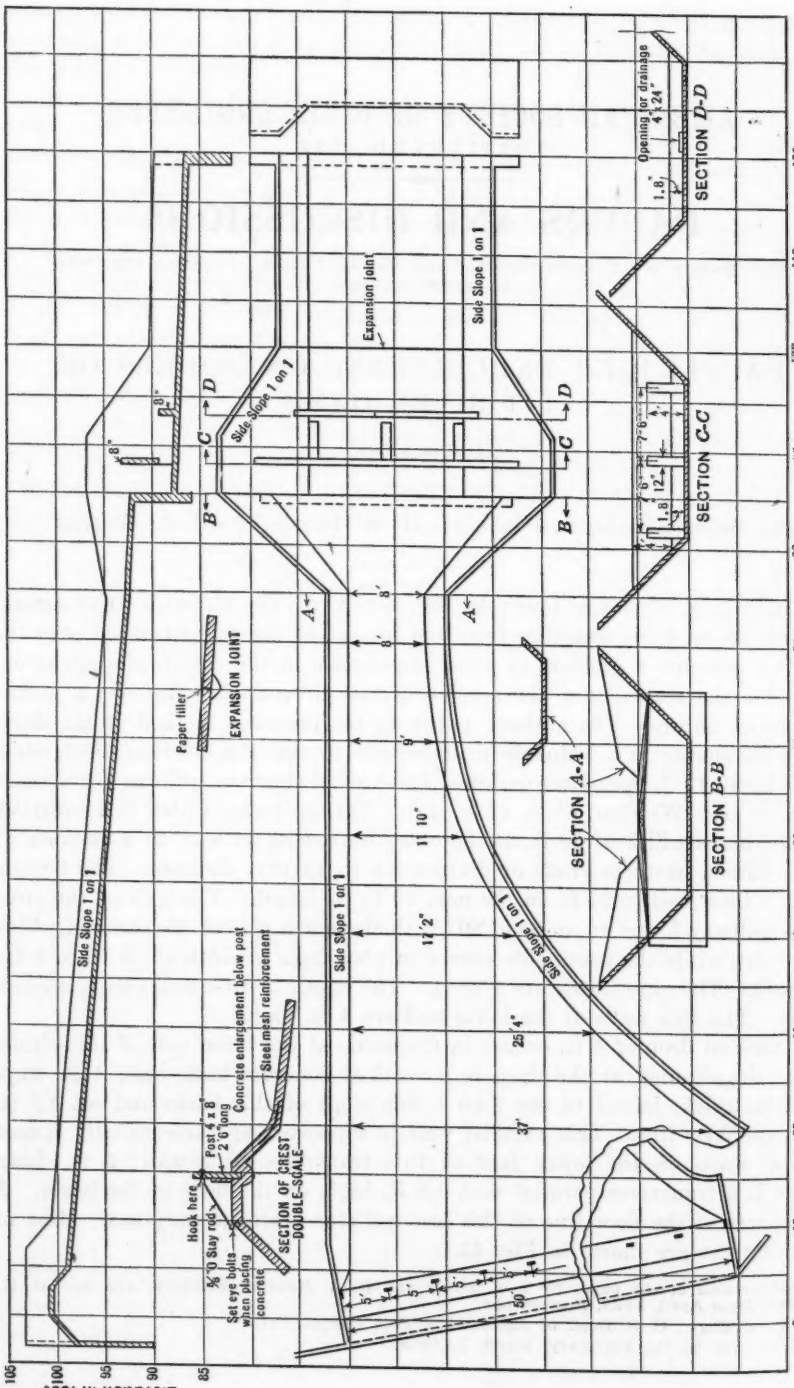

FIG. 45.—PLAN OF CONCRETE SPILLWAY FOR RESERVOIR NEAR WINDSOR, COLO.

FIG. 46.—HALLIGAN DAM, SHOWING SPILLWAY IN ACTION, WITH A DISCHARGE OF 700 SECOND-FEET, JUNE 24, 1917.

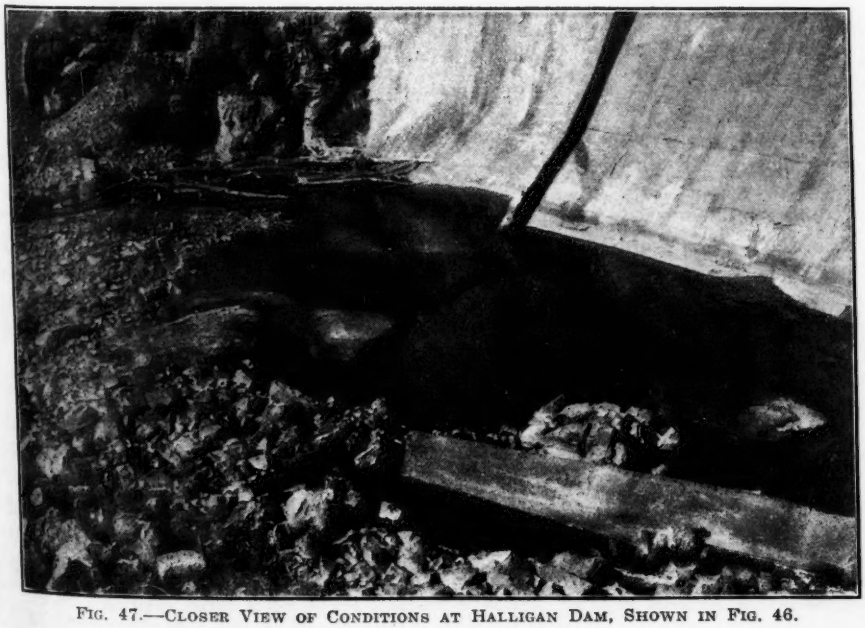

FIG. 47.—CLOSER VIEW OF CONDITIONS AT HALLIGAN DAM, SHOWN IN FIG. 46.

FIG. 45.—PLAN OF CONCRETE SPILLWAY FOR RESERVOIR NEAR WINDSOR, COLO.

These transverse walls are for dissipating the energy of the stream by impact. The lower or secondary wall creates a water cushion and chamber for the discharge as it passes over the end of the approach channel.

This spillway was built largely to forestall an emergency caused by a sudden rise of the water surface within the small reservoir, such as might be caused by a freshet in the stream. The writer has never seen this structure in action, but has been informed that for relatively small discharges it appears to function properly.

An interesting example of scour below dams is shown at the Halligan Dam, on the North Fork of Cache la Poudre River, about 35 miles northwest of Fort Collins, Colo.* This structure (Fig. 46) is of reinforced cyclopean concrete. The width of the base is 51 ft.; the extreme height, 94 ft.; and the length, along the top of the dam, 350 ft. The dam, completed in May, 1910, is of the arch type with a constant radius of 324 ft. The foundation is a hard seamy granite mixed with red talc formation. Table 11 shows the flow over the spillway for the period, 1919 to 1926, inclusive.

TABLE 11.—DISCHARGE OVER HALLIGAN SPILLWAY.

Year.	SPILLWAY IN ACTION.		DISCHARGE, IN SECOND-FEET.	
	Period.	Days.	Maximum.	Average.
1919	None	None	0	0
1920	May-June-July	57	850	420
1921	May-June	21	640	240
1922	None	None	0	0
1923	May-June-July	44	2 740	490
1924	May-June	29	710	385
1925	None	None	0	0
1926	April-May-June	40	570	170

These data were taken from a record of reservoir gauges showing the depth of water in tenths of feet above the crest elevation. The discharge was computed by the formula, $Q = C L H^{\frac{3}{2}}$, in which C is assumed to be 3.5. The length of crest is 110 ft. Since the completion of the dam there has been reported, on more or less reliable information, a depth of 4.7 ft. on the crest of the spillway. For this head the discharge would have been approximately 3 900 sec-ft.

After a period of about 18 years of service, there appears to be no scour below this dam. As indicated, the extent of the discharge over the crest of spillway is dependent on the annual run-off from the water-shed above the reservoir. In some years there has been no flow over the dam, while for others the maximum has been nearly 4 000 sec-ft. These maxima, as shown, occur for very short periods. Fig. 46 shows a discharge of about 700 sec-ft. passing the crest, while Fig. 47 gives a more detailed view below the dam. Both views indicate, when compared with photographs taken at completion of the

* "The Halligan Dam; A Reinforced Masonry Structure," by G. N. Houston, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), p. 112.

dam,* that there has been very little if any increase of scour, because of the nature of the formation.

H. W. DENNIS,† M. AM. SOC. C. E. (by letter).‡—There is a well-defined growing tendency to take advantage of experiments on models as a guide in the design of major structures, and the authors have indicated such an application and submitted their results for the benefit of the Engineering Profession. They have called attention to the question as to the quantitative interpretation of the results, although it is clear that qualitatively they are valuable.

The following may be a contribution to the subject from the standpoint of the quantitative interpretation of the hydraulic scouring action to be expected when a large quantity of water is discharged into an uncompacted material. In this case, it was the dump from the Florence Lake Tunnel, constructed by the Southern California Edison Company for conveying water in a comparatively large quantity from the Florence Lake Reservoir to the Huntington Lake Reservoir of that Company.

Although the outlet portal of the tunnel was excavated in earth and required a concrete lining for a distance of about 800 ft., the remainder of the tunnel was excavated in solid granite, and the material was deposited, as was convenient, near the portal of this tunnel. The elevation at the portal is 7 000 ft. above sea level, and Huntington Lake, when full, is at Elevation 6 950. Accordingly, provision was required that the water should be permitted to discharge without undercutting the tunnel portal. With an anticipated maximum discharge from the tunnel of about 2 300 sec-ft., a steel pipe 12 ft. in diameter was inserted in the end of the concrete lining and was constructed on a 7.85% grade for a distance of 258 ft., so as to bring the actual outlet at an elevation of 6 980 ft., where the pipe was supported by a concrete head-wall. The diversion into the tunnel is controlled by gates, and the first operation through this outlet structure afforded opportunity to take measurements of the scour which would be produced by flow up to a maximum available capacity. At the time of the experiment, this was limited to 1 875 sec-ft., due to a reduced elevation of the upper reservoir.

Contour surveys were made before any water was diverted and also after the maximum flow (1 875 sec-ft.) had been allowed to run for several hours, at the end of which time no further apparent scour of the channel was taking place at the outlet structure. The outlet structure, under conditions of minimum and maximum flow, is shown in Figs. 48 and 49, respectively. Fig. 50 shows the plan and profile, including contours so taken.

Fig. 50 shows a large rock located almost exactly on the center line of the pipe and about 55 ft. from the outlet. The presence of this rock was unknown at the time of laying out the work, and, of course, it functioned somewhat as a baffle and assisted in destroying the energy of the discharging water. The natural weir created below this pipe was such that, when no water was discharging, the water stood at Elevation 6 978, whereas, during the operation, the general elevation of the water on this weir was 6 987 ft.

* *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), p. 112.

† Chf. Civ. Engr., Southern California Edison Co., Los Angeles, Calif.

‡ Received by the Secretary, March 6, 1928.

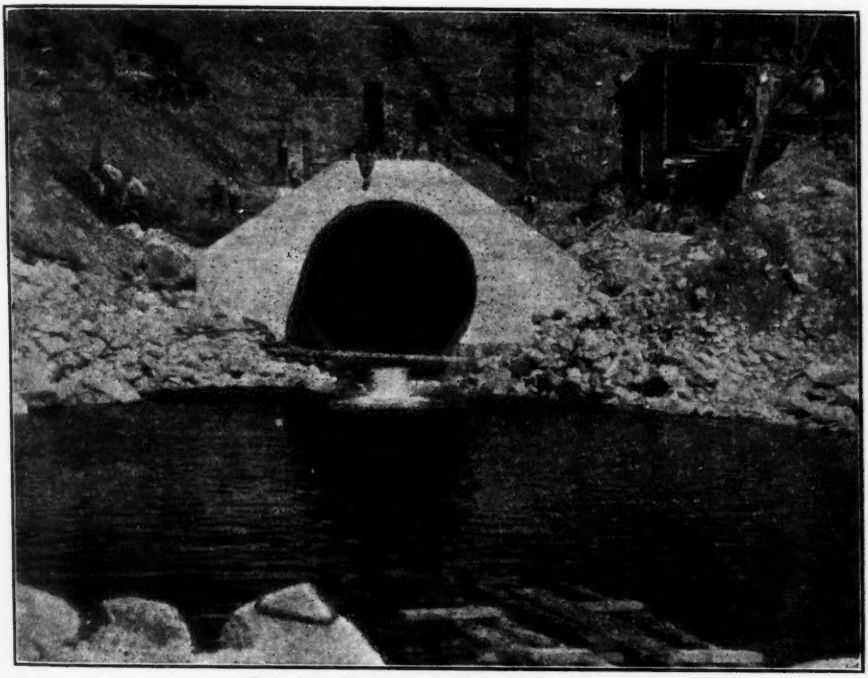

FIG. 48.—VIEW OF OUTLET STRUCTURE.

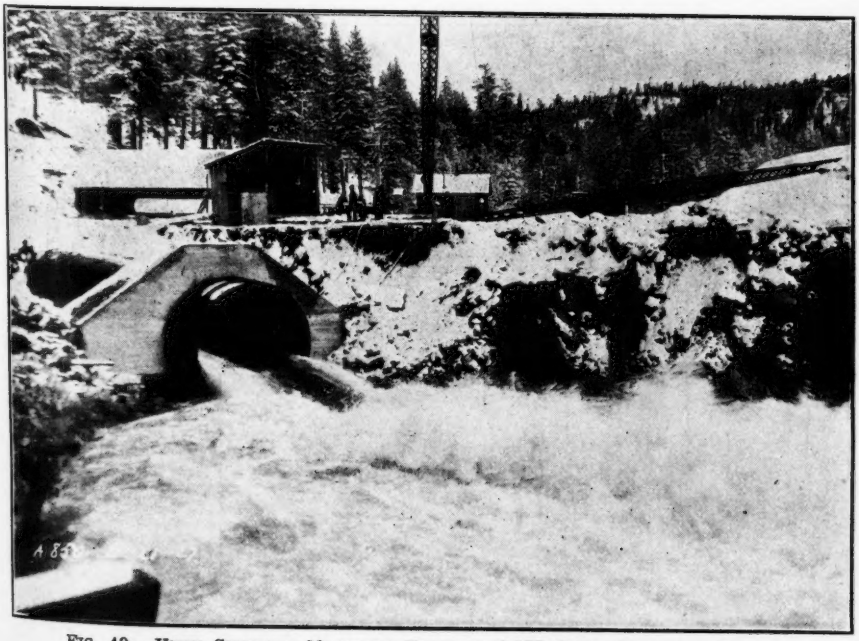

FIG. 49.—VIEW SHOWING MAXIMUM FLOW OF 1 875 CUBIC FEET PER SECOND.

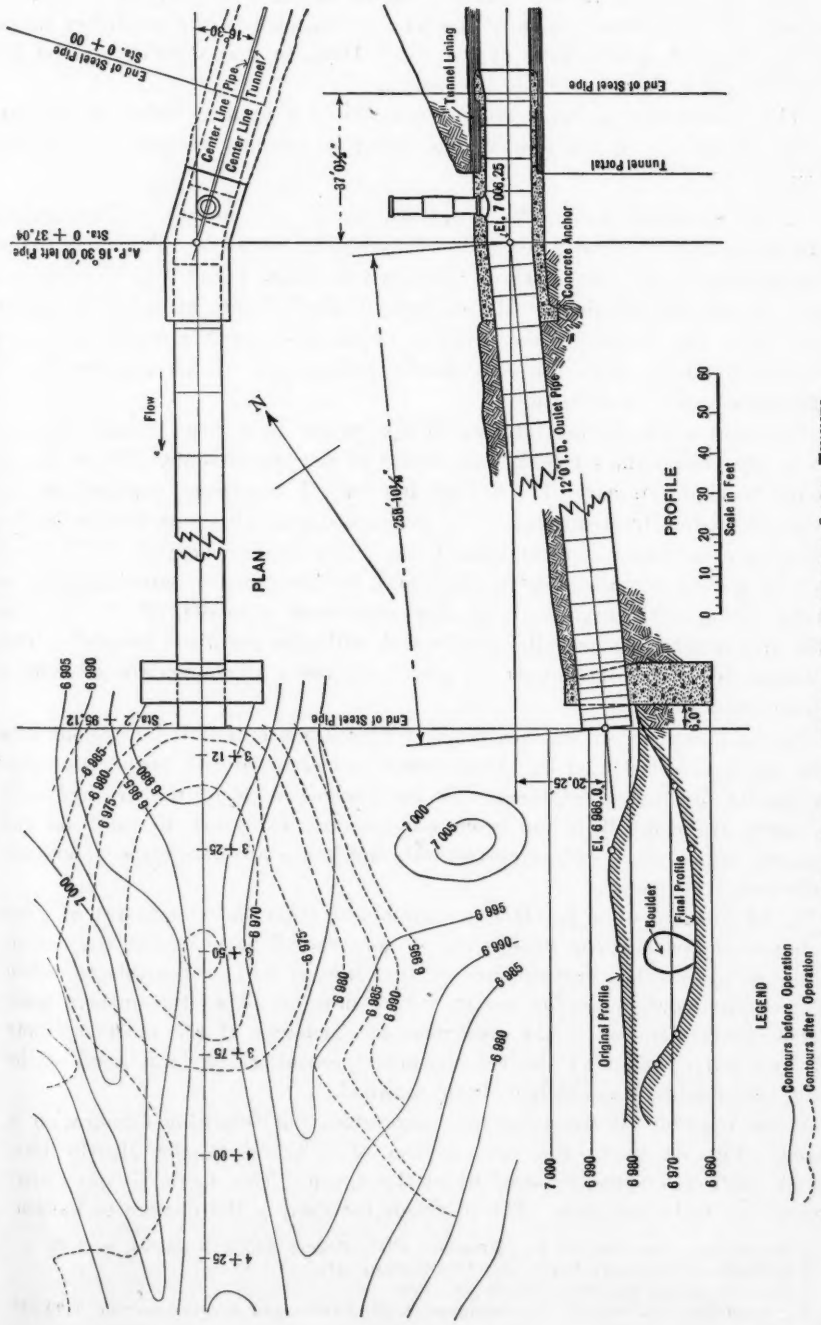

FIG. 50.—PLAN AND PROFILE OF FLORENCE LAKE TUNNEL.

No horizontal or vertical curve occurred in the pipe near the outlet, and accordingly the general slope of the jet corresponded to the condition recited by the authors* as obtaining at Pit No. 3 Dam, in that the direction of flow is downward rather than truly horizontal.

The size of the material which remained as a natural lining of the pool at the conclusion of the observation would be roughly classed as "one-man stone".

I. A. WINTER,† ASSOC. M. AM. SOC. C. E. (by letter).‡—The authors have presented a splendid example of hydraulic research. The final design adopted for the Pit No. 3 Dam, as shown in Figs. 1 and 10,§ illustrates a very satisfactory solution of the control of flood waters at spillway aprons. The writer has examined the original photographs of the model in action and has noted in detail the remarkable performance of the combination of baffle-piers and stilling-basin.

Two points of special interest to the writer have been clearly brought out in presenting the methods and results of the experiments. These factors should receive the careful consideration of all engineers engaged in the design of hydraulic structures. The authors state:‖ "It was finally decided that close observation and notation of the action of the water * * * were the best means of obtaining results", and, "while there is some question as to the action with any depth of flow over such a model, * * * , the baffle arrangement as actually constructed will give excellent results". This indicates the trend of thought of many engineers when confronted with a similar situation.

Over a period of several years, the writer has accumulated sufficient data to be convinced that models, when properly constructed and tested, give similar results for their prototypes. At the present stage of the art hydraulic engineers are compelled, for economic reasons, to resort to methods and apparatus which are purely experimental, and hence are of specific value only to the structure tested.

If the solution of a problem is approached from the standpoint of some fundamental law and the models are so constructed as to enable the experimenter to observe the performance of that law, it will be possible to obtain the necessary coefficients for use in future designs. Had the authors made precise observations as to the coefficient of discharge of the model spillway and back-water weir and the velocity-head percentage of total head at the apron, valuable data would have been obtained.

As an example of the procedure suggested, the following illustration is offered. Fig. 51 shows the cross-section of a model of the Martin Dam spillway with a return-flow conduit at the apron. This model is one-thirty-sixth of the full-sized dam. The principle involved is the change in momen-

* *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2191.

† Hydr. Engr., Alabama Power Co., Birmingham, Ala.

‡ Received by the Secretary, March 13, 1928.

§ *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, pp. 2190 and 2199.

‖ *Loc. cit.*, p. 2191.

tum caused by the introduction of an auxiliary jet under the spillway sheet. In effect, the device creates an incomplete hydraulic jump, reducing the effective head at the end of the apron to one-half the initial value.

The three factors that determine the design of the passages are: First, the coefficient of discharge of the spillway; second, the velocity-head percentage of total head of the spillway sheet approaching the retarding basin; and, third, the coefficient of discharge of the return flow conduit. The values used in the illustrated example were obtained from data on previous models.

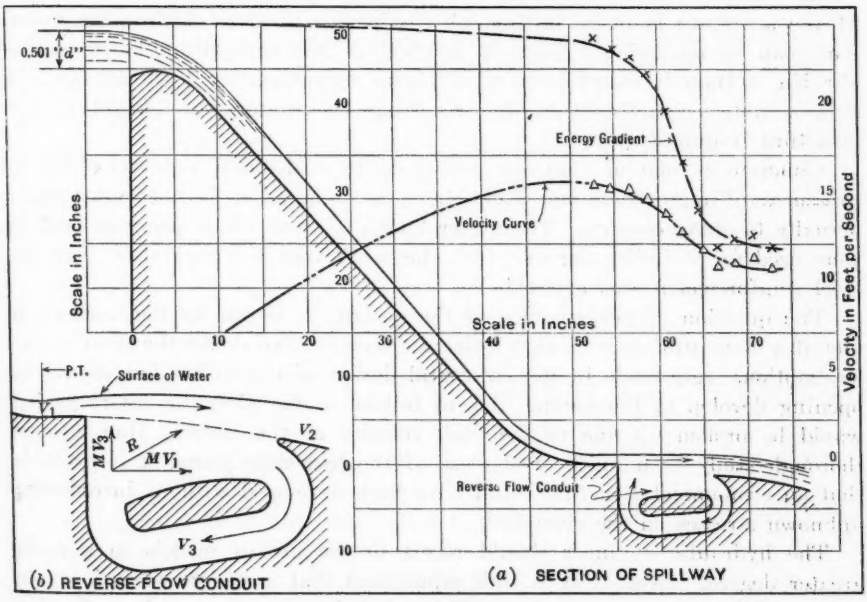

FIG. 51.—MODEL OF MARTIN DAM SPILLWAY.

Having determined these three factors, the design is computed by resolving the components of momentum of the two masses. The beginning of the computation and the completed design are illustrated in Fig. 51(*b*). The curve drawn through the Pitot tube readings indicates perfect agreement with the theory as established. It is reasonable to assume that similar structures can be designed for any height of dam, depth of overflow, and change in momentum, up to the maximum depth of the hydraulic jump.

Referring to the curve in Fig. 51(*a*), it is seen that the high velocity at the approach (40 to 55 in.) is effectively reduced during the travel across the basin (55 to 70 in.). The upper surface of the spillway sheet is parabolic in shape and its influence is manifested up stream to the point of tangency of the apron. It is possible to design the basin so as to give a horizontal surface, should it be advantageous to do so.

This illustration is offered because it explains one of the fine points of the design shown in Fig. 51. The fact that the most satisfactory results were obtained by using a 15-ft. step-off indicates that the resulting eddy under the

spillway sheet is an important factor in reducing the momentum. Also, the second row of baffles and the final back-water weir creates two additional basins which function in the same manner. Directing the sheet downward, against the baffle-piers is, as the authors point out, of great value, but only when there is sufficient relief below to permit a reversal of flow. This fact is clearly illustrated by the performance in Experiment 11, Fig. 4,* and in Table 3.†

The use of baffles alone often leads to serious complications. They merely change the direction of the water without effectively reducing the momentum.‡ If the structure is to be lasting, all dissipation must be within the water. This can be appreciated when it is realized that the stilling-basin at the Pit No. 3 Dam is called upon to dissipate approximately 1 000 000 h.p. If this is not accomplished by the resulting back-water, the hazard to the structure is indeed great.

Concrete is one of the best materials to withstand high velocities at pressures of more than one atmosphere, as long as it is not called upon actually to absorb energy. The writer knows of cases where concrete buckets were used for a similar purpose, with the result that in a very short time the steel reinforcement was exposed.

The question of performance of the design, as shown by the authors, is one of a structural as well as a hydraulic nature. No doubt the most severe assumptions were made in the structural design of the baffles, but should an opening develop in the paving, due to failure of the pier, the entire paving would be broken up due to the high velocity of the water. Had a more thorough study been made of the use of the hydraulic jump, it is probable that an economical structure could have been developed without introducing unknown stresses on the river bed.

The hydraulic engineer should resort to the use of models to a much greater degree. Even if it is well established that a design worked out on paper will function properly, it should be proved by experimentation. Testing engineering materials is an accepted practice, not necessarily to effect better and more economical designs, but to be certain that the standard is maintained. It is only fair that the hydraulic engineer be given the same opportunities to test his designs. The safety factor which he must use is frequently much less than that permitted in many of the associated branches of engineering.

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* *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2193.

† *Loc. cit.*, p. 2201.

‡ "The Hydraulic Jump as a Means of Dissipating Energy", Technical Rept., Pt. 3, Miami Conservancy Dist.

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PAPERS AND DISCUSSIONS

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THE SCIENCE OF FOUNDATIONS—ITS PRESENT AND FUTURE

Discussion*

By T. L. Condron, M. Am. Soc. C. E.

T. L. Condron,† M. Am. Soc. C. E. (by letter).‡—The writer contributes to the data on this subject, the results of recent soil tests which may prove helpful to others in deciding what loading may safely be used on a similar soil.

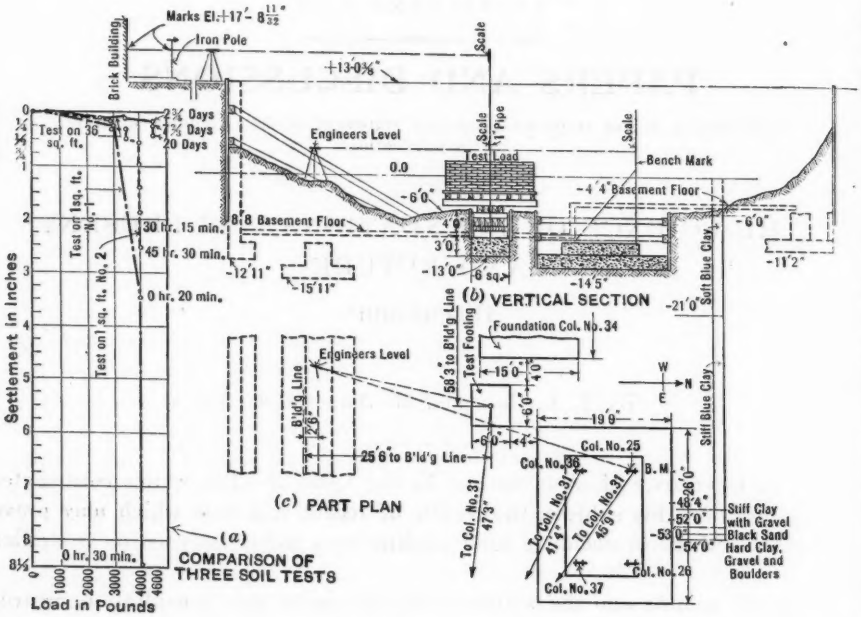
Several months ago the writer's firm was called into consultation regarding the foundations for a building then under construction. This building is located in Chicago, Ill., about two miles from the center of the "Loop District". The contract cost of the structure is about $750 000 and the ground area, about 13 000 sq. ft. The building had been designed with "spread footings", resting on clay, from 20 to 30 ft. below street grade, or from 6 to 16 ft. below Chicago City Datum (Lake level). At the time the consultants were called in, the general excavation was nearly completed down to Elevation — 6.0, or 20 ft. below street grade, and five of the footings, or about 20% of the foundations, were in place. Some question had arisen as to the bearing power of the soil and two soil tests had been made by the Contractor under the direction of the Architect. The results of these two soil tests caused great concern to all. For each test, a load of 4 000 lb. was placed on an area of 1 sq. ft. The settlement under the first test was reported as 8½ in. in 30 min., and for the second test, 2$\frac{9}{16}$ in. in 45 hours. The second test was reported as being made in accordance with the recommendations of the Society's Special Committee on the Bearing Value of Soils for Foundations. The results of these two tests as reported to the writer are shown in Fig. 24(*a*), as well as the result of a third test which is described later.

* Discussion of the paper by Charles Terzaghi, M. Am. Soc. C. E., continued from April, 1928, *Proceedings*.

† Cons. Structural Engr. (Condron & Post), Chicago, Ill.

‡ Received by the Secretary, February 14, 1928.

In view of the very unfavorable showing of these two tests, it was proposed to remove the five spread footings that were in place and substitute for all foundations, concrete cylinders extending down to hardpan, at an estimated extra expense of $55 000 to $60 000.



(d) DIAGRAM OF SETTLEMENT UNDER TEST LOAD ON 36 SQ. FT.

FIG. 24.

It seemed surprising that the clay, which was fairly stiff and difficult to excavate, offered no better support than was shown by the two tests reported, since they had presumably been made on undisturbed clay at the depth of the proposed footings. However, as the situation was apparently so serious and the expense for the proposed change so large in proportion to the total cost of the project, the writer proceeded to investigate the condition of other buildings of similar height and character in the immediate vicinity. The proposed building was intended for non-commercial use and was to be only six stories high, except for a tower to enclose tanks. Several buildings were found near-by resting on spread footings designed for a soil pressure of 4 000 lb. per sq. ft. and none of these buildings showed any evidence of unequal settlement although they were from 10 to 15 years old. The footings for the proposed building were to be loaded to not more than 3 000 lb. per sq. ft.

Consequently, the consultants recommended that a further soil test be made before arriving at a decision.

In making this test it was felt that, so far as possible, the conditions should be as nearly identical as possible with those for the designed footings. Consequently, a location was selected (see Fig. 24 (*c*)), where a test footing 6 ft. square could be built and left in place without interfering with any part of the general foundation work. An excavation was made from Elevation —6.0 to Elevation —13.0, a depth of 7 ft. The upper 4 ft. of this excavation was sheeted and braced and the lower 3 ft. carefully cut to size and filled with 1 : 2½ : 3½ concrete (Fig. 24(*b*)). The lower end of a 1½-in. steel pipe, 20 ft. long, to which a gauge was to be attached for readings, was embedded in the concrete block. When the concrete had hardened, a timber grillage was built on which was placed a timber platform to receive the test load of pig iron. Bench-marks were established on one of the footings in place and on near-by buildings and light poles. The arrangement of this test and its location with reference to two footings then in place and to proposed footings, is clearly shown in Fig. 24(*b*) and (*c*). On this diagram is also shown to scale the results of a typical boring which disclosed "blue clay" increasing in stiffness down to Elevation —52.0, where clay mixed with gravel was found.

Careful level readings were taken on this test footing as the test load was applied, as shown on Fig. 24(*d*), the initial reading being taken when the load of the concrete and platform was 23 970 lb., or 670 lb. per sq. ft. on the soil. On the third day the total load, including the pig iron, was 165 910 lb., or 4 608 lb. per sq. ft. on the soil. At that time the settlement recorded was one-thirty-second more than ⅛ in. After the test load had been in place 12 days, the total settlement was measured as one-thirty-second more than ¼ in. and no further settlement could be measured during the next 7 days. The test load was then removed and a rebound of ⅛ in. was recorded. It is interesting to compare the results of this test with the results of the first two tests, as shown in Fig. 24(*a*).

In the writer's opinion this test warrants the conclusion that 3 000 lb. per sq. ft. would be an entirely safe load on this soil. The client was so advised and work was resumed on the building. The full weight of the building has now been on the foundations for a month or two and no measurable settlement has been reported during the period of construction.

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FLOOD CONTROL WITH SPECIAL REFERENCE TO THE MISSISSIPPI RIVER

A SYMPOSIUM

Discussion*

By W. J. Schlick, M. Am. Soc. C. E.

W. J. Schlick,† M. Am. Soc. C. E. (by letter).‡—The careful study made by Professors Woodward and Nagler, furnishes valuable evidence on the much debated question of the effect of drainage development on flood flows. Although the available data are not extensive, the definiteness of the indicated conclusions warrants their general acceptance. It is doubtful if the effect of drainage will be more pronounced on any of the streams in the Mississippi water-shed, so there seems to be no question of the accuracy of the conclusions in the paper as applied to floods of that great river, particularly since only a small portion of any major flood flow can come from an area which is drained, or in which any extensive drainage development is economically probable.

It seems that if there are any inaccuracies in the conclusion, these are the results of insufficient data and apply to smaller water-sheds in which lateral under-drainage is, or may become, more nearly complete. The water-shed of the Des Moines River above Fort Dodge, Iowa, is of such character that a very large proportion of it probably will be underdrained eventually. The resulting effects of this drainage development on flood flows, and particularly on maximum flood flows, are problematical but are worthy of consideration.

In 1920 the Iowa Engineering Experiment Station instituted a study of the discharges from several of the larger drainage districts of the State. In two of these districts (with drainage areas of approximately 25 and 35 sq. miles, respectively), the outlet drains were just being completed. The fact that the rainfall during the spring and summer months has not been excessive,

* Discussion of the Symposium on Flood Control with Special Reference to the Mississippi River, continued from April, 1928, *Proceedings*.

† Drainage Engr., Eng. Experiment Station, Iowa State Coll., Ames, Iowa.

‡ Received by the Secretary, March 14, 1928.

and that there has been an economic depression in the agricultural States, has so retarded the installation of farm under-drainage systems that the flow records obtained to date represent very largely the surface run-off admitted to the open ditches directly and through inlets in the district tile lines. The curves of "surface flow", Fig. 47, are for one of these districts and a lateral area of it.

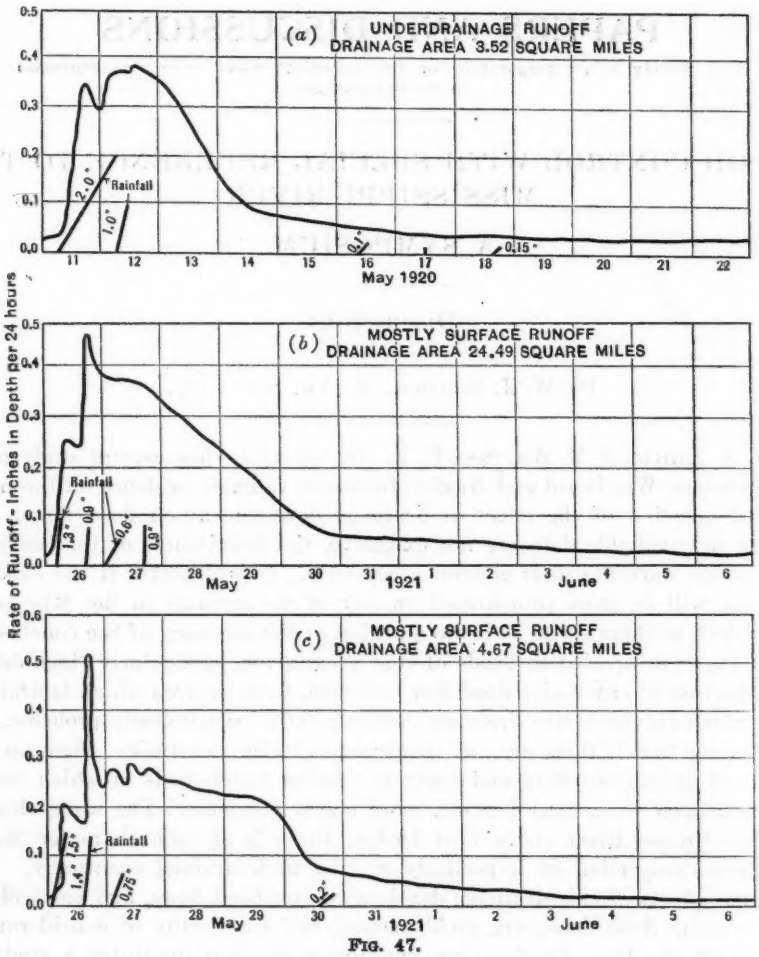

FIG. 47.

Another of the districts studied was rather completely underdrained, with no provisions for admitting surface run-off. The discharge from this system is entirely under-drainage flow, except for the surface run-off of a small area immediately adjacent to the outlet.

The shapes of the rate-of-run-off curves shown on Fig. 47 are illustrative of those obtained during this study. They present some interesting possibilities in connection with the study of the effects of drainage on flood flows.

The curves for the "under-drainage" area are always flatter and the peaks less sharp, than those for the "surface run-off" areas. Neither of the curves represents the run-off from the maximum storms that reasonably may be expected, although the maximum rates shown are as high as any during the period from 1920 to 1927, inclusive. Only one of these areas is in the water-shed of either of the streams studied by Professors Woodward and Nagler, but they are typical, in a general way, of the flatter areas of both those water-sheds.

The available data do not warrant any definite conclusion as to what effect the extensions of under-drainage may have on the correctness of the conclusions of Professors Woodward and Nagler. The percentage of the water-shed of the Des Moines River that probably will be underdrained is unknown, but it is doubtful if it will exceed 75% of the water-shed above Fort Dodge, 50% of that above Des Moines, and 25 to 33% of that above Keosauqua. Since the present drainage development has had no determinable effect on flood flows, it seems reasonable to expect that the effect of future under-drainage development on the flood flow at Des Moines and Keosauqua will not be material. However, the shapes of the curves do indicate the possibility of such an effect, especially on the medium high floods from the smaller water-sheds.

The present-day limits of the economical maximum intensity of under-drainage are such that the storms producing the maximum flood flows usually cause some surface run-off from underdrained areas. Also, the tile lines of nearly all drainage-district systems are now provided with inlets to hasten the removal of surface water. These two facts will tend to minimize any effect under-drainage may have on flood flows, particularly maximum flood flows.

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EMERGENCY DAM ON INNER NAVIGATION CANAL AT NEW ORLEANS, LOUISIANA

Discussion*

By T. B. Mönniche, M. Am. Soc. C. E.

T. B. Mönniche,† M. Am. Soc. C. E. (by letter).‡—This paper is of great interest to the profession. It describes a novel structure, and as the literature on emergency dams is very meager, any addition to this subject should be of value. The problems involved in the design of an emergency dam are far more complex than one would think at first.

The author claims that the New Orleans Dam is as reliable in operation as the Panama Canal dams; that it requires a smaller operating force; and that it has the merit of being lower in cost than any emergency dam previously built. The writer is of a different opinion. He also believes that it would have been entirely infeasible to construct a dam of the "stop-log" type for the Panama Canal, whereas the type adopted there could have been used to great advantage at New Orleans.

When the writer began designing the Panama Canal dams, no emergency dam had ever been tested under actual emergency conditions, so that there were no practical results to follow. Calculations were made of the flow of water that would occur at each site of the dams in case of an emergency. The most serious condition was found to be at Gatun (see Fig. 23), where there is a double flight of three locks, each 110 ft. wide and about 1 000 ft. long. The depth of water at the upper lock-sill is 49¾ ft. and the difference in elevation of the water at the top lock and the bottom lock is 88 ft. The computed result was a discharge of 95 000 cu. ft. per sec., with a velocity of 24.1 ft. per sec. It became apparent at once that in trying to stop a flow of such magnitude, it would be practically impossible to do so simply by enlarging upon the dams already constructed at Sault Ste. Marie. The

* Discussion of the paper by Henry Goldmark, M. Am. Soc. C. E., continued from April, 1928, *Proceedings*.

† Boquete, Panama.

‡ Received by the Secretary, March 21, 1928.

wicket girders became so long that in order to counterbalance their weight, the horiontal truss had to be placed on the down-stream side of the down-stream vertical truss instead of, as at the "Soo", between the two vertical trusses; and the rolling gates could not be made (as at the "Soo") in one length, but had to be made in sections and hung up under the floor so as to clear the flow of water. This is shown in Figs. 23 and 24.

The principal point considered in the design of the wicket girders and gates was to construct them so as to make their lowering in the swift current as reliable as possible. The possibility of the wicket girders meeting obstructions at the sill was guarded against by placing large concrete pockets, 32 ft. wide and 14 ft. deep, in front of the sill. These would have to be filled before any obstruction could settle on the sill. Furthermore, the shape of the sill was made such that any obstruction which might settle would be washed away by a current of 24 ft. per sec. If some obstruction should be caught between the sill and the wicket girders just as the latter are approaching the sill, the hoisting machines have been made so powerful that these girders may be raised against the current so as to allow the obstruction to pass. In lowering the gates in the current they are subjected to pressure caused by the static head and the kinetic energy of the water. In order to ascertain what the friction would be for each set of gates during lowering, tests were made on full-sized gates. These tests showed that the average load required to overcome the friction due to the loads produced by the current, was 3.2% of the weight of the gates.

Only a few months after the plans of the wicket girders and gates for the Panama Canal had been made, the accident at the "Soo" occurred. This was the first time an emergency dam had been brought into actual use and its successful operation proved its feasibility. The most serious mishap that occurred was that, in lowering one pair of wicket girders, they struck some obstruction so that they did not get properly seated and were warped into a twisted mass. However, the principal framework of the dam was intact and the opening was later closed by provisional means. The gates, which were provided with wheels (but without roller bearings), failed to function so they had to be jacked down into place. It was a satisfaction for the writer thus to obtain an actual proof of the importance of those points on which he had laid so much stress in his design.

The lock force on the Panama Canal is given a routine drill once a month in operating the emergency dams. The force which usually operates the dams at the Atlantic locks consists of ten "gold" and twenty "silver" employees, which is approximately 14% of the "gold" and 10% of the "silver" force. During the six months from August, 1927, to January, 1928, the average time for closing one dam, either day or night, was 25 min., the maximum time was 26 min., and the minimum time, 24 min. The force used in drilling during the same months at the Pacific locks consisted of eight "gold" and twelve "silver" employees. This represents, at Pedro Miguel, 16% of the "gold" and 12% of the "silver" force. The average time for closing was 29 min. for Pedro Miguel, and 24 min. for Miraflores. These

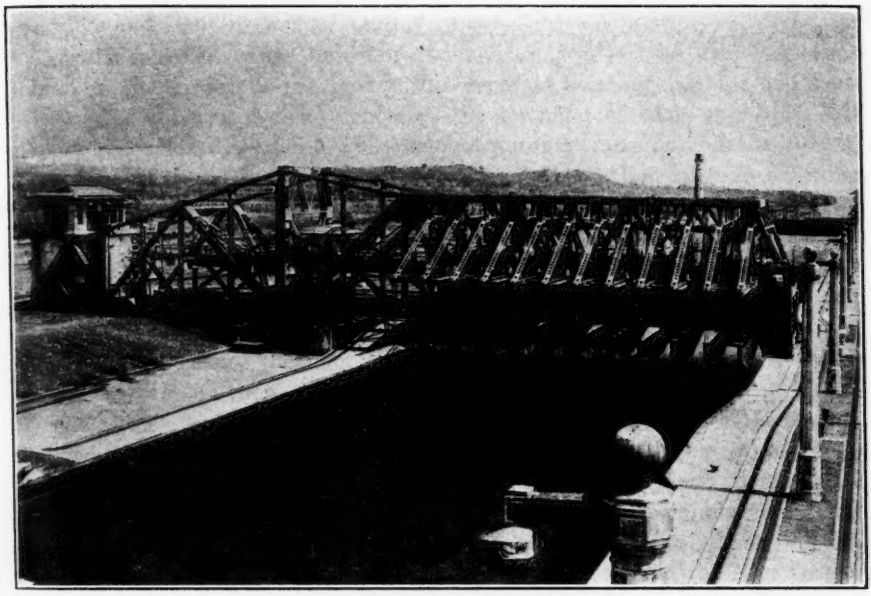

FIG. 23.—VIEW SHOWING WEST EMERGENCY DAM IN POSITION ACROSS LOCK CHAMBER, GATUN LOCKS, PANAMA CANAL.

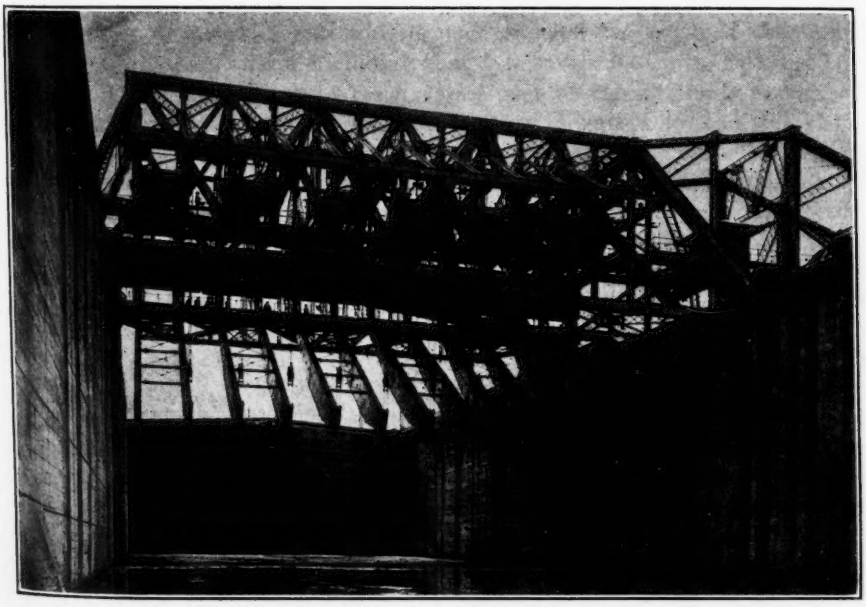

FIG. 24.—PANAMA CANAL EMERGENCY DAMS, PEDRO MIGUEL LOCKS, VIEW SHOWING LOWERING OF WICKET GIRDERS.

numbers of men are not the minimum forces required, as some of the men are oilers and stand-by men.

The lockmaster swings the dam and is in charge of the drill. As each pair of wicket girders is provided with a separate hoisting machine and limit switch, the lockmaster alone can start the lowering of each pair of wicket girders one after the other. The switches automatically cut off the current for each machine when the wicket girders reach the bottom sill. In a similar manner each horizontal set of gates may also be lowered. In lowering the gates, however, two additional men would be required to ascertain that the hooks for the gates are not interfering with the gates properly engaging the rails of the wicket girders. Thus, the dams for the Panama Canal may, if necessary, be operated with a minimum force of three or four men. The time for closing the dam in this manner would be increased to about double the present time.

The author states* that the principal operations of the New Orleans Dam are under the control of one man, although a few others are required in the storage yard and on the walls, and that "this does not mean any addition to the ordinary lock force." He also states† that a "large number of attendants" is required for the operating of the Panama Canal dams. As no addition to the ordinary lock force is required for operating the New Orleans Dam, and as only 14 to 16% of the lock force is ordinarily used for operating the Panama Canal dams, it seems to the writer more probable that it takes a larger force to operate the New Orleans Dam than those of the Panama Canal. It would be interesting to know the exact number of men required for operating the New Orleans Dam, and the recorded time used during the day and night, including that lost between each operation. The Panama Canal dams have to resist forces and to check kinetic energy of water of about twice the magnitude of the New Orleans Dam and, according to the available figures, this is done in less than one-half the time.

The author's statement that the New Orleans Dam is as reliable as those of the Panama Canal does not seems to the writer to be quite correct. The experience gained by the accident at the "Soo" shows the importance of guarding against improper seating of girders due to meeting obstructions. Possibly this point has been given little consideration by the author in his design, one of the principal features of which is the use of releasing hooks with which large box girders spanning the width of the lock chamber are lowered into place. The releasing of these hooks depends on the proper seating of each box girder. The chances for one of these box girders meeting obstructions while being lowered in a swift current are far greater than for all the wicket girders at the "Soo" or at the Panama Canal, because these box girders are to be seated across the entire width of the lock chamber, and as there are eight of them, the chances are still further multiplied.

No provision has been made for raising the box girders to allow obstructions to pass. With this type of emergency dam it would be practically

* *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2640.
† *Loc. cit.*, p. 2620.

impossible to do so on account of the enormous loads to which the hoisting machine would be subjected. It might happen, therefore, that the first girder to be lowered would meet an obstruction, so that the releasing hooks would not function and thus would put the dam entirely out of commission. A catastrophe of this kind could not occur at the Panama Canal. If a pair of wicket girders should not get properly seated, so that a set of gates could not be lowered, the principal structural framework of the dam would remain intact and the small opening in the dam could be closed later by some provisional means, as was actually done at the "Soo".

Unfortunately, the author has not given the hydraulic conditions for the New Orleans Dam. He simply states* that the computed volume of the flow of water will be "in excess of 70 000 sec-ft.", without giving the velocity. From this figure the writer has computed the entrance head to be about 16 ft.; and the entrance velocity, about 28 ft. per sec.

In computing the flow for the Panama Canal the surface drop at the end of the three flights of locks was not taken into consideration either, as it would only have complicated the problem and have had little effect on the final result, because this drop occurs about 88 ft. below the upper level. At New Orleans, however, the conditions are such that the surface drop at the end of the lock will greatly affect the computations. The writer, therefore, has made approximate calculations, the result of which is a discharge of 50 000 sec-ft., and a velocity of 16.5 ft. per sec. During the lowering of the stop-log girders at New Orleans the maximum static head imposed on them is estimated by the writer to be about 9 ft., whereas at the Panama Canal the maximum static head during the lowering of the gates is 24 ft.

The author ascribes† the low cost per square foot of lock opening at New Orleans mainly to the more economical type of design. The writer believes it is uneconomical, because it is subjected to the full static head, whereas at the Panama Canal two-thirds of the reaction of the static head is carried by the sill and only one-third by the dam itself.

In the author's comparison‡ of the cost of the New Orleans Dam with the Panama Canal dams, no account is taken of increased moments due to longer spans, nor to increased loads due to greater volume of flow, velocity head, and static head. This comparison is identical to estimating the weight of two bridges by simply calculating their weights in straight proportion to their respective spans without considering the different loads to which they are subjected. Evidently, in comparing costs it should be kept in mind that the moments vary in proportion to the square of the span; that the kinetic energy of the flow of water varies in proportion to the cross-section of the flow multiplied by the square of the velocity; and that the horse-power required and the weight of the machinery and its supports vary in inverse proportion to the time required for closing. In this connection it should be remembered that

* *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2620.
† *Loc. cit.*, p. 2643.
‡ *Loc. cit.*, p. 2642.

an emergency dam is practically nothing but one huge machine. The Panama Canal dams are subjected to:

Moment due to span..... $= \left(\frac{110}{75}\right)^2 = 2.15$ times New Orleans Dam.

Kinetic energy of flow $= \frac{4\,000}{3\,050} \times \left(\frac{24.1}{16.5}\right)^2 = 2.8$ " " " "

Machinery and supports..... $= \frac{60}{25} = 2.4$ " " " "

If proper allowance is made for these factors, the cost of each emergency dam for the Panama Canal ($432 000) compares very favorably with the cost of the New Orleans Dam ($353 000).

The greater reliability in the operation of the Panama Canal dams should also be taken into account. The use of releasing hooks was at first considered for the Panama Canal dams. A preliminary design was published in the Annual Report of the Isthmian Canal Commission for 1908. The principal feature in the operation of this dam was identical with that of the New Orleans Dam, because the whole operation of the dam was to be performed from a central power plant and releasing hooks were to be used. This design was abandoned because releasing hooks were considered unreliable on account of the possibility that they might not release if the wicket girders or the gates should meet an obstruction at their lower ends. The design was low in cost. Its estimated weight was 1 870 000 lb., whereas the dams constructed weigh 6 700 000 lb. each. Thus, the estimated weight of each emergency dam for the Panama Canal as originally designed with releasing hooks amounted to only 28% of the weight of each of the present dams. This large increase in weight was caused not only by abandoning the use of releasing hooks and substituting separate hoisting units, but also by enlarging the size of the gates, thus reducing the time of closing; by furnishing the gates with roller bearings; and by providing the hoisting machines with sufficient power to raise the wicket girders against the current. The increased cost due to this increase in weight was of minor importance on the Panama Canal where reliability was given first consideration.

If it is desired to make a comparison between the "stop-log" type of emergency dam as used at New Orleans with the wicket-girder type as used at the Panama Canal, it should be made with the original design for the Panama Canal, using releasing hooks. The result of such a comparison, making proper allowance for the different spans, loads, and time required for closing, will show that the stop-log type as built at New Orleans is several times as expensive as the wicket-girder type with releasing hooks would have been if built in its stead. The writer believes that the New Orleans Dam, instead of having "the merit of being lower in first cost" than any emergency dam previously built, has the defects of being higher in cost and less reliable.

All the merits claimed by the author in favor of the New Orleans Dam as compared with the Panama Canal dams are lost, when it is considered that the stop-log type used at New Orleans would not be feasible for use at the

Panama Canal. Following the author's design the stop-log girders, if used for the Panama Canal would have the following dimensions: Length, out to out, 120 ft.; horizontal depth at ends, 6 ft. 6 in., and at the middle section, 11 ft.; and vertical depth, 6 ft. 2 in. The horizontal area of each girder, A_h, would be 1 000 sq. ft. and the vertical area, A_v, would be 745 sq. ft. As the spans at the Panama Canal and New Orleans are 100 ft. and 75 ft., respectively, and as the static head, when the locks are empty, is practically the same at both places, the weight of the stop-log girders for the Panama Canal would be $\left(\frac{110}{75}\right)^2 = 2.15$, say, twice the weight of those for the New Orleans Dam. The sinker castings might be proportioned accordingly, and thus the weight of one stop-log girder with sinker castings for the Panama Canal would be $D = 2 \times 244\,000$ lb. $= 488\,000$ lb.

A study of the hydraulic conditions for the Panama Canal dams, during and after the lowering of each gate has been published by the writer.* In order to calculate the loads on the stop-log girders, if used for the Panama Canal, the writer has, for the sake of convenience, assumed that the upper edge of each stop-log girder will be lowered to the same depth as the upper edge of each tier of gates of the Panama Canal dams. In this manner the complicated computations made for these dams may be utilized for calculating the loads. The data thus utilized are given in Table 3, Columns (1), (4), (6), and (8). Fig. 25 illustrates the hydraulic conditions just before a stop-log girder reaches its final position, when its upper edge is at the same elevation as the upper edge of Gates No. 3 of the Panama Canal dams.

It may be seen that when the water rushes between the stop-log girders, a contraction will take place due to the sharp corners, and that a partial vacuum will be created on the under-side and down-stream side of the stop-log girders (shown at (*a*), Fig. 25). These girders are filled with water while being lowered; during the lowering, they will also be subjected to the load of water flowing over them. These loads are given in Table 3, Column (5). No account has been taken of the loads produced by the partial vacuum. The uplift in the girders caused by the down-stream pressure is given in Column (9). The load carried by the hoisting machinery, while lowering the stop-log girders, is equal to the loads given in Column (5), minus the loads given in Column (9), and minus the friction produced at the ends of the girders given in Column (3). While raising the girders the load carried by the hoisting machinery is equal to the loads given in Table 3, Column (5) minus the loads given in Column (9) plus the friction, Column (3). The resultant loads to be carried by the hoisting machinery when raising and lowering the girders are given in Columns (10) and (11), respectively.

It may be noted that the loads to be carried by the hoisting machinery when the stop-log girders have been lowered to the same elevation as Gate No. 2 will be, for lowering 1 985 000 lb., and for raising 2 401 000 lb., or in other words, more than the total estimated weight of each emergency dam

* *Transactions*, Inter. Eng. Congress, San Francisco, Calif, 1915, "The Panama Canal", Vol. 2, No. 18.

for the Panama Canal as originally designed with releasing hooks. These figures should be sufficient to show the infeasibility of using the stop-log type of emergency dam for the Panama Canal.

TABLE 3.

UP STREAM.

Gate No.	Horizontal pressure, p, in pounds per square foot. (1)	Horizontal load, $H = p \times A_v$, in pounds. (2)	Friction, $F = 0.2 \times H$, in pounds. (3)	Static head, H_u, in feet. (4)	Vertical load. $V = H_u \times A_h \times 62.5$, in pounds. (5)
1	1 120	835 000	±166 000	39.0	2 450 000
2	1 400	1 040 000	±208 000	32.5	2 030 000
3	1 375	1 020 000	±204 000	24.8	1 540 000
4	790	588 000	±117 000	15.5	970 000
5	190	141 000	± 28 200	4.8	0

DOWN STREAM.

Gate No.	Velocity, v, in feet per second. (6)	Velocity head, $h = \frac{v^2}{2g}$, in feet. (7)	Static head, H_d, in feet. (8)	Uplift, $U = (H_d + h) \times A_h \times 62.5$, in pounds. (9)	Lowering, $V + U + D - F$, in pounds. (10)	Raising, $V + U + D + F$, in pounds. (11)
1	22.9	8.2	−24.6	−1 020 000	1 752 000	2 084 000
2	21.4	7.1	−12.3	− 325 000	1 985 000	2 401 000
3	18.0	5.0	− 1.3	0	1 824 000	2 232 000
4	8.8	....	.....	0	1 341 000	1 575 000
5	0	....	.....	0	460 000	516 000

On the other hand, the stop-log type may be feasible for the New Orleans Dam, but it is difficult to form a definite opinion as to this because the hydraulic conditions while lowering the stop-log girders are not given. The author merely states*:

"When the girders are being lowered into place under emergency conditions * * * the horizontal pressures are a combination of the velocity head due to the flow and the static head at the various levels. The computation of the loads due to these conditions is quite involved. In this case, too, the worst possible conditions have been assumed."

The author mentions nothing about the vertical loads imposed upon the girders by the flow of water. For the Panama Canal these vertical loads, as may be seen from Table 3, are about twice as large as the horizontal loads. It would be interesting to know the magnitude of the vertical loads for the New Orleans Dam under emergency conditions. It would be especially so because it is due to these loads that an emergency dam of the stop-log type is infeasible for the Panama Canal.

The author states* that while lowering the girders they are filled with water, but while raising them, at a rate of 30 ft. per sec., the water is allowed

* *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2629.

to flow out at the proper rate, so that the girders will contain, for a few minutes only, 1.43 ft. of water, thus guarding against an overload in the hoisting machinery and excessive stresses in the girders. It appears to the writer, that the critical condition for the hoisting machinery and the girders is not during the raising of the girders in still water, but during the lowering of them under emergency conditions, when they are filled with water and

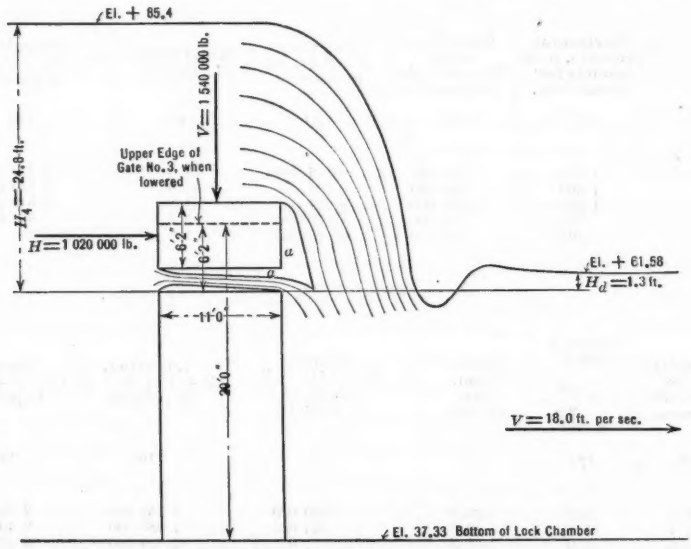

FIG. 25.—HYDRAULIC CONDITIONS FOR THE PANAMA CANAL IF STOP-LOGS WERE TO BE USED.

also have to carry a large volume of water flowing over them. The vertical loads thus carried by the stop-logs would be the equivalent of a head of several feet for a considerable period of time, instead of 1.43 ft. for a few minutes. The writer, therefore, fears that under emergency conditions excessive stresses may be produced in the hoisting machinery as well as in the stop-log girders of the New Orleans Dam.

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UNUSUAL ENGINEERING FEATURES OF AN IMMENSE THEATRE BUILDING

Discussion*

BY MESSRS. THEODORE BELZNER AND FRED A. NOETZLI.

THEODORE BELZNER,† AFFILIATE, AM. SOC. C. E. (by letter).‡—The author states§ that "a slight difference in pressure on the points of the gauge when applying the extensometer produced considerable variation in the readings". During 1913 and 1914, a comprehensive series of extensometer investigations was made on the important truss members at the main towers and legs of the Brooklyn intermediate towers, during the strengthening of the end spans of the Williamsburgh Bridge, in New York City.

A 20-in. Howard extensometer, with 55° tapered points, was used, and the writer does not recall such variations as the author indicates, except in cases where the pressure on the strain-gauge was not properly applied. When the pressure was always placed directly over the contact points and firmly held against the member to be measured, there could not possibly be any spring of the various parts in the strain-gauge, causing variations in the readings.

The gauge holes in the steel members were drilled with a hand ratchet, using a No. 57 combination drill bit with a 60° countersink. After the holes were drilled, the edge between the countersink and drilled holes was removed with a 55° center punch by tapping it with a hammer which, in turn, formed a conical seat to receive the points of the strain-gauge. A pointed aluminum rod, a small cedar stick, and some absorbent cotton were used for cleaning the holes before taking any readings. By using the aluminum rod, the softness of the material prevented any injury to the holes. The gauge holes were cleaned out at each reading.

The writer agrees with the statement concerning the reamed holes,§ and believes that with careful and accurate preparation, the application of proper pressure on the strain-gauge, and the extreme care taken by the author

* This discussion (of the paper by R. McC. Beanfield, Assoc. M. Am. Soc. C. E., published in December, 1927, *Proceedings*, but not presented at any meeting of the Society), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Insp. of Steel and Bridge Insp. in Chg., Brooklyn Bridge, Dept. of Plant and Structures, New York, N. Y.

‡ Received by the Secretary, January 27, 1928.

§ *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2659.

should have resulted in more accurate readings with the equipment used on the theatre building.

Detailed descriptions of precise extensometer investigations made on the Hell Gate Arch Bridge* and the writer's discussion of the Niagara Railway Arch Bridge,† may be of interest in connection with this paper.

FRED A. NOETZLI,‡ M. AM. SOC. C. E. (by letter).§—The design of the theatre building described by Mr. Beanfield involved some very unusual features. Especially noteworthy are the balcony truss of 168-ft. span, the balcony cantilevers with a 45-ft. overhang, the reinforced concrete proscenium arch of 100-ft. span, and the reinforced concrete frames over the Banquet Hall, having a center span of 90 ft.

The writer had the privilege of being connected with the design of some of the structural parts of the building, particularly those of reinforced concrete. In the original plans there were provided for the auditorium roof, two hinged reinforced concrete arches with horizontal ties spanned between the arch abutments. This type of construction would have involved some especially interesting features, but steel construction was subsequently used, as described in detail by the author.

The proscenium arch (Fig. 13)‖ is supporting, among other loads, the concentrated vertical reactions of four of the stage roof arches. Considerations from the architectural point of view prohibited the use of a horizontal tie to take the large thrust of this arch. On the other hand, the columns supporting the arch could not economically be constructed with sufficient lateral stiffness and strength for carrying this thrust. Consequently, the most economical solution appeared to be the design of this "arch" as a girder and for this purpose the great depth available for the girder (37 ft. at the center and about 60 ft. at the supports) was very desirable. Nevertheless, there were twenty 1½-in. bars necessary for the bottom reinforcement at the center of the span. The diagonal tension was taken care of by a combination of bent-up bars and stirrups.

The author's computation¶ of the secondary stresses in the steel balcony truss are of special interest. These studies indicated a way of eliminating part of the theoretical secondary stresses by proper erection methods. Such a procedure has been used very successfully also for other types of structures, especially bridge arches in which the effect of rib-shortening was partly compensated for by putting the arches under initial compression by means of screw-jacks and, subsequently, effecting the closure of the arches.

Mr. Beanfield's efforts to check, with extensometer readings, the theoretical stresses in important structural members is commendable. Information on the size of secondary stresses in such structures is especially desirable. It is to be hoped that such strain measurements will also be made to serve as a guide in the design of future structures.

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* *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), pp. 1047–1051.

† *Loc. cit.*, Vol. LXXXIII (1919–20), pp. 2004–2005.

‡ Cons. Hydr. Engr., Los Angeles, Calif.

§ Received by the Secretary, March 27, 1928.

‖ *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2663.

¶ *Loc. cit.*, p. 2653.

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THE VIRGINIAN RAILWAY ELECTRIFICATION

Discussion*

BY JOHN P. HOGAN, M. AM. SOC. C. E.

JOHN P. HOGAN,† M. AM. SOC. C. E.—The speaker feels that if there were more knowledge of the advantages of electric operation among civil engineers there would be more electrification. Mr. Gibbs has gone a long way toward making this knowledge available and all civil engineers owe him a great debt of gratitude.

The Virginian Railway has a very heavy traffic; in fact, the maximum of 110 000 tons total weight, in both directions, over a single-track line in a single day, is very close to the maximum in the United States, and, therefore, in the world. Outside this Continent a 2 000-ton train would be considered a curiosity, yet here is a system planned for 9 000-ton trains and the speaker understands that even heavier trains have recently been hauled.

Mr. Gibbs has touched generally on the conditions which led to the construction of an independent power plant.‡ While it is possibly true that, in the district in which the Virginian Railway operates, the existing utilities could not handle a load of this character, it may be stated in general that, in the more settled parts of the country, it should always be more economical to combine the railway load and that of the utilities. Whether it is possible to do so is always dependent upon the arrangements that can be made and whether a discussion of the relative costs can be brought about on a basis of perfect frankness and honesty. There is a great deal of misunderstanding between the power companies and the railroad companies on the question of power supply which is about equally distributed between the two and is generally due to misinformation.

The power companies do not often take the trouble to acquaint themselves with the load characteristics, nor do they do the necessary work to determine

* Discussion of the paper by George Gibbs, M. Am. Soc. C. E., continued from April, 1928, *Proceedings*.

† (Parsons, Klapp, Brinckerhoff & Douglas), New York, N. Y.

‡ *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 16.

the effect of this load on their system, and the result is that uncertainty naturally causes high quotations.

The railroad people, on the other hand, frequently impose restrictions on power company supply far in advance of the service that they themselves can obtain from independent stations. This increases the cost of the service out of all proportion to the benefit that can be derived from such restrictions. The railroad companies frequently do not keep sufficient records of cost of power from their independent stations to enable them to determine the real cost of the service, and such figures as they do keep are usually shrouded in deep secrecy. In some cases, railroad companies, having independent power stations, later make connection with utilities and throw on the latter the service burden, power-factor correction, reserve capacity, etc., and then pride themselves that power from their own stations is cheaper.

An examination of Fig. 4* discloses the fact that the general shape of this curve would be very advantageous for combination with that of a utility with its maximum peak around 7 o'clock and its maximum hour between 8 and 9 o'clock. While the load itself is somewhat ragged and subject to violent fluctuation, it would be submerged in the daily load of a large utility with a good inherent system regulation. It is evident, however, that a power system with poor regulation would have difficulty in meeting the service demand of the railroad with a load of this character.

Mr. Gibbs has stated,† in part, as follows,

"Unfortunately, in the Eastern United States, available water powers are few, and such as exist are not generally near the consumer, and are often not dependable in capacity."

This statement should not be applied to the entire Eastern United States because all through Northern New York and New England there are large districts which have excellent service from water power, and which are not normally supplemented by fuel-burning plants.

Mr. Gibbs also states,† in part:

"As a matter of fact, the load conditions * * * are so exceptional that it is doubtful whether any existing local company * * * could afford to take on a load of this character, except by devoting special generators and transmission circuits to the railway supply."

Was this entirely due to the deficiencies of the local utilities or was it partly caused by the desirability of single-phase generation?

Referring to the author's statement‡ that "the alternating current system is the nearest approach so far to a general one," the speaker does not think that any trend has developed toward any particular system. England, France, Holland, Belgium, Spain, Australia, South Africa, and South America are using direct current; Switzerland and Germany are using single-phase, alternating current; and Italy is divided between direct and three-phase current. European conditions do not apply to American railways and, in general, the street car and interurban systems in this country are using direct current

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 13.
† *Loc. cit.*, p. 16.
‡ *Loc. cit.*, p. 15.

at varying voltages, and such changes as have been made in existing systems, have all been from alternating current to direct current. The largest main-line electrification in this country is that of the Chicago, Milwaukee and St. Paul Railroad using direct current at 3 000 volts. The most recent electrifications (except that of the Pennsylvania Railroad) have been the Illinois Central at 1 500 volts direct current, and the Great Northern, with alternating-current distribution and direct-current locomotives. Most American terminal electrifications, including that of the Pennsylvania, have been at 750 volts direct current. On the other hand, the New York, New Haven and Hartford, the Pennsylvania, and the New York Connecting Railways are equipped with alternating-current distribution and single-phase motors; and, finally, the electrifications of the Norfolk and Western and the Virginian Railways are based on alternating-current distribution and contact wires and three-phase motors. It can hardly be said, therefore, that any system has been general, or that railroads are even approaching a general system.

The author states* that the generators in the power-house are rated at 10 000, single-phase, continuous rating, and the speaker inquires as to what the equivalent rating of these generators would be if operated at three-phase. Was consideration given to the operation of motor-generator locomotives, or of single-phase motors instead of the split-phase motors, and, if so, what considerations led to their being discarded?

Mr. Gibbs has given an extremely interesting and clear description of the balancing wires and balancing transformers which are unique and useful features of this system and for which he is largely responsible, if not for their actual invention at least for their practical application. The speaker would like to ask whether the balancing transformers, which are almost equal in total capacity to the main transformers, are not (to all intents and purposes) useful and necessary, and whether, in the event of their omission, it would not be necessary to put in equivalent capacity in the main transformer stations? Would it have been possible to space the transformer stations 20 miles apart if it had not been for the addition of these balancing stations?

In addition to these questions, the following are also submitted:

(1) What normal pressure is maintained between the trolley wire and the pantograph as used on the Virginian Railway?

(2) Mr. Gibbs states,†

"Under these conditions the motors are subjected to high-current inputs and may be damaged unless they are of a type which will meet this severe requirement."

Can a single-phase motor be constructed that will meet these requirements?

(3) The speaker notes‡ the connection of two driving wheels to a single motor. Is any difficulty experienced in the distribution of the load between the two motors included in each unit?

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 22.
† *Loc. cit.*, p. 45.
‡ *Loc. cit.*, p. 47.

(4) The split-phase motor is limited to two speeds except when operated in connection with a rheostat. This would seem to limit its use to heavy freight traffic where a comparatively low rate of speed was sufficient and would not make it suitable for general railway operation under varying conditions. How much is the rheostat used in ordinary operation other than in acceleration?

(5) Rheostatic operation is presumably rather expensive in power consumption and in this connection the speaker would also like to know how much the speed limitations influence the character of the load curve, since it would appear that a train might be required to go up a heavy grade at a higher speed than was absolutely necessary.

The speaker wishes to congratulate Mr. Gibbs on the accuracy of his preliminary power estimates and on the very brief but satisfactory statement* of the general results.

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 59.

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ANALYSIS OF ARCH DAMS BY THE TRIAL LOAD METHOD

Discussion*

BY MESSRS. C. A. P. TURNER, F. H. CONSTANT, AND F. W. HANNA.

C. A. P. TURNER,† M. AM. SOC. C. E. (by letter).‡—The authors are entitled to appreciation of the profession for their laborious effort to place the design of curved dams on a more rational basis. The increase in strength and economy of the curved vertical dam over that of the straight gravity type is little appreciated because of a fundamental misconception regarding its mode of resistance. In elementary mechanics the student is taught that the applied moment of an external force is the product of that force times its lever arm. Applied moment is an abstract quantity, balanced either by internal bending or twisting resistance, depending on the manner of support, restraint, and the form of the resisting section.

In many cases the internal resistance of the member considered is entirely bending resistance. In others, such as the shaft, the internal resistance is almost wholly that of twisting resistance. In the square plate supported on four sides the resistance to flexure is half bending and half twisting. A curved dam, because it is fixed at the base so that it cannot deflect down stream, tends to oppose water pressure predominantly by twisting, rather than by arch action. The applied external force consists of the normal hydraulic pressure that increases uniformly, downward from the top, along the vertical up-stream face.

The horizontal reaction between the dam and its base is represented by the sum of the horizontal down-stream components of the hydraulic pressures on a vertical up-stream face. The components of these pressures parallel to

* Discussion of the paper by C. H. Howell, M. Am. Soc. C. E., and the late A. C. Jaquith, Esq., continued from April, 1928, *Proceedings*.

† Cons. Engr., Minneapolis, Minn.

‡ Received by the Secretary, February 8, 1928.

the chord of the arc of the dam furnish the external reactions to the internal ring compression in the arched masonry or concrete of the dam. The intensity of these ring stresses for an economic design are small relative to those developed by the twisting shears.

A curved dam may be considered analagous to a vertical curved plate rigidly restrained at the base and ends, and subject to the pressure of parallel forces. These tend to twist the plate along its length, causing a maximum rotation at the deepest part. As the action is a twisting one there must be both a positive and a negative twisting deformation. Were the plate free at the base, restrained at its end, and then twisted from the center, the points of equilibrium would be at the quarter-points and a vertical plane at a quarter-point would be the locus of equilibrium of the positive and negative twisting shears. However, because the arch is rigid both at its base and at its ends at the quarter-point of the free crest, the direction of the plane of equilibrium will rotate from the vertical to an angle of 45° with the horizontal projection of the crest of the dam.

In a plate deformed by twisting and bending, contours of curvature may be developed by graphic methods as applied to uniform elastic plates supported, restrained, and bent in any manner as previously outlined by the writer. To apply the method to the curved plate of the dam, treat the down-stream component of the pressures separately as these component pressures and the reactions thereto constitute the applied torsional forces.

The first step in this analysis is to fix the travel of the pressure toward its support. This travel of the load or pressure follows a straight line and in the case of plates which are twisted these lines intersect at some given point which may be termed a focus. The plane of equilibrium of the positive and negative twist would represent one of these planes of zero shear and the focus of intersection of the other planes will lie along this line at some point to be determined. Curves of greatest shear will be normal to those of zero shear and the co-ordinated intersections will determine one series of the twisting shear curves. The co-ordinated intersections of the planes of zero shear and of greatest twist will fix another or complement series of twisting shears in the opposite direction from the first set. Symmetrical intersections of the twisting shears will fix the contours of curvature which are principal stress lines at the mid-thickness of the plate.

Because when an element is twisted in one direction on one side of the plate and the other direction on the other equilibrium will occur at mid-length, there are two independent series of shear curve intersections which locate the same principal stress line or contours of curvature.

In Fig. 18, a graphic analysis is offered of an opening between rock walls similar in form to the experimental dam at Stevenson Creek, in which the sides have a slope of 45 degrees. The first problem is to locate the foci of the planes of zero shear representing the transference of the down-stream hydraulic pressure to its reaction at the base. Because the crest of the dam is free to deform in an elastic manner, the length of the projected positive and negative twisting deformation will be equal and the deflection of the

crest will pass through zero at the quarter-point of its down-stream projected length. The equilibrium of the positive and negative twisting, that occurs at the mid-thickness on a line at 45° to the crest, will represent one plane of zero shear and the focus of the series or pencil of planes of zero shear will lie somewhere along the length of this line.

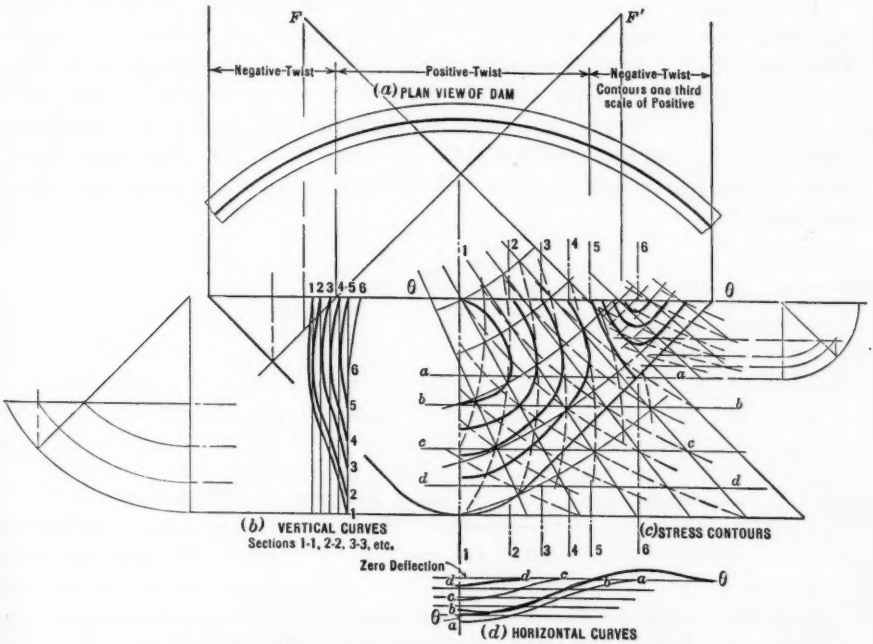


FIG. 18.—GRAPHIC ANALYSIS, STEVENSON CREEK DAM.

The abscissa of the focus for each respective side of the dam about its center line will lie at the same distance from the center as the center of the line of equilibrium of the positive and negative twist, because twisting deformation accumulates along the length of the member twisted and this length fixes the horizontal position of the focus.

It is then necessary to co-ordinate the lines of greatest shear, greatest twist, and zero shear. For this purpose it is generally convenient to use four equal divisions for the co-ordinates in the positive and negative areas, respectively, treating the twist about the vertical as productive of equal shifts. Equal pressures must be represented by the spacing of the horizontal planes of greatest twist; that is, take the water pressure increasing from zero at the crest to a maximum at the bottom, and the projected crest of the dam will be the first plane of greatest twist. The half depth will represent the second plane, and 0.7071 times the depth at the center will represent the third, etc. The vertical areas between the planes of greatest twist will represent equal areas of pressure. Having divided the vertical pressure (for the positive area twisted) into equal pressure areas or parts, it is necessary to co-ordinate the circular curves of greatest shear by dividing the vertical

height (from the crest to the base) into four equal parts and drawing circles from the foci as a center to represent the curves of greatest shear. Then the twisting spirals are drawn as shown in Fig. 18 (*b*) and (*d*) and the contours of curvature, Fig. 18(*c*) are located therefrom. The area of negative twist is treated in like manner.

In treating the negative area of curvature from twist by proceeding along like lines the contours would represent a different magnitude of negative deflection. This difference would vary in proportion as the depth of the base of the negative area varies with the depth of the base from the water level in the positive area. On this scale, the deflection curves may be readily coordinated and contours interpreted. As applied to the Stevenson Creek Dam, the graphic method develops deflections in striking agreement with those measured.

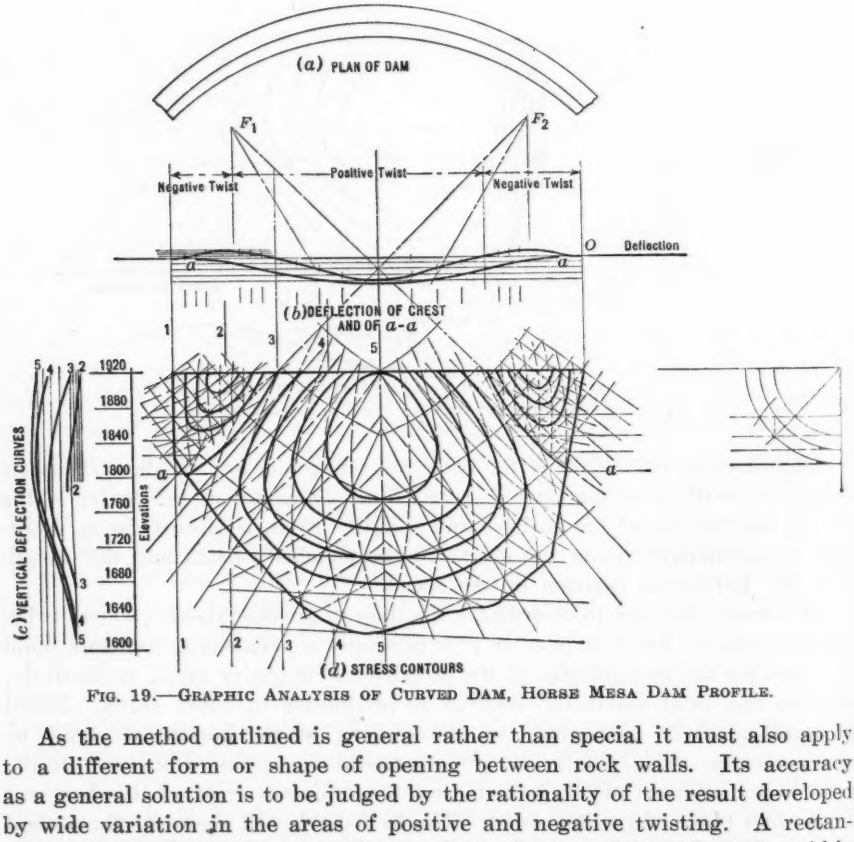


FIG. 19.—GRAPHIC ANALYSIS OF CURVED DAM, HORSE MESA DAM PROFILE.

As the method outlined is general rather than special it must also apply to a different form or shape of opening between rock walls. Its accuracy as a general solution is to be judged by the rationality of the result developed by wide variation in the areas of positive and negative twisting. A rectangular opening between rock walls, nearly as deep or deeper than the width, will give a satisfactory opportunity for the application of the method. This is presented in Fig. 19, an analysis of Horse Mesa Dam.

In the case of an opening with 45° sides it will be noted that, on the same method of reasoning, the abscissa of the focus occurs at the three-eighths

point of the half chord of the arc. For the rectangular opening it occurs at the quarter-point measured, in each case, from the end of the arc. Thus, the focus shifts horizontally and vertically with the change in the configuration of the opening. In like manner the relative spacing of the planes of greatest twist change, in the positive and negative areas, with the change in configuration of the opening.

It is now possible to compare the contours determined by this method and the exact method of torsional analysis. The authors have assumed that all the load is carried by the horizontal and vertical elements instead of diagonally in the most direct line to the support. The contours in Fig. 9* form a cusp at the vertical center line of the dam. It is a fundamental property of internal elastic stress as stated by Eddy† that the stress on an ideal plane of division that traverses any given point of a body cannot change suddenly, either as to direction or magnitude, while that plane is turning in any way about the given point. A sudden change can only take place at a surface where there is change in the material.

Because a contour of curvature is identical with the principal stress line at the mid-thickness of the plate, the error embodied in the authors' assumption is apparent. The direction of the contour of zero deflection in Fig. 9 differs from the true contour by convex instead of concave curvature toward the negative area. In direction it differs from the true direction by an angle equal to from 45 to 80 degrees. Accordingly, as principal stress lines, some of them may occupy the position of shear curves. Hence, if interpreted as tensile stress, the strength of concrete in shear resistance may be mistaken for the weakness of concrete in pure tension. Furthermore, in their computations of the Horse Mesa Dam, no recognition of the area of negative twist appears and the formation of the contours is quite different from those which may be correctly developed by the graphical method or by experiment, as was done at the Stevenson Creek Dam.

It may be objected that in presenting graphic methods, the writer has failed to take into consideration the change in the torsional moment of inertia incident to change in the thickness of the dam from the top to the bottom and that any method which would disregard such variation cannot be considered approximate even to a moderate degree. Such criticism fails to take into consideration the fact that the graphic method determines relative not actual deflections; that a fairly wide variation in the torsional moment of inertia (due to increase in the thickness of the dam from the top toward the bottom) does not affect the relative position or the location of deflection contours. For that reason the graphic analysis presents a simple and expeditious manner of determining elastic behavior and a correct method of finding the distribution of the reaction along the length of the dam. With this distribution known it should not be difficult to compute the vertical couples which oppose overturning and the stresses resulting therefrom.

The trial-load method endeavors to treat a torsional phenomenon as if there were a flexural equivalent thereto, to wit, cantilever action and arch

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 73.
† "Graphical Statics", 1878.

action. As the trial-load method depends on distribution of the applied pressure toward the support in vertical and horizontal directions, the resultant of these distributions at each unit of vertical height may readily be determined. It will then be found that the resultants do not follow a regular law of distribution and do not, in the most direct manner, convey the pressure to its support as required by the law of least work.

The twisting moments brought into action by the restraint of the dam at its base and ends takes effect by changing the direction of the pressure and altering the distribution of the reaction thereto along the base of the dam. The effect of any variation in thickness under such action differs wholly from the phenomenon of flexure and the graphic method permits an analytical investigation of the variation in contours of relative deflection to the extent that they are affected by the variation in form and thickness of the structure.

The curves of greatest shear are, of necessity, circles drawn from the foci as centers. The planes of greatest twist are fixed by the intensity of the applied hydraulic pressure such that their spacing, per unit, represents equal pressure areas. The only variation then, that may be effected by a change in the thickness of the dam, is the position of the focus and spacing of the radials of zero shear and the depth of the negative area. Provided the crest of the dam is made of a constant width, the point of inflection (where the positive twisting changes into negative) will remain substantially fixed and the spacing of the radials (because they depend on this locus) remain unchanged even though the lower portion of the dam is increased in thickness.

The deflection is reduced by thickening the dam at the base, but the locus of the contours of equal deflection remains substantially fixed because the co-ordinates that determine the twisting shears remain substantially fixed. An abrupt change in the thickness would affect the accuracy of this generalization. That the function of the curved dam is to oppose twisting moments rather than flexure substantiates the trend of practice toward a more uniform section along the length of the dam than has been practiced in many examples of older construction.

An acceptable elastic analysis must be verified by its elastic deportment. Unfortunately, the elastic behavior of curved dams has been investigated systematically only in the single example of the Stevenson Creek Dam.*

Deflection of the dam is shown graphically in diagonal projection in Fig. 20. A similar diagonal projection of the relative deflections, determined graphically from Fig. 18, may be compared therewith. It will be noted that the contours of negative twist fix a slightly different curvature from that shown in the report of Engineering Foundation, referred to, in that the restraint at the end of the dam manifests itself in an additional point of contraflexure neglected by the draftsman in drawing the curves through measured points on the experimental dam. This does not show a difference in relative deflections where measured, but rather tends to correct an error of interpretation of the measurements made. The curved contour

* See Progress Rept. No. 3, Engineering Foundation, Fig. 2.

of zero deflection, measured at Stevenson Creek, matches those determined by the graphic method.

Every elastic plate or curved dam, in which sudden jogs in the thickness of the material are avoided, will deflect in a smooth regular curve and contours, whether determined graphically or by computation, and, if rational, should determine smooth consistent flexure curves. The results, by graphic analysis, treating the arch resistance as torsion (Fig. 18), present such curves. The diagonal projection of Fig. 20 may be compared to the Progress Report 3, of Engineering Foundation, and the agreement between the relative theoretical and measured deflections is sufficiently close to carry conviction.

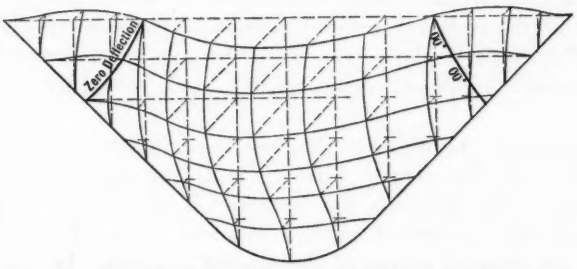


FIG. 20.—DIAGONAL PROJECTION OF CONTOURS OF DEFLECTION. GRAPHIC ANALYSIS, STEVENSON CREEK DAM.

In the experiments on the Stevenson Creek Dam, a slight shrinkage crack on the surface was noted, which was observed to close up as the water pressure approached the crest of the dam. This results from the ring stresses by the component of the hydraulic pressure parallel to the chord of the arch.

The graphic method of analysis permits the simple derivation of the curves of flexure of a plate bent in any manner and subjected to any kind of restraint, manner of support, or applied load. This solution is not always obvious but, when once worked out, the procedure is simple and the method gives the investigator an exact conception of the displacement curves and equi-potential curves, in addition to the curves of shear and principal stress. It permits the development of the contours of distortion which the plane normal section undergoes during the twisting action.

For example, in a round shaft under torsion, half the end area is increased in length and half decreased. The entire area within a radius of $0.7071R$ is elongated by the radial compressions and circular tensions developed in the twisting process. If the cross-section is square, the rotational couples about consecutive radii are not equilibrated by uniform intensity of the shearing stress in the normal planes as is the case in the circular shaft. The result is a deformation consisting of alternate octantal bulge and depression which, in a particular octant, is dependent on the direction of the twisting force.

When the shape of the specimen is changed to rectangular, and the long side is one and one-half times the breadth, the alternate bulge and depression occur in alternate quadrants and in the case of a curved dam the area of depression would be the upper down-stream quadrant. The quadrants of

bulging distortion would be the upper up-stream and the lower down-stream quadrants. This torsional distortion would manifest itself in a thrust at the base, although the elastic strain therefrom would be relatively slight as measured in the Stevenson Creek experiments.

Torsional analysis of the curved dam indicates that material has been used with lavish wastefulness in the deeper portion of the dam and, in general, it has been thinned down too rapidly toward the ends. A more uniform section is better adapted to resist the torsional stresses and would result in greater strength with less material.

The economic form of the arch in a V-opening is a third degree curve rather than a circular one. Such a structure, to be well proportioned, should cut the cost of the ordinary gravity dam in half allowing a greater degree of safety. This is particularly true when the concrete is properly reinforced, thereby insuring toughness, dependability, and resistance to temperature stress.

In pure bending, the distance the load is shifted in its travel to the support, multiplied by the weight shifted, equals the applied moment. Therefore, in torsion, the distance the pressure is shifted by twist, times its weight, equals the portion of applied moment balanced thereby when the shift lies in the plane of action of the applied moment. In the straight dam this shift does not lie in the plane of the over-turning moment and therein the curved dam derives its great advantage over the straight dam.

Safe economic design of great concrete dams is of the utmost importance to the public when flood control and the reclamation of waste areas by irrigation are considered. The writer is pleased to contribute his effort in substantiating the authors' correct judgment regarding the superior merit of the curved dam as opposed to the straight dam. While he differs with them in the details of the demonstration, his expressed difference of opinion is intended as constructive criticism in presenting a theory in accord with experiment.

It will be observed that the torsional graphic analysis does not primarily take the curvature of the dam into consideration; that is, it is applicable alike to the straight and curved dam, but it deals only with the torsional effects in distributing the load along the base. Torsion shifts pressure or shear in its transfer to the support. In the curved dam there is a down-stream shift by virtue of curvature, accompanying the longitudinal shift.

In a V-opening the longitudinal shift distributes the reaction at a higher elevation and may thereby reduce the applied moment a substantial amount in either a straight or a curved dam. The shift of the reaction along the base when the dam is curved, has a down-stream component and this adds to the lever arm of the reaction in opposing over-turning moment. The sharper the curvature the greater the arm; also the greater the length of the dam. A balance between the disadvantage of increased cost of additional length and the advantage of greater lever arm and reduced thickness, constitutes the problem of economic design.

In an examination at a school of engineering, if the student should undertake to compute the strength of a shaft by a beam formula, he would

be considered deficient in knowledge of the principles of elementary mechanics; and yet many post-graduate engineers, with years of experience, undertake to calculate the torsional problem of the curved dam by the ordinary old beam formula for a gravity type structure unadapted to the conditions of the problem.

In conclusion, the credit for this graphic solution is in a large measure due to the laborious and painstaking effort of Messrs. Howell and Jaquith in their endeavor to reduce curved dam analysis to a rational basis. Interest in their work and an attempt to analyze it led to the writer's first application of graphical principles to the problem under discussion. Ernest McCulloh, M. Am. Soc. C. E., assisted in the preparation of this discussion and furnished many helpful suggestions.

F. H. CONSTANT,* M. AM. SOC. C. E. (by letter).†—The authors present four general conclusions to which the writer wishes to add comments, as follows:

1.—If the cantilever acts, the arch elements are not under uniform loading. With this conclusion the writer is in hearty accord. Any treatment of arches which considers only uniform loading is of little practical importance in the solution of arch dams.

2.—Computing proportionate loads and deflections in the two systems at one vertical section only, such as at the crown of the arches, is not sufficient for the solution of the problem as a whole. A complete solution should find coincident deflections at every point in every section. It is clear that the method of trial is not an easy one, but the writer has gone far enough with it to feel convinced that in spite of its real difficulties, it is practical. He found (as did also the authors) that the deflections of the cantilevers are very sensitive to slight changes of loading, while, of course, the stresses are not. Hence, it is not necessary to carry the trials so far as to secure absolute coincidence of arch and cantilever deflections in order to arrive at a close approximation of the actual distribution of the water load.

3.—Rational formulas, for a complete solution, must take account of the shape of the canyon. The trial method does this by using a great many vertical and horizontal sections. It is not possible to do it by direct mathematical means. The rational formulas that have been heretofore devised fit only special and very simple cases, and any attempt to adapt them to some other type of dam section makes them hopelessly complex.

4.—In place of the authors' fourth conclusion‡ the writer presents somewhat simpler formulas for the special (and yet very common) case of the circular dam of constant thickness throughout any horizontal section and symmetrical relative to the vertical section at the crown. Unsymmetrical dams do occur, of course, but on the whole there is a general symmetry about most narrow profiles. The dams cited by the authors are quite symmetrical.

The writer also feels justified in treating the case of arches of constant thickness. The curve of pressure in an arch dam does not deviate from the

* Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.
† Received by the Secretary, March 2, 1928.
‡ *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 78.

axis of the arch nearly as much as in arch bridges, and there would not seem to be any necessity for varying the thickness, except possibly at the very end. In an approximate solution of the Stevenson Creek Dam, assuming a parabolic distribution of the water load, and neglecting temperature effects, the writer found that the curve of pressure remained practically within the middle third throughout, at least in the upper 40 ft. of the dam. It would be possible to change the shape of the arches slightly to conform more closely with the curve of pressure, and thus eliminate all but temperature moments. The error thus introduced in using the formulas for circular arches would be negligible.

The following formulas were derived from the principle of work by well-known methods and the derivations will not be given. They have been carefully checked by independent methods and are believed to be correct. For example, the deflections check by Maxwell's principle of reciprocal deflections.

The following notation (see Fig. 21), has been used. All angles refer to the half arch. They are in radians, when they occur alone; in degrees, when they occur in functions:

ϕ_1 = the central angle between the center and the end of arch;
ϕ = the central angle to the point at which the moment is required;
θ = the central angle to the point of application of the concentrated load, P;
θ_1 = the central angle to the point at which the deflection is required;
D = the radial deflection of the arch at θ_1 due to a pair of symmetrical radial loads, P;
R = the radius of the axis of the arch;
I = the moment of inertia of the arch, with a vertical width of 1 ft.;
t = the constant horizontal thickness of the arch;
M_0 = the bending moment at the crown, produced by a pair of symmetrical radial loads, P;
H_0 = the horizontal thrust at the crown, produced by a pair of symmetrical radial loads, P;
M = the bending moment at a section, under the same loading;
P = the applied radial force, taken as unity. There is a symmetrical force on the other half of the arch (not shown in Fig. 21);
p_1, p_2, etc. = intensities of water pressure on the arch at Points 1, 2, etc.;
E = the modulus of elasticity of concrete. If the same value of E is used for the cantilevers, it may be omitted from the equations in equalizing deflections. The deflections found, then, will be E times the actual deflection;
$A = t$ = area of cross-section of an arch element, in square feet; and,

$$C = \frac{\sin \phi_1}{\phi_1}.$$

$$M_0 = R \frac{1 - \cos (\phi_1 - \theta)}{\phi_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

$$H_0 = \frac{\frac{1}{2}\left[\sin (\phi_1 - \theta) \sin \phi_1 - (\phi_1 - \theta) \sin \theta\right] - C\left[1 - \cos (\phi_1 - \theta)\right]}{\frac{1}{4} \sin 2\phi_1 - 2C \sin \phi_1 + \phi_1 \left(\frac{1}{2} + C^2\right) + \frac{I \phi_1}{A R^2} \left(= \frac{t^2 \phi_1}{12 R^2}\right)} \quad \ldots\ldots (59)$$

$$M = M_0 - R \sin (\phi - \theta) - H_0 R (\cos \phi - C) \quad \ldots\ldots\ldots (60)$$

$$\frac{E\,I}{R^2}\,D = (M_0 + C\,R\,H_0)\left[1 - \cos(\phi_1 - \theta_1)\right] - \frac{R}{2}\left[\left(\phi_1 - \begin{matrix}\theta\\ \theta_1\end{matrix}\right)\cos(\theta - \theta_1)\right.$$
$$\left. - \sin\left(\phi_1 - \begin{matrix}\theta\\ \theta_1\end{matrix}\right)\cos\left(\phi_1 - \begin{matrix}\theta_1\\ \theta\end{matrix}\right)\right] - \frac{R\,H_0}{2}\left[\sin(\phi_1 - \theta_1)\sin\phi_1\right.$$
$$\left. - (\phi_1 - \theta_1)\sin\theta_1\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

In the double value expressions, $\begin{matrix}\theta\\ \theta_1\end{matrix}$ and $\begin{matrix}\theta_1\\ \theta\end{matrix}$, use the upper values if $\theta > \theta_1$; the lower values if $\theta_1 > \theta$.

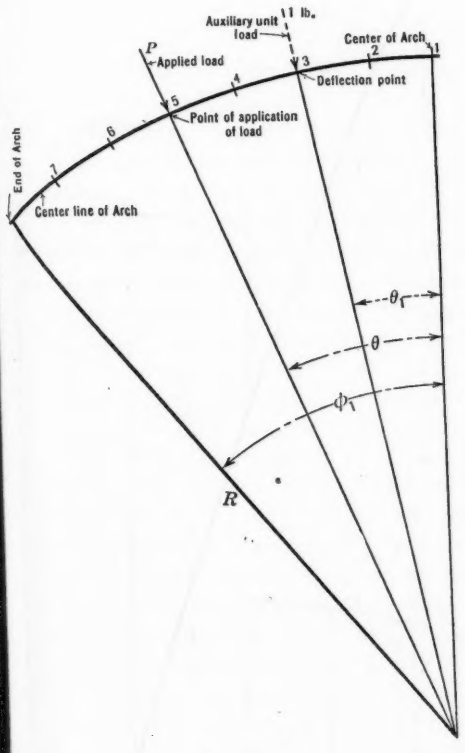


FIG. 21.

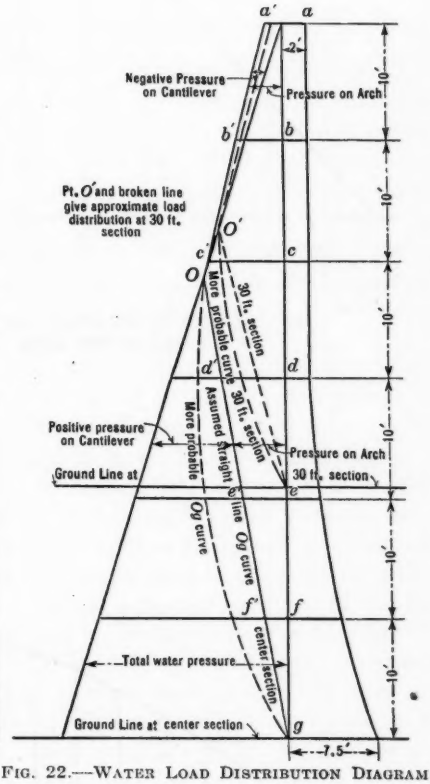
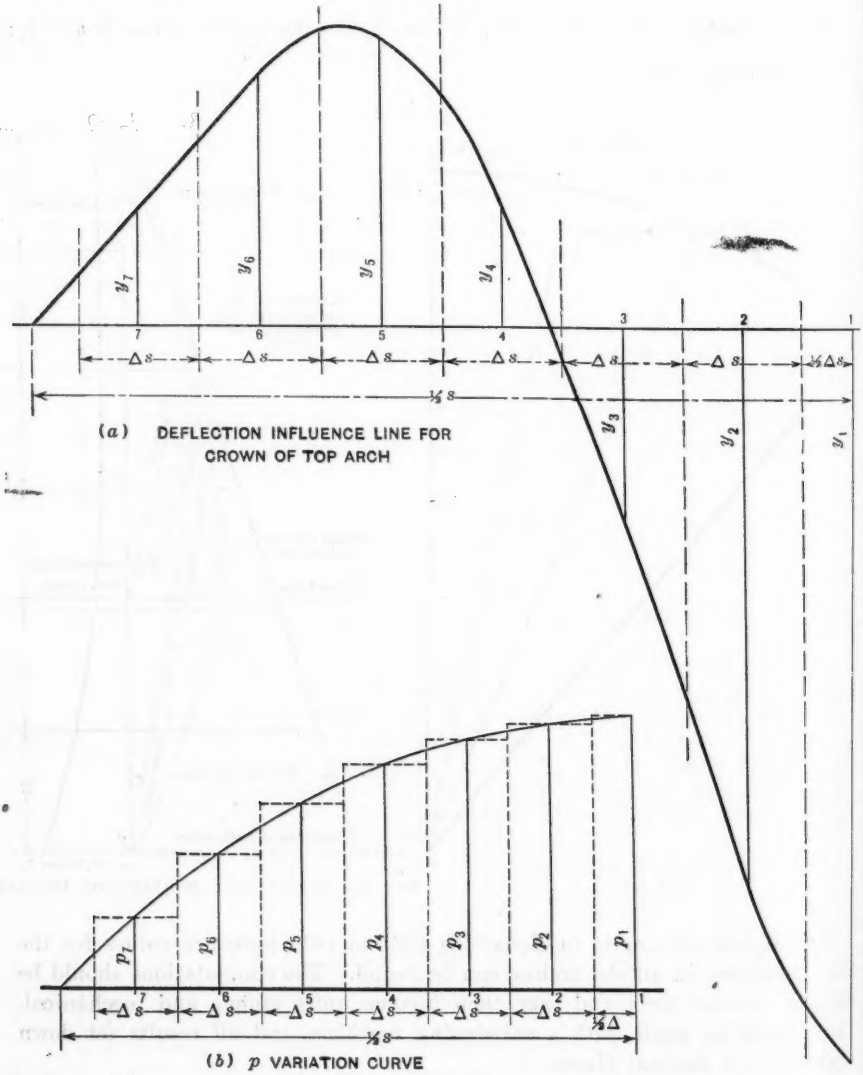

Negative Pressure on Cantilever
Pressure on Arch
Pt. O' and broken line give approximate load distribution at 30 ft. section
More probable curve
30 ft. section
Assumed straight line Oy curve
Positive pressure on Cantilever
Pressure on Arch 30 ft. section
Ground Line at
Oy curve
Total water pressure
center section
Ground Line at center section
7.5'

FIG. 22.—WATER LOAD DISTRIBUTION DIAGRAM.

By varying the angles in Equations (58) to (61) influence values for the four quantities, in all the arches, can be found. The computations should be made in tabular form and they then become quite simple and mechanical. They should be made with a calculating machine, and all results set down to at least six decimal places.

Fig. 23 is a typical deflection influence line. It shows the change in deflection at the crown of the top arch as a pair of radial unit loads move from the crown symmetrically over the left and right halves of the arch, respectively.

An assumption is now made in regard to the variation in radial water pressure along any horizontal arch. If the intensity of the radial water pressure, p, is uniform along the arch, the deflection at any point would be the area of the corresponding influence line multiplied by p. If p is not uniform, but varies in some assumed manner, as shown in Fig. 23(b), the deflection of any arch at any point can be found as follows:



(a) DEFLECTION INFLUENCE LINE FOR CROWN OF TOP ARCH

(b) p VARIATION CURVE

FIG. 23.

Let p_1, p_2, etc., represent the intensities of water pressure carried by the arch at the points, 1, 2, 3, etc., where the horizontal base line represents the actual length of one-half the arch, and Δs equal the distance between the charted

points on the arch. Then the actual loads at the points, 1, 2, 3, etc., are $\frac{1}{2}p_1 \Delta s$, $p_2 \Delta s$, $p_3 \Delta s$, etc. The deflection at some point, 3, due to the total load on the arch will then be the sum of the products of these loads into the corresponding ordinates of the influence line for Point 3 of that particular arch. It is best to express p_2, p_3, etc., as some fraction of p_1. For example, to find the deflection, D_3, at the point, 3, of an arch, let the ordinates in the influence line at 1, 2, 3, ... etc., be y_1, y_2, y_3, ... etc. Then,

$$D_3 = \Delta s \sum \left(\frac{1}{2} p_1 y_1 + p_2 y_2 + \ldots.\right)$$

or, if $p_2 = k_2 p_1$, $p_3 = k_3 p_1$, etc.,

$$D_3 = \Delta s\, p_1 \sum \left(\frac{1}{2} y_1 + k_2 y_2 + k_3 y_3 + \ldots.\right) = K_3 p_1$$

For the assumed distribution and the given deflection influence lines values of K can be obtained for each point of each arch, and the deflection of that point will be known as soon as p_1 is found. It might be convenient to construct a complete table of the values of K to include all the charted points, using some notation such as K_{13}, K_{25}, etc., meaning thereby, Point 3 of Arch 1, Point 5 of Arch 2, etc. The values of K_{11}, K_{21}, etc., will give the crown deflections of the several arches as soon as the corresponding values of p_1 are found. For a first trial the curve of variation of p might be assumed to be a parabola, for all the arches. Further modification will then be suggested by the deflections which parabolic distribution produces.

The next step is to find such numerical values of p_{11}, p_{21}, p_{31}, etc., as will cause the vertical deflection curve of the crowns of all the arches approximately to coincide with that of the center cantilever.

To find such values, p_1, the semi-graphical method of Cain and Noetzli is used. Fig. 22 is a vertical section of the dam at the crown; $a\ g'\ g$ is the total water-pressure triangle; and by trial the positive and negative areas, $O\ g'\ g$ and $O\ a'\ a$, respectively, are found by the Noetzli method so as to make the deflection curves of the center cantilever and of the crowns of the arches coincide approximately. The points, O and a', and the curvature of the line, $O\ g$, are found by trial. The writer used a straight line for $O\ g$, but the authors found that $O\ g$ has a considerable outward curvature, thus increasing the loading on the lower arches. The writer's investigation confirms this conclusion. The horizontal ordinates of the area, $g\ O\ a'\ a\ g$, represent the p_1-values, or crown pressures, on the arches. Thus (Fig. 22), $aa' = p_{11}$; $bb' = p_{21}$; etc. Then, the crown arch deflections are, $D_{11} = K_{11}\ p_{11}$; $D_{21} = K_{21}\ p_{21}$; etc., where the first subscript indicates the arch and the second, the point on the arch.

After a coincidence of arch and cantilever deflection curves at the crown has been obtained, on the basis of the assumed variation of pressure throughout the length of each arch (which for short may be called the p-curve (Fig. (23(b)), proceed to another vertical section, such as at Point 3. At this point all the ordinates representing the loading on the arches (Fig. 22), namely, aa', bb', etc., must be reduced in the fixed ratio $\frac{p_3}{p_1} = k_3$. The point, O, will

accordingly lie nearer the top, and the lower part of the diagram is cut off to conform to the ground level. The dotted lines in Fig. 22 represent the approximate condition. The cantilever and arch deflection curves are constructed as at the crown in the usual manner. Thus, the deflection curve of the arches at Section 3 can be drawn from $D_{13} = K_{13}\ p_{11}$; $D_{23} = K_{23}\ p_{21}$; etc., in which, $p_{11} = aa'$; $p_{21} = bb'$; etc. (Fig. 22). If the p-curve has been correctly assumed these arch and cantilever deflection curves should be coincident at Section 3. If they are not, within reasonable limits, a new p-variation curve, or a different center section distribution (Fig. 22) will have to be assumed until a fair check is obtained. For accurate results several sections should be tested in this manner.

Instead of a parabolic variation of water pressure on a horizontal arch the authors have used Fig. 5(*c*), "Final Proportional Load,"* which shows a nearly uniform distribution over the middle two-thirds of the arch, decreasing rapidly to zero near the ends. As the area enclosed is greater than that by a parabola having the same center height, the use of such a curve instead of the parabola would result in increased calculated compressive fiber stresses.

The writer is led to believe, as a result of his own investigations on a similar dam, that the proportionate load distribution found by the authors, and shown in their Figs. 5 and 7,† must be nearly correct. It is obvious, as experience is acquired in the use of this method, closer first assumptions will be made and the goal reached much more quickly than at present.

The writer wishes to express his appreciation of the valuable assistance given by Joseph B. Reynolds, Professor of Mathematics, Lehigh University, and D. B. Sloan, Jun. Am. Soc. C. E. Professor Reynolds checked all the mathematical work and made valuable suggestions which led to the present compact form of the working equations.

F. W. HANNA,‡ M. AM. SOC. C. E. (by letter).§—This able presentation of the distribution of stresses in arch dams is commendable and timely. The paper shows that an immense amount of careful study has been given to the subject. The trial and error method of analysis used by the authors conforms to the customary mathematical practice of solving involved equations by successive approximations. The method, therefore, will produce reliable results, if the fundamental equations are based on sound assumptions. However, in any mathematical problem, it is more satisfactory to proceed by direct methods if it is economical and feasible to do so.

The authors have given a comprehensive and clear development of stress and deflection formulas comprising the effects of bending moment and thrust, but have omitted the very important effects of shear which, in the lower arches of a high arch dam, are too great to be neglected. The writer has found, in dams of this type, that the effects of shear amount to as much as 12½%, or more, on computed compressive stresses, 146% on tensile stresses, and 3% on crown deflections.

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 68.
† *Loc. cit.*, p. 70.
‡ Hydr. and Designing Engr., East Bay Municipal Utility Dist., Oakland, Calif.
§ Received by the Secretary, March 10, 1928.

Using the authors' nomenclature and Fig. 14,* the movement of C about D due to shear distortion may readily be found and combined with the movements due to bending moment and thrust. Consider the voussoir, the center of which is D. Let Δr be the radial movement of its upper end relative to its lower end; V, the radial shear at D acting negatively inward; E', the modulus of elasticity of the masonry in shear; and V_L, the shear at D due to the water load between C and D. The following relations may now be written:

$$\Delta y = \sum_C^A \Delta r \cos \alpha \quad \ldots\ldots (62)$$

$$\Delta x = -\sum_C^A \Delta r \sin \alpha \quad \ldots\ldots (63)$$

$$\Delta r = \frac{n V s}{E' T} = \frac{3 V s}{E T} \quad \ldots\ldots (64)$$

$$V = H_C \sin \alpha + V_C \cos \alpha + V_L \quad \ldots\ldots (65)$$

$$\Delta y = 3 \sum_C^{A_s} \frac{\cos \alpha}{E T} (H_C \sin \alpha + V_C \cos \alpha + V_L) \quad \ldots\ldots (66)$$

$$\Delta x = -3 \sum_C^{A_s} \frac{\sin \alpha}{E T} (H_C \sin \alpha + V_C \cos \alpha + V_L) \quad \ldots\ldots (67)$$

The value of Δy and Δx given in Equations (66) and (67) may now be added to Equations (22) and (23),† respectively. Proceeding in a manner similar to that followed by the authors in the development of Equations (28)‡ and (29),‡ and noting that Δx and V_C should have their signs changed for the right half of the arch, new general equations, Equations (68) and (69), have been derived that include the combined effects of bending moment, thrust, and shear. Since shear does not affect the value of $\Delta \phi$ in Equation (10),§ Equation (27)‖ remains unchanged; but Equations (28) and (29) are changed as follows:

$$M_C \left(\sum_L \frac{x s}{I} - \sum_R \frac{x s}{I} \right) + H_C \left(\sum_L \frac{x y s}{I} - \sum_R \frac{x y s}{I} + 2 \sum_L \frac{s \sin \alpha \cos \alpha}{T} - 2 \sum_R \frac{s \sin \alpha \cos \alpha}{T} \right) + V_C \left(\sum_L \frac{x^2 s}{I} + \sum_R \frac{x^2 s}{I} + \sum_L \frac{s \sin^2 \alpha}{T} + \sum_R \frac{s \sin^2 \alpha}{T} + 3 \sum_L \frac{s \cos^2 \alpha}{T} + 3 \sum_R \frac{s \cos^2 \alpha}{T} \right) = \sum_L \frac{M_L x s}{I} - \sum_R \frac{M_R x s}{I} + \sum_L \frac{H_L s \sin \alpha}{I} - \sum_R \frac{H_R s \sin \alpha}{T} - 3 \sum_L \frac{V_L s \cos \alpha}{T} + 3 \sum_R \frac{V_R s \cos \alpha}{T} - c t E (h_L - h_R) \quad \ldots\ldots (68)$$

* *Proceedings*, Am Soc. C. E., January, 1928, Papers and Discussions, p. 79.
† *Loc. cit.*, p. 82.
‡ *Loc. cit.*, p. 84.
§ *Loc. cit.*, p. 81.
‖ *Loc. cit.*, p. 83.

$$M_C\left(\sum_L \frac{y\,s}{I}+\sum_R \frac{y\,s}{I}\right)+H_C\left(\sum_L \frac{y^2\,s}{I}+\sum_R \frac{y^2\,s}{I}+\sum_L \frac{s\cos^2\alpha}{T}+\sum_R \frac{s\cos^2\alpha}{T}+3\sum_L \frac{s\sin^2\alpha}{T}+3\sum_R \frac{s\sin^2\alpha}{T}\right)+V_C\left(\sum_L \frac{x\,y\,s}{I}-\sum_R \frac{x\,y\,s}{I}+2\sum_L \frac{s\sin\alpha\cos\alpha}{T}-\sum_2 \frac{s\sin\alpha\cos\alpha}{T}\right)=\sum_L \frac{M_L\,y\,s}{I}+\sum_R \frac{M_R\,y\,s}{I}-\sum_L \frac{H_L\,s\cos\alpha}{T}-\sum_R \frac{H_R\,s\cos\alpha}{T}-3\sum_L \frac{V_L\sin\alpha}{T}-3\sum_R \frac{V_R\sin\alpha}{T}+\frac{c\,t\,E}{s}(l_L+l_R)\quad\ldots\ldots(69)$$

M_C, H_C, and V_C, having been computed by means of Equations (27), (68), and (69), the moments, thrusts, and shears may be computed by Equations (7),* (18)† and (65).

If the arch is assumed to be symmetrical, Equations (27), (68), and (69) may be greatly simplified; and inasmuch as generally in the design of an arch dam it is possible and desirable to make each arch symmetrical, it will be of interest and of value to impose the conditions of symmetry on these equations. In this procedure the $\sum_L$ and $\sum_R$ for like quantities are equal and algebraically additive. Having due regard for signs, dividing by 2 in each case, and omitting the effects of temperature, Equations (27), (68) and (69) assume the following forms:

$$M_C\sum_L \frac{s}{I}+H_C\sum_L \frac{s\,y}{I}=\sum_L \frac{M_L\,s}{I}\quad\ldots\ldots(71)$$

$$V_C=0\quad\ldots\ldots(72)$$

$$M_C\sum_L \frac{y\,s}{I}+H_C\left(\sum_L \frac{y^2\,s}{I}+\sum_L \frac{s\cos^2\alpha}{T}+3\sum_L \frac{s\sin^2\alpha}{T}\right)=-\sum_L \frac{M_L\,y\,s}{I}-\sum_L \frac{H_L\,s\cos\alpha}{T}-3\sum_L \frac{V_L\sin\alpha}{T}+\frac{c\,t\,E\,l_L}{s}\quad\ldots(73)$$

M_C, V_C, and H_C, for the left half of the arch, having been found from Equations (71), (72) and (73), the moments, thrusts, and shears for the left half of the arch may be found by use of Equations (7) and (18) and Equation (65), by noting that the V_C-terms become zero. The moments, thrusts, and shears will be the same of course, for corresponding voussoirs for the right half of the arch.

The authors have extended the customary method of distributing the computed water load on an arch dam between the horizontal arches and the single vertical cantilever through the crowns on the basis of relative rigidity. In making this extension, the cantilevers are assumed to deflect in radial planes perpendicular to the extrados of the dam. There is considerable error involved in this assumption. The sections of the horizontal arches cannot deflect in a

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 80.
† *Loc. cit.*, p. 82.

radial direction except at the crown, and the divergence from a radial direction increases from the crown of the arch toward the abutments. All the lateral cantilevers, therefore, will be affected by the torsional force of each intersecting arch that tends to twist them around into coincidence with the motion of the arch. This tendency will be different for the horizontal arches because of their variable lengths, thicknesses, and loads. Moreover, the components of the arch-deflecting forces perpendicular to the radial plane through the central axes of the cantilevers have different intensities in the various arches, and they will tend to distort the cantilever tangentially and warp it out of shape in this direction also. The lateral cantilevers are, therefore, not only called upon to carry their share of the water load in the assumed radial directions, but must resist torsional stresses and lateral distortion stresses of considerable amounts. In this they are not dependent on their resistance alone, but on the enormous torsional and tangential resistance of the dam itself.

In order to show the directions and amounts of the deflections for various points in any single arch element and in different arch elements of the same dam, the writer has had the deflections computed for two circular symmetrical arches each of 100 ft. mean radius and 120° central angle; one with a uniform thickness of 4 ft. and a normal load of 2 500 lb. per sq. ft.; the other with a uniform thickness of 20 ft. and a normal load of 12 500 lb. per sq. ft. The results are given in Table 10 and Figs. 24 and 25 show the deflection of the 20-ft. arch ring, as an example.

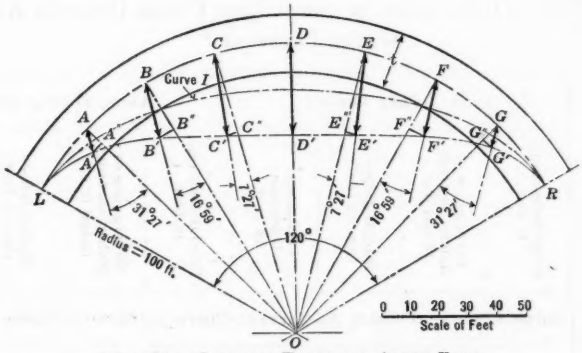


FIG. 24.—SECTION THROUGH ARCH RING.

It will be noted that the points, *A*, *B*, *C*, and *D*, at 15° intervals on the center line of the arches, *L R* (Fig. 24), are deflected to the points, *A′*, *B′*, *C′*, and *D′*. To measure the deflections divide the scale values by 1 000. Curve *I* is the locus of points bisecting the deflections *A A′*, *B B′*, etc. The deflection of *D* is coincident with the assumed radial deflection of the cantilever, but those at *C*, *B*, and *A* diverge from radial deflections by 7° 27′, 16° 59′, and 31° 27′, respectively. This lack of direction coincidence will cause torsional stress in the cantilevers. *A″ A′*, *B″ B′*, and *C‴ C′*, perpendicular to the respective radii, may be taken to represent graphically the peripheral deflections necessary to make the movements of the cantilevers coincide

with those of the arches. Since the points, *A*, *B*, *C*, and *D*, must deflect to the same respective points in the arches and cantilevers, the cantilevers must be distorted laterally by considerable amounts. In Fig. 25 there are plotted four curves that are of interest in connection with this subject, Curve *A* showing the total deflections, *AA'*, etc.; Curve *B*, the radial deflections, *AA''*, etc.; Curve *C*, the peripheral deflections, *A'' A'*, etc.; and Curve *D*, the divergence of the arch deflections from radial lines.

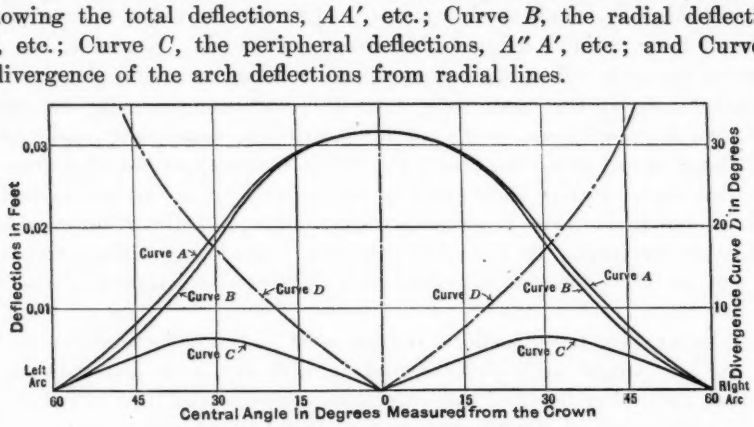


FIG. 2.—DEFLECTIONS OF ARCH RING.

In addition to the torsional and lateral distorsion forces already referred to as affecting the lateral radial cantilevers, these cantilevers are also affected by the moment, thrust, and shearing forces of the arches, gravity, and water pres-

TABLE 10.—DEFLECTIONS OF ARCH RING UNDER UNIFORM NORMAL WATER PRESSURE.*

Angular distance from crown, in degrees.	Arch, 4 Feet Thick.†				Arch, 20 Feet Thick.‡			
	Total deflection, in feet.	Divergence of deflection from radial line.	Radial deflection, in feet.	Peripheral deflection, in feet.	Total deflection, in feet.	Divergence of deflection from radial line.	Radial deflection, in feet.	Peripheral deflection, in feet.
Abscissas.*	Curve *A*.	Curve *D*.	Curve *B*.	Curve *C*.	Curve *A*.	Curve *D*.	Curve *B*.	Curve *C*.
0	0.0334	0	0.0334	0	0.316	0	0.0316	0
15	0.0285	7° 5′	0.0283	0.0035	0.289	7° 27′	0.0286	0.0038
30	0.0187	16° 9′	0.0180	0.0052	0.192	16° 59′	0.0184	0.0056
45	0.0072	32° 59′	0.0060	0.0039	0.0078	31° 27′	0.0066	0.0040

* See values plotted in Fig. 25.
† $t = 4$ ft.; $w = 2\,500$ lb. per sq. ft.
‡ $t = 20$ ft.; $w = 12\,500$ lb. per sq. ft.

sure, all of which must be taken into consideration in determining their value in the distribution of stresses in the dam. In view of these conditions, it appears that cantilevers other than the crown cantilever must be used with great care and keen judgment in order to appraise their true value in determining stress distribution in arch dams.

While the writer prefers to use the elastic center, rigid bracket, unit load method of developing arch deflection and stress formulas and a series of cantilevers of varying widths, each symmetrical with respect to a vertical section through the arch crowns of the dam, he feels that the authors have presented an excellent method of arch analysis and have rendered the Engineering Profession a distinct service for which they deserve its thanks.

Acknowledgment is hereby made to T. L. E. Haug, M. Am. Soc. C. E., Designing Engineer of the East Bay Municipal Utility District, Oakland, Calif., for assistance in checking the formulas and computations used in this discussion.

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PAPERS AND DISCUSSIONS

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STREAM FLOW IN GENERAL TERMS

Discussion*

BY MESSRS. FRANK S. BAILEY AND CECIL E. PEARCE.

FRANK S. BAILEY,† ASSOC. M. AM. SOC. C. E. (by letter).‡—The author seems to have accomplished his purpose, as stated in his Synopsis,§ "to present a workable general method for the analysis of stream flow in irregular channels". His fundamental assumptions are generally accepted as fair ones; his development of the equations showing the relations between the various factors governing the flow are logical, and, in so far as the writer has discovered, his mathematics are correct. Diagrams showing the relations of d plus h, and Q have been used for several years,‖ but the addition of the f-curves by the author is a valuable new development.

Of the numerous papers and articles which have been published in the past with the object of determining the location of the water surface for a given rate of flow in an irregular channel, few, if any, are so comprehensive and clearly stated as that of Mr. Casler. His definition§ of the Chezy slope factor is commendable.

The fact that a given quantity of water can flow at different depths in a channel, may result in considerable damage if care is not exercised in the design of a channel. Suppose that a spillway channel for an earth dam is located on the side of a hill which rises from the bed of the stream, and that it is necessary to build a wall on the lower side of the channel, of sufficient height to prevent any flood flow from overtopping the wall with possible injury

* This discussion (of the paper by Melvin D. Casler, M. Am. Soc. C. E., published in January, 1928, *Proceedings*, but not presented at any meeting of the Society) is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

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‡ Received by the Secretary, March 14, 1928.

§ *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 97.

‖ See, "The Hydraulic Jump and Critical Depth in the Design of Hydraulic Structures," by Julian Hinds, M. Am. Soc. C. E., *Engineering News-Record*, November 25, 1920, Vol. 85, p. 1034.

to the down-stream slope of the dam. A spillway channel wall might be designed on the basis that in all cases the water would flow at high velocity at a low depth and if, due to some unexpected cause, the actual velocity were lower and the depth higher than anticipated, the wall itself might act as a spillway, with undesirable consequences.

Karl R. Kennison, M. Am. Soc. C. E., mentions* the case of two high flood flows of not greatly different amounts which ran at different depths in a wooden flume, and in his discussion of that paper, the late Frederic P. Stearns, Past-President, Am. Soc. C. E., described† at greater detail the same phenomenon. If a horizontal deflection or a curve exists in the channel, the chances of a mistake in the design are increased, and more knowledge, on the subject of loss of head in curves, and differences in elevation of water on the inner and outer edges of curves, would be helpful.

An entering or discharging stream of water at the intersection with a branch channel may also have to be taken into account. Losses due to intersecting streams have been ably treated‡ by J. C. Stevens, M. Am. Soc. C. E., from a theoretical standpoint.

The author's treatment of: (*a*) flow over weirs; (*b*) the effects of height of weir above the bottom of the channel; (*c*) area of channel above the weir as affecting the velocity of approach; and (*d*) flow over the weir, is novel and ingenious.

Cecil E. Pearce,§ Jun. Am. Soc. C. E. (by letter).‖—This excellent paper should be welcomed by all hydraulic designers. Basically, nothing new is presented, but the treatment is exceptionally clear, and presents the facts in a readily usable form. Having set up the fundamental energy equation, which is well known and is commonly called "Bernoulli's theorem", the author bases the larger part of his mathematics upon it, of necessity. The treatment of the critical section, however, and the curves used, form valuable tools which lead to a clearer understanding and shorten the work involved in computing the water surface in an irregular channel or conduit for a given discharge.

The writer, in calculating the flood discharges through the Exchequer Dam and Power House, in 1926, used a step-by-step process, with Chezy's formula, $c = C\sqrt{r\,s}$, obtaining the proper elements by trial and error. The determination of the proper trial was based on the same fundamental reasoning as that of the author.

The conduit was very irregularly shaped. The steps were as follows:

1.—Assume a given value of Q, say, 300 sec-ft.

2.—For a first trial, assume that, with 300 sec-ft. flowing, the water will have a certain depth at the lower end. Since $Q = a\,v$, the corresponding velocity at that lower end may be computed.

* "The Hydraulic Jump in Open-Channel Flow at High Velocity," *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 338.

† *Loc. cit.*, p. 368.

‡ "Theoretical Energy Losses in Intersecting Pipes," *Engineering News-Record*, July 22, 1926, p. 140.

§ Designing Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

‖ Received by the Secretary, March 7, 1928.

3.—With this as a basis, calculate by trial and error what the water surface elevation would be at every point along the irregular conduit, and, finally, in the reservoir.

4.—Repeat this process a number of times for $Q = 300$ by starting with different assumed depths of water at the lower end.

A number of such computations gives an equal number of possibilities for the water-surface profile. Any assumed elevation of the water surface at the lower end determines the water elevation in the reservoir. Which of these profiles is the proper one?

To answer this question, consider what holds the water surface where it is. If a glass of water is emptied on a flat surface, it immediately tries to flatten out to zero thickness. The same is true of the water in a channel; all that holds the water surface where it is is the back pressure. The water, in attempting to flatten out, meets with obstacles (resistance). Considering that the only resistance causing a reaction to the flow of the water is the frictional force and the resistance of inertia in producing velocity, it is those two forces that supply the back pressure to restrain the water surface.

Also, with any given surface elevation in a reservoir, there must be one maximum value of Q which can be discharged through a given channel. Referring to Fig. 10, F represents energy consumed in overcoming friction; and h_y represents the energy consumed in producing velocity. Therefore, $(h_y + F)$ is the total energy consumed in overcoming friction and producing velocity.

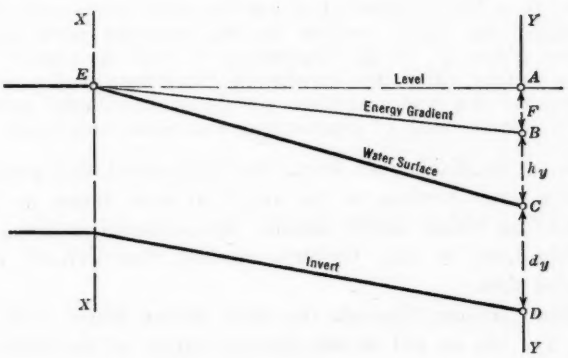

FIG. 10.

By the theory of least work, however, the total energy consumed must be a minimum. Therefore, $(h_y + F)$ must be a minimum, and, since Points (A) and (D) are given, Point (C) must rise as high as possible. For a higher portion of Point (C) the value of Q would be less, or the elevation at Point (E) would increase.

Hence, for a given maximum discharge, Q, and a given water surface elevation at Point (E), the water surface, E-C, will be as nearly level as possible, or for a given Q and the water surface as nearly level as possible

(in order to comply with the minimum energy loss requirement), the water surface elevation at Point (E) will be the minimum which would discharge the given Q. This is necessary in order to make Q the maximum for that elevation.

If it is hard to visualize the theory entirely, a simple calculation will make the statement clearer:

(1) Consider a short channel leading from a reservoir.
(2) Assume a given discharge Q.
(3) By successively taking assumed depths at the lower end, calculate what the water surface elevation in the reservoir will be in order to give that discharge.
(4) Suppose at first shallow depths are assumed at the lower end; then, in successive trials somewhat greater and greater depths at the lower end are taken.
(5) For the first trial the very shallow depth requires a great velocity; consequently, the friction loss and velocity head loss are great, and a relatively high water elevation in the reservoir is required to furnish the necessary heads.
(6) For the next trial, with a greater depth at the lower end, the velocity is less, and consequently the friction loss is less. The elevation of the water surface in the reservoir then need not be as great as in the first trial. Finally, after repeated trials the reservoir elevation reaches a minimum, and then begins to rise again. Or it may be found that, with the minimum head in the reservoir, the total energy loss is a minimum in order to discharge the given Q, and that with another trial calculation, assuming the water surface at the lower end of the conduit to be a little higher than for the minimum reservoir elevation, either the water surface in the reservoir must rise to create the given Q, or the discharge, Q, will decrease. This shows, as before, that this minimum elevation in the reservoir represents the water surface profile of minimum energy loss for the specific task of discharging the given maximum Q.

In the case of the Exchequer Dam, the trial calculation giving the least elevation of the water surface in the reservoir was chosen as representing the actual condition which would obtain. For comparing these calculations with results obtained by Mr. Casler's method, the writer's computation will be presented first.

A longitudinal section through the dam, power house, and channels is shown in Fig. 11. As an aid to calculation, curves of the hydraulic radius and the area of each cross-section were plotted, but are not reproduced.

Various constants were adopted as follows:

(1) At the entrance, A, three piers project inward. The coefficient for entrance loss, K_0, was therefore taken as 0.78.*

(2) For the loss due to obstruction and sudden contraction at F, the factor, 0.25, was assumed.

(3) The coefficient, n, for friction or rough concrete was taken as 0.015.†

Further, a curve was plotted for determining the coefficient, C, for various values of hydraulic radius, r, when $S = 0.001$ and $n = 0.015$ for a rough

* See King's Handbook, p. 152.
† *Loc. cit.*, p. 191.

concrete channel. The values of C for a given r vary little between the limits, $S = 0.0004$ and $S = 0.01$; so that using values of C when $S = 0.001$ (or 0.0004 or 0.01) does not materially affect the answer if the actual water slopes are within those limits.

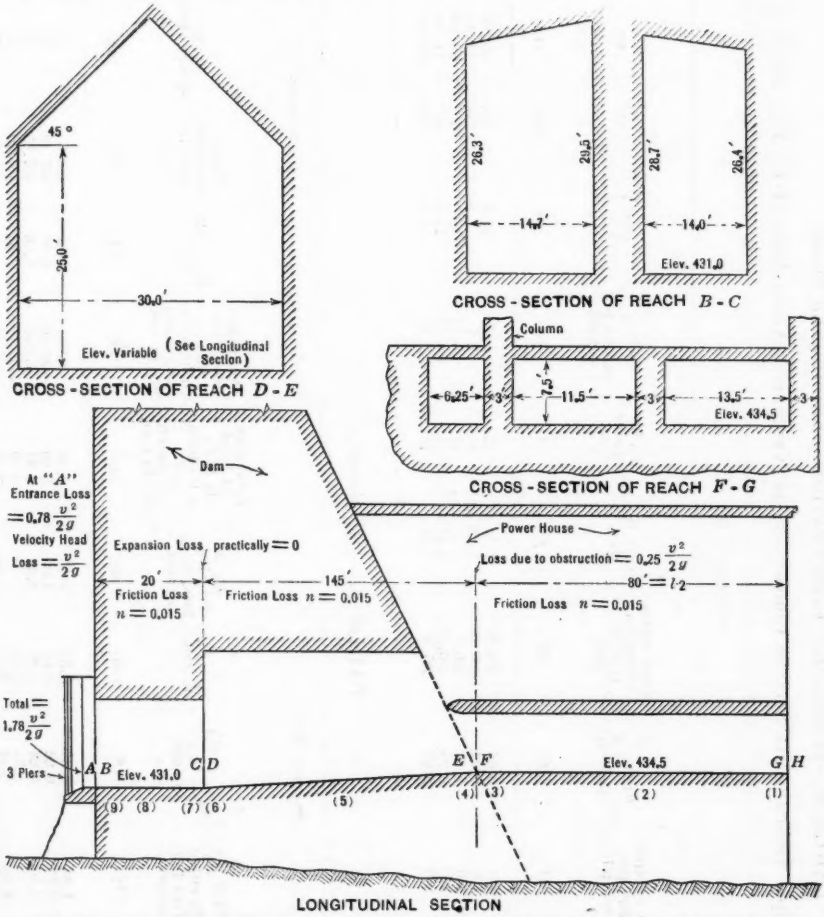

FIG. 11.—SECTIONS OF FLOOD CONTROL CHANNEL, THROUGH DAM AND POWER HOUSE.

Using these various curves, Table 9 was prepared, showing the various trial computations. Many of the notations and derivations will be apparent from Fig. 11. The terms, A, v, h, r, l, S, and H, give the area, velocity, velocity head, hydraulic radius, length of channel, hydraulic slope, and differential head, respectively. The subscripts refer to the parts of the channel concerned, as follows: 1, for Point G; 2 for the reach, GF; 3, for Point F; 4 for Point E; 5 for the reach, E-D; 6 for Point D; 7 for Point C; 8 for the reach, C-B; and 9 for Point B (all these points are numbered to correspond in Fig. 11).

The values of "water elevation" (Columns (1), (6), (15), (24), (31), etc.) are assumed in each case, whereas other "computed elevations" (Columns (22), (30), etc.) are derived from the tabular values. Ordinarily, for the

TABLE 9.—HYDRAULIC COMPUTATIONS, EXCHEQUER DAM.

(Various Subscripts Indicate the Section Concerned, as Numbered Along Line *A-H*, Fig. 11.)

SECTION *F-G.*

Water elevation, at *G*, in feet. (1)	A_1, in square feet. (2)	$v_1 = \frac{Q}{A_1}$, in feet per second. (3)	Q, in cubic feet per second. (4)	$h_1 = \frac{v_1^2}{2g}$, in feet. (5)	Mean water elevation *G-F*, in feet. (6)	(r_2), in feet. (7)	A_2, in square feet. (8)	$v_2 = \frac{Q}{A_2}$, in feet per second. (9)	$\left(\frac{v_2^2}{r_2}\right)$. (10)	C. (11)	C^2. (12)	$S_2 = \frac{\left(\frac{v_2^2}{r_2}\right)}{C^2}$. (13)
436	46	6.52	300	0.66	436.30	1.33	56	5.35	21.6	104	10 800	0.002
437	78	3.84	300	0.23	437.10	1.73	81	3.7	7.91	110	12 100	0.000655
435.50	31	9.68	300	1.46	436.0	1.15	46	6.52	37	102	10 400	0.0036
436.50	62	4.84	300	0.36	436.57	1.47	64	4.69	14.9	107	11 400	0.0013
435.25	23	13.0	300	2.63	437.0	1.69	78	3.85	8.75	110	12 100	0.00072

TABLE 9.—(*Continued.*)

SECTION *F-G.*									SECTION *D-E.*			
l_2, in feet. (14)	Water elevation at *F*, in feet. (15)	A_3, in square feet. (16)	$v_3 = \frac{Q}{A_3}$, in cubic feet per second. (17)	$h_3 = \left(\frac{v_3^2}{2g}\right)$, in feet. (18)	$h_1 - h_3$, in feet. (19)	H_2 = friction, loss, $S_2 l_2$, in feet. (20)	Total gain of head in *G-F*, = Column (19) + Column (20), in feet. (21)	Computed elevation at *F*, Column (1) + Column (21), in feet. (22)	$H_3 = 0.25 h_3$. (23)	Water elevation at *E*, in feet. (24)	A_4, in square feet. (25)	$v_4 = \frac{Q}{A_4}$, in cubic feet per second. (26)
80	436.50	62	4.84	0.36	0.30	0.16	0.46	436.46	0.09	436.6	65	4.61
80	437.06	80	3.75	0.22	0.01	0.05	0.06	437.06	0.06	437.10	78	3.84
80	437.0	78	3.84	0.23	1.23	0.28	1.51	437.01	0.06	437.05	78	3.84
80	436.64	67	4.47	0.31	0.06	0.10	0.16	436.66	0 08	437.70	95	3.16
80	437.8	106	2.83	0.12	2.51	0.06	2.57	437.82	0.03	437.8	106	2.83

TABLE 9.—(*Continued.*)

TABLE 9.—(*Continued.*)

SECTION *D-E.*

$h_4 = \left(\frac{v_4^2}{2g}\right)$, in feet. (27)	$(h_3 - h_4)$, in feet. (28)	Total gain, *F-E*, = Column (23) + Column (28), in feet. (29)	Computed water elevation at *E*, = Column (22) + Column (29), in feet. (30)	Mean water elevation, *E-D*, in feet. (31)	Mean elevation of bottom for *E-D*, in in feet. (32)	Mean depth, *E-D*, in feet. (33)	A_5, in square feet. (34)	$v_5 = \frac{Q}{A_5}$, in cubic feet per second. (35)	r_5. (36)	$\left(\frac{v_5^2}{r_5}\right)$. (37)	C. (38)	C^2. (39)
0.33	+0.03	+0.12	436.58	436.75	432.75	4.00	120	2.5	3.15	1.98	120	14 400
0.23	−0.01	+0.05	437.11	437.21	432.75	4.46	135	2.22	3.45	1.43	121	14 600
0.23	0.00	+0.06	437.07	437.21	432.75	4.46	135	2.22	3.45	1.43	121	14 600
0.16	+0.15	+0.23	436.89	436.95	432.75	4.20	127	2.36	3.3	1.69	120	14 400
0.12	0.00	+0.03	437.85	437.92	432.75	5.17	155	1.93	3.85	0.97	123.55	15 210

TABLE 9.—(*Continued.*)

SECTION *D-E.*										SECTION *B-C.*	
$S_5 = \frac{\left(\frac{v_5^2}{r_5^2}\right)}{C^2}$. (40)	l_5, in feet. (41)	$H_5 = S_2 l_2$, in feet. (42)	Water elevation at *D*, in feet. (43)	A_6, in square feet. (44)	$v_6 = \frac{Q}{A_6}$, in cubic feet per second. (45)	$h_6 = \left(\frac{v_6^2}{2g}\right)$, in feet. (46)	$(h_4 - h_6)$, in feet. (47)	Total gain, *E - D*, Column (42) + Column (47), in feet. (48)	Water elevation at *D* = Column (30) + Column (48), in feet. (49)	Water elevation, at *C*, in feet. (50)	A_7, in square feet. (51)
0.000137	145	0.02	436.85	176	1.70	0.04	0.29	0.31	436.89	436.9	170
0.000098	145	0.01	437.31	190	1.58	0.04	0.19	0.20	437.31	437.30	180
0.000098	145	0.01	437.31	190	1.58	0.04	0.19	0.20	437.27	437.26	180
0.000117	145	0.02	437.0	180	1.67	0.04	0.12	0.14	437.08	437.03	175
0.000064	145	0.01	438.00	210	1.43	0.03	0.09	0.10	437.95	488.95	228

TABLE 9.—(*Continued.*)

SECTION *B-C.*

$v_7 = \left(\frac{Q}{A_7}\right)$, in cubic feet per second. (52)	$h_7 = \left(\frac{v_7^2}{2g}\right)$, in feet. (53)	$h_B - h_7$, in feet. (54)	Computed water elevation at C = Column (49) + Column (54), in feet. (55)	Mean water elevation, C-B, in feet. (56)	A_8, in square feet. (57)	$v_8 = \frac{Q}{A_8}$, in cubic feet per second. (58)	r_8, in feet. (59)	$\left(\frac{r_8^2}{r_8}\right)$. (60)	C. (61)	C^2. (62)	$S_8 = \frac{\left(\frac{v_8^2}{r_8^2}\right)}{C^2}$. (63)
1.76	0.05	−0.01	486.88	486.88	170	1.76	3.24	11.01	125	15 625	0.000065
1.66	0.04	0.00	487.31	487.31	180	1.66	3.35	0.822	121	14 640	0.000056
1.66	0.04	0.00	487.27	487.31	180	1.66	3.35	0.822	121	14 600	0.000056
1.71	0.05	−0.01	487.02	487.02	175	1.71	3.30	0.69	120	14 400	0.0000615
1.31	0.03	0.00	488.95	488.95	228	1.31	3.67	0.467	122	14 900	.....

TABLE 9.—(*Continued.*)

SECTION *B-C.*

l_8, in feet. (64)	H_8, in feet. (65)	Water surface at B, in feet. (66)	A_9, in square feet. (67)	$v_9 = \frac{Q}{A_9}$, in cubic feet per second. (68)	$h_9 = \left(\frac{v_9^2}{2g}\right)$, in feet. (69)	$h_7 - h_9$ = velocity head gain from C to B, in feet. (70)	Computed water elevation at B = Column (55) + Column (70). (71)	Total loss at entrance = 1.8 h_9, in feet. (72)	Computed water elevation in reservoir = Column (71) + Column (72). (73)	Point number for plotting. (74)
20	0.00	486.88	169	1.78	0.05	0.00	486.88	0.09	486.97	1
20	0.00	487.31	180	1.66	0.04	0.00	487.31	0.07	487.38	2
20	0.00	487.31	180	1.66	0.04	0.00	487.31	0.07	487.38	3
20	0.00	487.02	175	1.71	0.05	0.00	487.02	0.09	487.11	4
20	0.00	488.95	228	1.31	0.03	0.00	488.95	0.05	489.00	5

first trial, the elevation in Column (22) will not check that assumed in Column (15). The values given are for the final results and similarly for the other reaches.

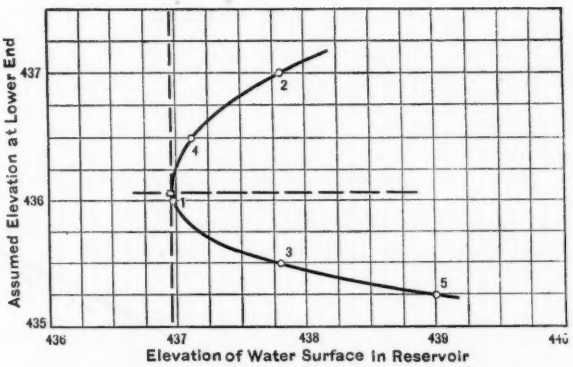

FIG. 12.—CURVE SHOWING MINIMUM ELEVATION OF WATER SURFACE IN RESERVOIR. PLOTTED FROM TABLE 9.

Then, the curve in Fig. 12 was plotted from Column (73), Table 9. Evidently, the minimum elevation of the water surface is at Elevation 436.95. This same process may be repeated for other discharges, Q, of 500, 1 000 sec-ft., etc., which will give the data for plotting another curve which would show the discharge through the channel for various elevations of the water surface in the reservoir.

A check will now be applied to those calculations using Mr. Casler's method, to compare not only the results, but also the advantages and time saving. Fig. 13, showing values of Q and f for Section F-G, is plotted from values computed according to the author's method, for $Q = 300$ sec-ft. Similar curves were plotted for each of the other two sections, but are not presented.

By inspection, it is probable that the critical section is at (or near) the lower end. If it is, then the procedure would be as follows.

1.—Determination of Water Depth at Critical Section.—

$$\frac{a^3}{b} = 0.0311\ Q^2 = 2\ 799$$

$$\frac{a^3}{b} = b^3 \times \frac{d^3}{b} = b^2 \times d^3 = 2\ 799$$

but,

$$b = 31.25, \text{ and } b^2 = 976$$

therefore,

$$d^3 = \frac{2\ 799}{976} = 2.87$$

or,

$$d = 1.42$$

$$h = \frac{d}{2} = 0.71$$

Therefore,

$$(d + h) = 2.13$$

Then the elevation of the water surface at G, is,

$$434.50 + 1.42 = 435.92$$

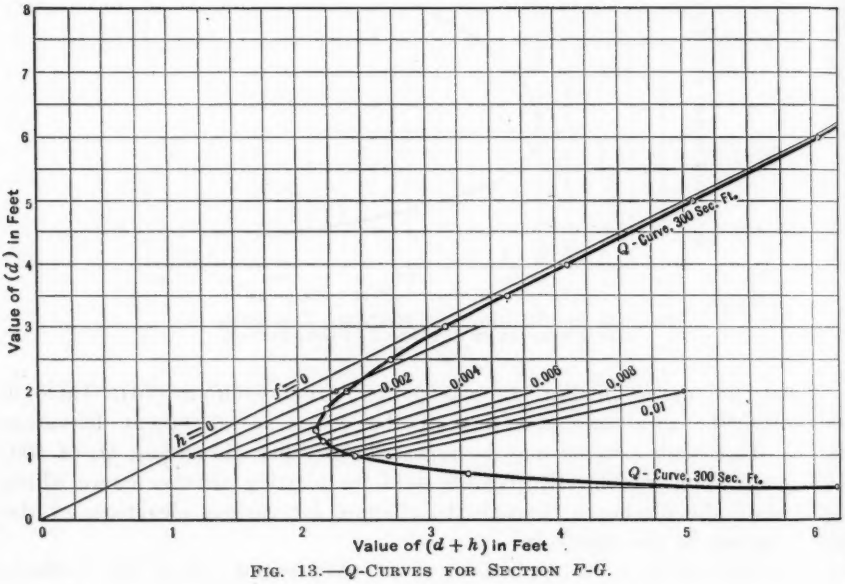

FIG. 13.—Q-CURVES FOR SECTION F-G.

The value of f may be found from the formula, $f = \frac{p\,g}{b\,C^2}$. Since,

$$p = 31.25 + (6 \times 1.42) = 39.77$$
$$b = 31.25$$

and,

$$C = 115$$
$$f = 0.0031$$

2.—Consideration of Reach G-F.—The fundamental hydraulic relation between two planes, X and Y (Fig. 10), is:

$$(d_x + h_x) - \frac{l}{2} f_x = (d_y + h_y) + \frac{l}{2} f_y - l\,i$$

In this case: $l = 80$; $f_y = 0.0031$; $i = 0$; and $(d_y + h_y) = 2.13$.

Thus,

$$(d_x + h_x) - 40\,f_x = 2.13 + (40 \times 0.0031) - 0 = 2.25$$

but $f > i$; therefore, follow the Q-curve upward from the vertex (Fig. 13).

After successive trials, assume that $(d_x + h_x) = 2.31$. Then, from Fig. 13, $f_x = 0.0015$, and $40\,f_x = 0.06$; or $2.31 - 0.06 = 2.25$ (which checks, as it should).

Then,

$$(d_x + h_x) = 2.31$$

From Fig. 13, $d_x = 1.90$

or,

$$h_x = 0.40$$

$$\text{Elevation of invert} = 434.50$$

$$d_x = 1.90$$

Therefore, water surface elevation at $F = 436.40$

3.—Consideration of Losses at Point E-F.—The loss due to obstruction at $F = 0.25$ times the velocity head at the point, $F = 0.25 \times 0.40 = 0.10$.

As before,

$$(d_x + h_x) - \frac{l}{2} f_x = (d_y + h_y) + \frac{l}{2} f_y - l\, i + \text{other losses}$$

$$= 2.31 + 0 + 0 + 0.10 = 2.41$$

$$i = 0,$$

and,

$$f = 0.0015$$

As $f > i$, follow the Q-curve upward from the vertex. Since $l = 0$,

$$(d_x + h_x) = 2.41$$

From a curve for Section D-E (similar to Fig. 13), $d_x = 2.03$ ft., and $f_x = 0.001$. Then,

$$\text{Elevation of invert at } E = 434.50$$

$$d_x = 2.03$$

Therefore, elevation of water surface at $E = 436.53$

In a similar manner, the water surface elevations (and the other concurrent functions) were calculated for each section to the reservoir.

Check on Flow at Point B.—It was found by calculation that, at B, $h = 0.05$ and $d = 5.93$. Therefore,

$$v = 1.79$$

$$a = 28.7 \times 5.93 = 170.5$$

$$Q = a\, v = 170.5 \times 1.79 = 3.05 \text{ (check)}$$

Comparison of Results.—A comparison of the water surface elevations at each point along the channel, calculated by the two different methods (Table 10), shows that one is a substantial check on the other. The writer's method gives a water surface elevation varying from about 0.08 ft. higher to 0.07 ft. lower than that of Mr. Casler.

Discussion.—Judging from experience in calculations of this sort, evidently Mr. Casler's method is much shorter and more direct, and gives a clearer vision of the whole problem. The writer's method is one of trial and error, and is very laborious.

In many hydraulic problems, the critical section may be located from a superficial glance at the physical conditions. After this section is determined, the solution, by Mr. Casler's method, is almost entirely direct.

TABLE 10.—Comparison of Results by Two Methods.

Point.	Elevation of Water Surface.		Difference from Casler's method.
	Casler's method.	Writer's method, using Chezy formula.	
G	435.92	436.00	+0.08
F	436.40	436.46	+0.06
E	436.53	436.58	+0.05
D	436.93	436.89	−0.04
C	436.93	436.88	−0.05
B	436.93	436.88	−0.05
A	437.02	436.95	−0.07

It will be noticed that Reach *F-G* is a three-barreled conduit, and that these calculations assume that the water surface elevation will be the same in all three. Actually, this will not happen, for the hydraulic radii are not the same, and, therefore, the friction loss will differ. If one cared to follow out the calculations, considering each barrel separately, it could be done, but the practical difference would be negligible.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

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DESIGN AND CONSTRUCTION OF CONCRETE PAVEMENTS

Discussion*

By Messrs. G. E. Hawthorn and H. O. Root.

G. E. Hawthorn,† Assoc. M. Am. Soc. C. E. (by letter).‡—The writer is in agreement with Mr. Older's general conclusions and recommended construction practice, but does not believe that the stresses obtained by the use of Equations (1), (2), and (3)§ will, in all cases, be the maximum stresses developed by truck loads.

Two other factors should be considered in connection with these equations, namely, the size of the pavement slab and the maximum and minimum concentration of wheel loads.

In the light of developments in pavement design in the past few years it is not necessary to present any argument to justify the use of some kind of a longitudinal center joint. Such a joint will divide the slab and greatly reduce the transverse bending moment, due to wheel loads, as well as curling, due to sudden temperature changes.

The satisfactory length of pavement slabs is not as well established. Experience in the State of Washington seems to justify a short panel length of not more than 20 ft. If longer panel lengths are used they will be shortened in a few years by transverse cracks. It should be emphasized that expansion joints would not be necessary at every panel end, but only every three or four panels. The writer believes that the "dummy joint," as used by the City of Seattle and the State Highway Department of Washington, to divide intermediate panels, with expansion joints every 60 ft., is very satisfactory. These joints control the transverse cracks and, at the same time, offer considerable frictional resistance to deflection.

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* Discussion of the paper by Clifford Older, M. Am. Soc. C. E., continued from March, 1928, *Proceedings*.

† Instr., Civ. Eng., Univ. of Washington, Seattle, Wash.

‡ Received by the Secretary, February 14, 1928.

§ *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 147.

With the present axle width it is only on rare occasions that a single wheel load acts alone and the deflection under any wheel will usually be due to at least two wheel loads.

The writer would like to call attention to one feature of design which has received little attention in recent discussions of pavement-slab design; that is, the width to which the edge thickness should be extended toward the center. The writer has spent considerable time in the study of slab design, and, during 1927, conducted a series of tests on small concrete slabs of various designs. As a result he believes that most edge thickening in use is of practically no value in reducing the maximum stress in the slab due to edge loads as it is effective only when the tension is in the top of the slab. Consider the fundamental equation, $f = \frac{M\,C}{I}$, which gives the stress, f, due to a bending moment, M, caused by a single wheel load, W.

The measure of the strength of the slab depends on the section modulus, $\frac{I}{C}$, which for a rectangular section becomes $\frac{b\,d^2}{6}$, in which, d is the depth of the pavement slab and b is the width over which each wheel load is distributed.

With axle widths as at present, of from 5 to 6 ft., and a narrow slab obtained by the use of a longitudinal center joint, it is apparent that b can be assumed to vary from one-half the slab width as a maximum, when the slab is symmetrically loaded with a truck load, to one-half the axle width as a minimum when one wheel load is at the outside edge. Thus, with 5-ft. axle widths, the section modulus of the strip at the edge of a 10-ft. slab with uniform depth would be only one-half that obtained when the slab is symmetrically loaded.

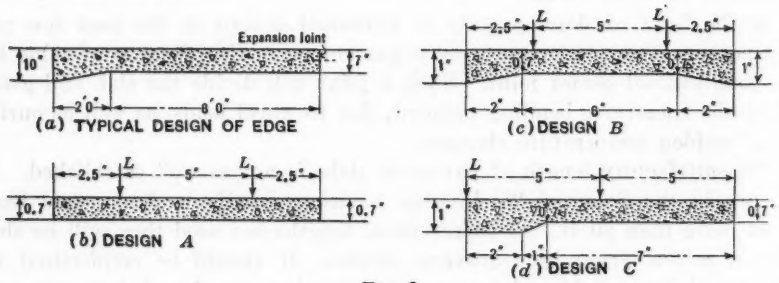

FIG. 3.

The section modulus of a pavement in which $d = 7$ in. throughout and $b = 60$ in., is $\frac{60 \times 49}{6} = 490$ in.^{3}, while with $b = 30$ in., it is 245 in.^{3}

Fig. 3 (*a*) shows an edge thickening typical of that so generally used, and which follows Mr. Older's recommendation * that the interior depth be made 0.7 that of the edge depth. The maximum tensile stress will usually occur under a wheel load and will be at the bottom of the slab. The section

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 149.

modulus of this section using $b = 30$ in., is $\frac{I}{C} = 257$ in^3. If the section modulus of the uniform slab, with $b = 60$ in., is equal to 1, the relative strength of the special section is $\frac{257}{490} = 0.525$ times as great, which is only a little better than the uniform section.

The section modulus for stress in the top of the slab is $\frac{1\,500 \text{ in.}^4}{4.16 \text{ in.}} = 360.6$ in.^{3}, and its relative strength is $\frac{360.6}{490} = 0.735$.

Assuming that the maximum stress due to a corner load will fall at a point half-way between the wheel loads, or 30 in. from the corner as is normally the case, the section modulus of the special section (Fig. 3 (*a*)), will be 1.57 times that of a slab with constant depth.

Since the deflection of the slab will vary inversely as the cube of the depth it is probable that the stress will not vary directly as the section modulus, which varies as the square of the depth. This, however, would tend to increase the unit stresses due to edge loads, rather than decrease them, since the maximum moment would be increased.

The writer, therefore, believes that if the slab is to be designed to take care of edge and corner loads properly, the outside edges should be thickened for a width approximately equal to one-half the axle width, or usually about 30 in.

Tests made by the writer on small slabs, 10 by 10 in. and 10 by 20 in., with varying cross-sections (see Fig. 3 (*b*), (*c*), and (*d*), and Table 2) seem to support these conclusions.

The slabs were made of 1 part cement to 3 parts dry sand, with a water-cement ratio of 0.8. They were cured for 30 days under wet burlap and then broken on a bed of dry sand.

The slabs were loaded, longitudinally at the middle, along two lines, 5 in. apart, as shown by the load arrows in Fig. 3 (*b*), (*c*), and (*d*). The tests were made with the idea of obtaining the relative strength of the different sections.

Design *A* and Design *B*, as was to be expected, showed about the same strength when loaded symmetrically. Design *C* when loaded with one joint at the edge also checked very closely, in both cases, the symmetrical strength of Designs *A* and *B*.

It should be noted that the 20-in. slabs did not develop the strength of the 10-in. slabs of the same cross-section. This was probably due to a larger moment in the longer spans because of the greater length over which the loads were distributed.

It should be pointed out here that while Design *B* did not show any advantage over Design *A* in these tests, it is in fact an improvement over Design *A* when the moment is positive, giving about 50% additional strength over the uniform section. This is a very important advantage because it is due to positive moment from corner loads that the most serious failures occur. It does not, however, give a balanced section.

The writer believes that concrete pavement slabs, with a cross-section, proportioned about as shown in Design *C*, give a well-balanced design and would well justify the slight additional cost.

TABLE 2.—TESTS ON PAVEMENT MODELS.

10 BY 20-INCH SLAB.

	DESIGN *A*.		DESIGN *B*.		DESIGN *C*.	
Slab No.	Total load, in pounds.	Point load, in pounds.	Total load, in pounds.	Point load, in pounds.	Total load, in pounds.	Point load, in pounds.
1	680	340	565	283	542	271
2	460	230	710	355	561	280
3	655	328	785	392	650	325
4	605	302	595	298	597	298
5	675	338	....	....	604	302
Average	615	307	664	332	591	296

10 BY 10-INCH SLAB.

	DESIGN *A*.		DESIGN *B*.		DESIGN *C*.	
Slab No.	Total load, in pounds.	Point load, in pounds.	Total load, in pounds.	Point load, in pounds.	Total load, in pounds.	Point load, in pounds.
1	1 030	515	946	473	1 098	549
2	1 020	510	870	435	1 082	541
3	905	452	938	469	900	450
4	952	476	1 065	532	846	423
5	.....	....	.....	....	1 025	512
6	.....	....	.....	....	800	400
Average	977	488	955	477	958	479

Equations (1), (2) and (3) are interesting in that they support design theory developed from tests. It is to be noted that Equation (1) becomes

$$S_c = \frac{3\,P}{h^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

when k, the subgrade modulus, becomes zero, and $h = \sqrt{\frac{3\,P}{S_c}}$. Equation (4) is the well-known formula for edge thickness, first proposed by Mr. Older*.

H. O. ROOT,† ASSOC. M. AM. SOC. C. E. (by letter).‡—Many engineers who are responsible for the design of concrete pavements as one item among many other duties and responsibilities, will doubtless appreciate Mr. Older's presentation of formulas, tables, and recommendations on the subject. These, if followed intelligently, will serve to reduce the number of totally inadequate, as well as the number of excessively extravagant, designs that are now being used.

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1180.

† Inglewood, Calif.

‡ Received by the Secretary, March 26, 1928.

The writer still feels, however, that insufficient field data are on record to justify the use of formulas, tables, and "rule-of-thumb" methods in designing concrete pavement slabs. There are still too many variables and unknowns, and many of the most careful and elaborate calculations are yet to be checked and verified by field tests. While such a design as that recommended by Mr. Older will probably be adequate, it still might not be as finely balanced as possible and, therefore, not quite as economical as possible.

One instance illustrating the need of further field studies is brought out very clearly in the paper*:

"In these [Bates Road] tests the same tire equipment passed over both edges, the corners, and the mid-portion of all test sections, and no serious damage occurred in sections of uniform thickness, except at corners. Evidently, then, the actual value of the edge stress could not have been as great or, at any rate, as serious as would be indicated by results obtained by Equation (2) (edge stress) as compared with Equation (1) using the same value of a in each case."

There may be, then, a difference in the "seriousness" of a certain stress according to whether it is developed at the corner, edge, or mid-portion of the slab. Perhaps it is even far less serious at the mid-portion than at the edge. In these tests, as far as they went, both the mid-portions and the edges of sections of uniform thickness were found to be strong enough, but the corners failed. It would be interesting to know just how much stronger the corners should have been to have caused the initial failure to result from the edge stress.

Another question might be raised as to the recommended use of steel wire mesh or closely spaced bars for the purpose of taking up temperature stress by dragging half the width or length of the slab (while contracting) across the sub-grade. The choice between slip dowels and thickened edges at joints would be an important factor in determining the amount of steel required. It would seem to be cheaper to cut down the length of the slabs so as to eliminate the need of steel for this purpose. The effective life of such steel acting as a doweling across an open crack would be comparatively short in any event. Closer spacing of joints would tend at least to reduce the number of "subsequent cracks", if not to eliminate them entirely. The fact that certain 25-ft. sections of the Bates Road developed transverse cracks under heavy traffic may have been due to a combination of temperature stress with that of transverse bending, neither one of which, acting alone, would have caused failure. Still closer spacing of joints, such as the Seattle type "dummy", might have so minimized the temperature stress that failure under traffic would not have occurred.

It seems logical to continue working with the object of developing a design in which all corners and edges would be definitely predetermined. Then it would no longer be necessary to strengthen the entire mid-portion for the purpose of protecting edges along subsequent cracks, or to provide all edges with such tremendous strength as to enable them to hold up corners formed by such subsequent cracks.

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 149.

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PAPERS AND DISCUSSIONS

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GENERAL CONTRACT SYSTEM VERSUS SEGREGATED CONTRACTS

Discussion*

By Messrs. C. C. Muhs, W. J. Barney, Lazarus White, L. S. Stiles, Gustave R. Tuska, T. Kennard Thomson, and H. M. Dougherty.

C. C. Muhs,† M. Am. Soc. C. E. (by letter).‡—Contrary to the author's opinion, the number of contractors with technical training is legion. His statements§ that:

"* * * many factors affecting the ultimate cost of a project transcend entirely the field of design." * * * "In some instances, engineers or architects have successfully performed this management function * * * not by reason of their technical skill and training, but in spite of it." * * * "When an engineer or architect awards interlocking portions of a project to several contractors * * * , he is not exercising professional duties, but the duties of a business executive"

show a bias that is, fortunately, rarely held by a contractor with technical training and somewhat infrequently by the "practical" contractor.

In the early history of construction work, plans, in the sense of drawings, were few and sketchy, and the architect (master builder) or engineer (one who devises and controls) was practically the general foreman on the job and controlled in detail the processes used and the manner of their execution. During that epoch there was little segregation of the crafts, and even later when a contract was "letten" to John Doe, carpenter, to build a certain bridge, it was not assumed, therefore, that he knew more about building bridges than the man who designed them, but merely that by this device less detail supervision would be required by the engineer. At present, in Continental

* Discussion of the paper by Ward P. Christie, Assoc. M. Am. Soc. C. E., continued from April, 1928, *Proceedings*.

† Const. Mgr., Hooper & Januseh, Inc., Chicago, Ill.

‡ Received by the Secretary, March 10, 1928.

§ *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, pp. 395, 398 and 399.

Europe, Asia, South America—in fact, practically all over the globe except North America—all engineers and architects are contractors, in the sense used by Mr. Christie, and all contractors are either engineers or architects. In view of this fact, Mr. Christie's statement* that "he [the designer] might as well repaint his shingle to signify that he is a * * * 'Construction Contract Broker', for he is no longer strictly speaking a professional man", loses its significance.

The problem then "boils down" to a matter of expediency, because engineers are not inclined to cavil about contractors having encroached upon their field of management.

As to things affecting expediency, which Mr. Christie denominates the owner's interest, the author believes† that some architects or engineers "unquestionably lean toward it [segregation] to escape the backfire of inadequate plans and specifications." The writer believes that those guilty of such omissions lean to the general contract as more likely to cover up their sins. Both practices, of course, are based on a fallacy, because the owner pays either directly or indirectly.

It is admitted that "mere legal or contractual control without physical facilities for performing almost any part of the work does not assure completion within the time or at the cost intended",‡ but since no general contractor in building work performs more than a few of the many details involved he is not in much better case in this respect, than the engineer or the architect.

The writer takes issue with the statement‡ ending, "* * * because a broker does not customarily offer effective guaranties".

Further, the author states,§ "* * * to let an owner proceed in such cases without informing him of his liabilities, does not have the earmarks of fair play". If such practice exists, this statement would be a serious indictment. A considerable experience has failed to show that it is at all frequent.

There is a fallacy in the statement‖ that, "the true obligations and equities of the various agencies of construction are so confused in the minds of American Courts that there is little hope of pinning liability where it belongs".

The owner's liability to sub-contractors and the public is only slightly affected by the existence of a general contract, his security in that case being the reliability of the general contractor—not the contractor's existence; and there are also reliable sub-contractors.

Furthermore, the writer questions the logic of the following statements:¶ (*a*) "If the various operations are not 'time-scheduled' in advance and constantly synchronized on the job, delay is inevitable"; (*b*) "Without an exist-

* *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, p. 399.
† *Loc. cit.*, p. 396.
‡ *Loc. cit.*, p. 398.
§ *Loc cit.*, p. 399.
‖ *Loc. cit.*, p. 400.
¶ *Loc. cit.*, pp. 400, 401 and 403.

ing organization trained for the work, * * * the latter [the adviser] is practically helpless to control the time of completion"; (*c*) "The complications of * * * labor * * * may readily paralyze the separate contract job"; (*d*) "* * * cutting and patching * * * is extremely difficult to produce"; and, (*e*) "* * * safety measures are practically impossible without some centralized management * * *."

These are the functions that an engineer performs as an agent of the owner, and, with the exception of patching, he can perform them as well, inherently, as the contractor.

To state* that "the professional man is not in a position to handle the situation", is merely to express an opinion.

As to the general contract case, Mr. Christie states:†

"* * * The essence of general contracting lies * * * in the highly specialized function of management, the success of which depends on the personal skill and direction of capable executives."

This is wholly true, but since most of the superintendents, estimators, and contract men who have become executives in this sense are employees of technical talent and since such talent may be just as readily employed by the practicing engineer, it makes, not a case for general contracting as a whole, but for each individual concern.

The following statements appear in the paper:‡

"Assuming for the sake of analysis that plans and specifications are correct and complete, then, under a general contract, the contract price is all that the owner is required to pay; * * * over-run, if present, is absorbed by the general contractor."

This is only partly true, because the greater part of such excess is extra work or "unforeseen delays", which will be charged to the owner in any case.

Certainly, "he [the contractor] has his regular contacts with sub-contractors",‡ but so has the engineer.

It is probably true that "this amount [5%] * * * is in reality a service charge".§ However, the inference that it includes such items as superintendent, watchman, protection, temporary heat, etc., is wrong, as all engineers, architects, and contractors know. Hence, a greater part of this 5% is profit than would at first appear.

If the fact that income tax reports indicate only a 2% average profit for contractors means anything exceptional, it means that contractors are not handling their work well. This, the writer doubts. The same conservative source of information forces the gloomy conclusion that "nobody never made no money nohow".

Many of Mr. Christie's observations are true, but many others contain grossly exaggerated and unfair inferences, as when he states:‖ "Conduits must be placed, not when the plumber or the electrician feels like doing so, but

* *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, p. 401.
† *Loc. cit.*, p. 398.
‡ *Loc. cit.*, p. 401.
§ *Loc. cit.*, p. 402.
‖ *Loc. cit.*, p. 403.

when forms and structural parts are ready to receive them". The inference that, under the control of an engineer or architect, the project would run "hogwild" is so often repeated throughout the paper that it should not go unchallenged. The plaint amounts to this, that engineers, by a system of segregating contracts, are usurping the functions and encroaching upon the field of the general contractor. This is measurably true, but it is not all the truth. Under such conditions engineers would be usurping the functions of the general contractor as defined fifty years ago which he, in turn, usurped from the engineer a hundred years ago. There is ample field for the general contractor without his embarking on a campaign of belittling the ability and integrity of engineers to whom he owes at least as much as they owe him.

W. J. BARNEY,* M. AM. SOC. C. E.—Although connected with a building organization, the speaker would like to discuss this subject from the standpoint of an engineer. There is no question that, after reading this paper, engineers and contractors will realize that there is little economic justification and little administrative justification—taking the broad picture as a whole—for attempting to break up a general contract into a number of separate contracts. On the other hand, this problem has been discussed because it exists, and since it exists there must be some causes.

The speaker believes that the practice of segregating and letting separate contracts on building operations has arisen largely through a weakness in the Engineering Profession. This does not apply to a great many engineers; but there are a number in the profession to-day, particularly in the building industry, who are selling their services, not as engineers, not as technical advisers, not as men who can lay out a plant for a better operation, but purely and simply as a type of super-purchasing agent.

The speaker has known cases where engineers have said:

"You give me the commission to build this building—not because I can lay out a plan so that you will have the best building for your money, not because I can design a more beautiful building; but because I can buy this building cheaper for you than you can buy it for yourself, or because I can buy it cheaper than my fellow engineers; I am the super-purchasing engineer."

A man selling on that basis grasps at straws to help him bolster up his argument. The factory owner says: "How can you buy this building cheaper?" And the engineer replies:

"I can buy the building cheaper because I am familiar with sub-contracts. I can save the general contractor's profits; you can eliminate the general contractor. For my fee I will furnish you the plans and specifications. I will do all the buying and I will let the contracts separately and there will be none of this large, wasteful general contractor's profits in this enterprise."

Often the factory executive accepts this type of argument; and, consequently, the Engineering Profession to-day is losing its stability and professional character because of the introduction of this element of purchasing agent.

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As to this question of the segregation of contracts, it will work itself out in the long run. If it has an economic justification, the speaker will cease to exist as a general contractor and will become a sub-contractor on the concrete skeleton. However, there is something foreboding to the Engineering Profession in this tendency, which, as a member of the Society, and as one who has been an engineer, the speaker dreads to consider. The profession should stop selling itself as a purchasing agent and go back to selling itself as a technical adviser.

Look at the subject from the standpoint of facts. Suppose that an engineer has sold the owner on the basis that he can do the work cheaper. What are the facts in the case? He sends out and gets bids on the electrical work, on the plumbing work, and on the elevator work; and he says to the general contractor: "I have no objection to your doing the same thing, and we will see if you can buy as cheaply as I can."

On a preliminary estimate no sub-contractor to-day practically ever gives his lowest price, because he has been trained in the hard school of experience, and he knows that when the contract is closed and the buying begins he is going to be asked to reduce his price somewhat, perhaps to justify the existence of a purchasing agent, and perhaps also to justify the idea that the engineer is a clever fellow who can always take a few dollars off. Therefore, the engineer is asking for a price and the contractor is also asking for a price. The chances are that the estimates will be the same, although the following possibility enters: The sub-contractor may sense that he is not going to deal finally with a general contractor, but with an engineer purchasing agent, and in order to put himself in a better position, he will give a price closer to his final proposition than he gives to the general contractor; but this seldom happens.

With these separate estimates—which the engineer totals as against the general contractor's bid—the general contractor guarantees his estimate and the engineer does not. The engineer then calls in the plumber and the heating man and the others, whose estimates may have amounted to $25 000, and says to them: "You can have this job for $20 000." Perhaps they haggle awhile and it is finally closed for $23 000. Then the purchasing engineer goes to the owner and says: "Here are the original bids that I received from the general contractor, and here is what I purchased it for; I have saved you 5 or 10%, and I have more than justified my fee." He has not done anything more than the general condition permitted both to the owner and the general contractor; but he has set up a false picture of conditions, and has created a myth that he can buy more cheaply.

There is no need to pursue the aftermath of this method of handling a building operation—its costly lack of co-ordination—because Mr. Christie has covered this phase fully. The speaker has no intention of suggesting that the engineer is not essential; he is very necessary. Nothing is more unpleasant for a general contractor than to be left to face the whims and misunderstandings of an owner without a competent engineer as an arbitrator; and the speaker always prefers to have a contract under the supervision of an engineer or an architect, provided he is competent and provided he has put his services on a professional basis and not on that of a purchasing agent.

LAZARUS WHITE,* M. AM. SOC. C. E.—The speaker has had approximately thirty years' experience, about half of which has been as an engineer administering general contracts—mostly public works—and the remainder as a contractor—partly as a sub-contractor and partly as a general contractor; so perhaps his experience has been more rounded, or has covered a larger field than others.

As a sub-contractor, naturally the speaker might want to deal directly with the owner, so as to escape some of the hard general contractors; but in the interests of the owner himself and disregarding the special interests the speaker believes that the owner is better off with the general contractor. He certainly would be with the average class of general contractor.

The speaker's firm has often taken work directly with the owner, using the owner's architect (who is really his representative) or an engineer purchasing agent. They are very punctilious, and after making a plan, they are so proud of it that they will not depart from it. The general contractor has not prepared the engineer's or the architect's plans, and he is, therefore, much more free to depart from them. The result is that the owner does not get what he ought to get; he does not get the experience and advice of the different specialists that are on his building.

In one very glaring case, an architect prepared a plan for a contract which was taken directly from the owner. This plan passed the Building Department; it provided foundations for the larger columns, but not for the smaller columns loaded to 40 or 50 tons each. The owner would not put those in. He said the plan had passed the Building Department and must be all right, and this was an extra and he was not going to pay for any extras. What should be done in a case like that? A general contractor would have "ironed" this out very quickly. There would be a new foundation provided for the loads and an "extra" that would be in the interest of the owner.

Of course, this whole subject comes down to the beginning of the matter; that is, has the contractor an economic justification? If the contractor cannot justify himself economically, he deserves no special consideration; but after long experience the general contractor has survived. The reason is this, that by and large, the general contractor's profits, although in special cases they may be large, will average a rather low percentage.

The efficiency of the general contractor—a man of peculiar type, who has usually fought his way up and is one surviving of a great many—generally is much higher than that of the owner, and is greater than the difference indicated by his fee. Suppose that he makes 5% on the general contract (3½% is more nearly correct). The owner, although he may be a very good clothing manufacturer, is not a very good builder. The fact that he is very good at something else probably means that he has not the ability of a general contractor, although he often thinks he has. He cannot make up that difference; he cannot make up the contractor's profit.

It is the same way with the engineer, who, of course, will naturally think that he can make up the difference of a contractor's profit because he designed the structure, and he has investigated conditions and thinks he knows all

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about it. Perhaps he does; but he lacks certain abilities. If the engineer designs the job himself, he may want to save the contractor's profits.

This also applies to public works, where the community, through the engineer, endeavors to do the work directly. If they can do it by that means more cheaply than through a general contractor, they ought to do it. The speaker will say that although it may destroy his business, but he does not believe they can do it more cheaply. He has seen too much of public work and the way the public business is conducted.

Necessarily, when he starts a job, the contractor has more freedom. He selects his men; he does whatever is necessary to get the particular men he wants. He builds up an organization, and that organization has to make good and knows it, and usually does. Public work is not managed on that basis. It has to be done with a public organization, plus a few additions, and the same efficiency cannot be obtained that is usually obtained by a general contractor.

In connection with public works the speaker would say that the general contractors on subway works are really general contractors. It would be difficult to segregate the different contracts. The excavation, steel work, etc., could not be segregated underground. The only alternative to a general contract would be for the public to do its own work, which has the objections that have been stated.

L. S. Stiles,* M. Am. Soc. C. E.—Some time ago, the Company with which the speaker is associated, was having a large structure erected under a general contract. The general contractor, naturally, brought in several miscellaneous trades. Had it not been for the experience of the men handling this contract, in co-ordinating the trades so that all lines worked together without overlapping or lagging behind, it is the speaker's opinion that progress with the structure would not have been as satisfactory as it was.

The speaker's work is not confined to building projects, but also includes equipping plants with all kinds of machinery and apparatus and other operations of a miscellaneous nature. Very often, under these conditions, it is found that the general contractor fits the situation. The work can be started later. The beginning can be postponed until such time as it is felt that the adequate facilities for proceeding with the work are there, and if it is then to be in the hands of a general contractor—a good co-ordinator—progress is made.

The speaker knows of many cases of owners of different kinds of business who have been very successful in their particular line. Some of them have attempted to do the requisite building themselves when expansion was necessary. They thought that the general contractor would be somewhat in their way and that he would be collecting money for work that they could do themselves.

However, he has never known an instance where the owner did not get into "hot water" before he was finished with the work. The owner has his sphere; the engineer has his sphere; and so has the general contractor; and they should not clash at all. It is true the owner should have a proper engi-

* Constr. Engr., The Brooklyn Union Gas Co., Brooklyn, N. Y.

neer, but what is really important is that none of the parties should overstep its own bounds. Companies that are enlarging their plants and facilities by extensions may engage, now and then, in the building business, but the general contractor is always in the business and, because of this, he has better relations with the trades. The average sub-contractor, in working on an extension or a new project, is apt to feel that, while he secured this order, he may not get another from the same owner. However, if he is working for a general contractor, he feels that he will have to make good, because when that particular work is finished, the contractor will have other projects on which he would like to be considered.

The speaker thinks that the attitude of the sub-contractor is different when he is doing business through the general contractor and when he is doing business through the owner, and that this is one of the great advantages of having a general contractor. He has never known a set of plans and specifications to agree, in every detail, with the finished job. The specifications never exactly include all the features that the owner hoped eventually to have built. Because of this, there is bound to be some over-reaching and some under-reaching, and the general contractor is far better fitted to "iron out" these difficulties than an owner himself would be.

The cost to install a certain item under a general contract is always more than the credit which would be allowed if it were eliminated. The speaker has had many particular instances of this. Not long ago, on a contract, it was decided not to buy a certain quantity of a certain item. The contractor was quite agreeable to the cancellation, but he still insisted that he should receive his profit on the cancelled item. This does not seem a reasonable demand.

The speaker has handled millions of dollars of contract work. It has been let out, sometimes by segregated contracts and sometimes through a general contractor, depending on the particular conditions, and both methods have proved satisfactory. The important thing is to be sure that both parties comprehend the project and that the work is organized in the proper manner. When, in the judgment of the owner's engineer, the scope of the proposed work warrants it, the speaker believes that a high-grade general contractor should be allowed to handle the job.

GUSTAVE R. TUSKA,* M. AM. SOC. C. E.—The question has been raised, as to whether the client is to rely on the general contractor or on the engineer for success in the work undertaken. The speaker has, at various times, acted as general contractor and, at other times, as engineer. The differences in the qualifications necessary for a successful engineer and a successful general contractor are fully appreciated. Nevertheless, it must be remembered that many successful contractors were previously successful engineers and received an important part of their training as such. There is considerable doubt as to whether they would have been equally successful contractors without such previous training.

In discussing this subject, it should be emphasized that all building construction does not belong to one class of work. The importance and magnitude

* Cons. Engr., New York, N. Y.

of metropolitan building construction is fully appreciated, but there is a large amount of construction work concerned with special industries. This includes the various lines of factory construction where special machinery and special types of construction are required. In such cases the special experience required to handle the work successfully is more likely to be found in the experienced engineer than in the general contractor.

From the discussion and from the cases cited by the author, one would be likely to reach the conclusion that most engineers handling construction work for their clients are exceptionally incompetent and that, on the other hand, most general contractors doing similar work are invariably absolutely reliable. Experience will show that there are efficient and inefficient engineers and capable and incapable general contractors and that, in the main, the success obtained will depend to a great extent on the kind of each class that has been employed on the work rather than on the class itself.

It may be of interest to contractors and engineers to know that both contractors and engineers have been described as experts using their best judgment, with the difference that the contractor backs his judgment with his own money while the engineer backs his judgment with his client's money.

T. KENNARD THOMSON,* M. AM. SOC. C. E.—This entire discussion reminds one of the discussions thirty-five to forty years ago when the statement was frequently heard that "Engineers could not make good contractors". This was before so many engineers had become very successful contractors. At that time the speaker answered such a statement by saying that he could sit down and typewrite a letter, taking a long time to do it; but if he did so for months, he would be as quick as an ordinary stenographer; and, by the same principle, a good engineer was not a good contractor until he had learned contracting by hard practical experience. If he then found that he had good business as well as engineering ability there was no reason why he should not excel as a contractor.

This has been abundantly proved by the experience of many engineers. The main reason why general contractors are worth while is that they have learned their business, have their organization, and, in addition, are financially responsible for what they do.

On the other hand, if there is no general contractor, the person who does the work of a contractor is merely an agent of the owner, who could not be successfully sued by his client, and is often untrained for the job.

H. M. DOUGHERTY,† M. AM. SOC. C. E.—The author has covered part of a very large subject, but his paper seems to have been limited largely to the construction of buildings (which the speaker has always looked upon as the province and problem of architects), and to construction in the Metropolitan District. Possibly, this is only natural in New York; but it must be remembered that the broad field of engineering and the allied business of contracting extends throughout the United States and Canada; and that the American engineer, with New York as his base, does work in South America, Europe,

* Cons. Engr., New York, N. Y.
† New York, N. Y.

Asia, and Africa. The speaker is possibly qualified to comment by twenty-five years' experience which, starting as a Constructing Quartermaster in the Army, leads successively through Engineer in Charge for the Contractor on Government contracts with the Bureau of Yards and Docks, and Engineer in Charge and Resident Engineer for the Contractor on public utility works, to Construction Engineer and Chief Engineer in charge of large construction works for operating corporations—this throughout the United States, in Hawaii, and in South America.

The speaker believes that the viewpoint of many contractors may be expressed as follows:

"We have grown into large contracting firms, and the Engineer, as an individual, is setting himself up as competent to do what we, as a corporation, advertise ourselves able to do; and what he, on account of professional ethics, may not advertise himself able to do. We feel that the Engineer as an individual must stay off the ground which we desire for ourselves. We as General Contractors will as, and if, necessary hire the Engineer; but we must be left alone in the field and the Engineer must be satisfied to be our employee; or the adviser and buffer between us and the owner."

In the case where the owner is not organized, or he cannot enlarge to take in construction; where he has not a purchasing department that he may bring to bear on the purchasing involved in construction; and where the life and size of the work are not such as to warrant him in purchasing or hiring the necessary equipment; the general contractor has not only a proper place, but a place which he fills with profit to himself and to the owner.

Where, however, the owner is an operating corporation which has a purchasing department capable of enlargement to take care of construction problems; where there is an organization which may be enlarged; and where there is work in hand which is either so continuous or of such size that there is warrant for investing in or renting proper equipment; a general contractor is out of place. For him to enter such situations complicates them and adds to the expense of the owner. The general contractor in these cases cannot use the knowledge of the operating force to the same full advantage as the individual engineer either in design or in construction; and the end is dissatisfaction both to the owner and to the contractor on account of the friction engendered.

That an engineer of wide experience may, with profit to these owners, be placed in charge of an engineering and construction department, taking the place of the general contractor, is to the speaker's mind not open to doubt or question. To lay down as a principle that the engineer of wide experience must move to the rear to make room for the general contractor, under any conditions, takes very debatable ground.

The experienced engineer of to-day is not simply a man of science, not a man skilled in one particular line. He is an organizer; a man competent to carry on the entire business of design and construction; to originate methods; to carry them out; and to protect his principal in the matter of contracts, purchases, and labor contacts.

In the field described as not suitable for the general contractor, the writer holds a distinct brief for the individual engineer; and that his ideas are not entirely wrong is attested by the fact that so many public utility corporations, mining corporations, motor corporations, steel corporations, etc., find it to their advantage to have a department in charge of a competent and experienced engineer, to carry on the work the general contractor carries on in the contractor's proper field, for the owner who requires and needs the contractor's facilities.

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PAPERS AND DISCUSSIONS

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THE O'SHAUGHNESSY DAM AND RESERVOIR

Discussion*

By W. H. Dittoe and G. Gale Dixon, Members, Am. Soc. C. E.

W. H. Dittoe† and G. Gale Dixon,‡ Members, Am. Soc. C. E. (by letter).§—For the past twenty years the City of Columbus has presented an outstanding example among the moderately large American cities in its progressive attitude toward public water supply, an example established by the excellent initial development involving softening works completed in 1908, and maintained by the further improvements recently completed.

The authors have completed a thorough-going record of the system by adding this paper to the earlier exhaustive description‖ of the initial development.

The architectural effects of the O'Shaughnessy Dam are dignified and appropriate, and evidently the construction work is excellent; the design appears properly conservative in flood-discharging capacity and in the static resistance of the masonry overflow dam.

The "cascade" treatment of the natural rock slopes of the valley below the dam, for returning the spillway discharge to the channel of the river, is of special interest. Under flood condition similar to that of 1913, the spillway will pass a 9.5-ft. depth of water over its crest, and for about 49% of its length will discharge directly upon the unprotected limestone floor surfaces of the cascade at elevations above tail-water level; this discharge will travel over the cascade floor in thin high-velocity sheets at "the lower alternative stage" of flow. During the discharge of smaller flows, this condition will rule over a still greater width of cascade floor, while for the maximum anticipated flood—a depth of 12.5 ft. over the crest of the spillway—the higher stage of

* Discussion of the paper by John H. Gregory, C. B. Cornell, and C. B. Hoover, Members, Am. Soc. C. E., continued from April, 1928, *Proceedings*.

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§ Received by the Secretary, March 13, 1928.

‖ *Transactions*, Am. Soc. C. E., Vol. LXVII (June, 1910), p. 206.

tail-water will result in only a small reduction of width of floor unprotected by any back-water. It is to be hoped that a thorough record by photograph and measurement will be made of the behavior of this design, both in regard to hydraulics in flood, and to the ability of the rock to withstand the effects of flood and weather.

The writers have been impressed by the excellent opportunity for a full-scale study of the occurrence of the hydraulic jump and related phenomena bearing on the subject of erosion below over-fall dams, which exists at the Julian Griggs Dam. Considerable experimental work with small-scale models has recently been done along these lines, but there is a very definite need for systematic observations by coincident photographs and measurements of the performance of actual structures. The Julian Griggs Dam is of the ogee type, with a uniform height of 33 ft., and is subject to overflow depths which have ranged to 13 ft. Marked erosion of the limestone river bottom beyond the apron has occurred. At the lower stages, a well-defined "standing wave" appears just at the end of the short apron, which at higher stages is drowned out by the natural back-water of the channel.

It is believed by the writers that a far-sighted policy in the management of domestic water supplies in populous districts requires the acquisition of a more liberal width of marginal land surrounding reservoirs than has apparently been considered necessary at Columbus, and also a rigorous control of the use of land and water surface, to afford positive protection against serious pollution. In recent years a growing tendency has manifested itself, in the vicinity of centers of population, toward the development of summer cottage colonies near all bodies of water within easy reach which, in any measure, might afford recreational opportunities. Unless the responsible authorities take steps calculated definitely to discourage such developments in the vicinity of water supply reservoirs, the value of these reservoirs for their primary purpose may, in time, become seriously impaired.

It is the understanding of the writers that the use of the O'Shaughnessy Reservoir for recreational purposes has already gained headway through an ordinance of the Columbus City Council permitting boating and fishing, and that land immediately abutting the reservoir property has been allotted and is being sold by realty companies for residential purposes.

A striking example of the results which may follow the adoption of a lenient policy in regard to recreational use of the water and the marginal lands of a reservoir is to be found in the Milton Reservoir on the Mahoning River in Northeastern Ohio, built by the City of Youngstown in 1917 in connection with its water supply system. The Milton Reservoir is located about 16 miles west of the city and at a distance of about 40 miles up stream from the Youngstown Water-Works Intake from the Mahoning River within the city. The reservoir was constructed for use in regulating the dry-weather flow of the river, not only for the benefit of the municipal supply, but also for industrial purposes and, incidentally, for preventing nuisance from the discharge of the city's untreated sewage into the stream. The purpose of construction was therefore not parallel with that under discussion, but the result

of the policy adopted for control of the reservoir is none the less an outstanding lesson.

The Milton Reservoir is formed by a dam 55 ft. high, flooding 1 657 acres of land and impounding 9 000 000 000 gal. of water. The initial purchases of land by the City comprised 3 344 acres, or about double the flooded area. Much of this property was acquired under deeds which reserved to the grantors the right of access to and the use of the reservoir for boating, fishing, bathing, and other uses normal to an ordinary body of water. Subsequently, the City established a "permanent holding line" at a relatively short distance from the water's edge enclosing about 2 500 acres—about one and one-half times the flooded area—and the "salvage land" lying outside this line was divided into blocks and offered for sale. Under these conditions, an extensive development of the surrounding lands for summer-resort purposes has naturally followed, and the reservoir has become an important recreational center. About twenty-five separate allotments offering summer camp and cottage sites have been put on the market, and restaurants, bath and boat houses, dance halls, and other semi-public buildings have been constructed for the service of a summer population of several thousand people.

TABLE 7.—WIDTH OF MARGINAL LANDS ACQUIRED ABOUT VARIOUS IMPOUNDING RESERVOIRS.

Reservoir.	Land acquired, in acres.	Area flooded, in acres.	Ratio of land acquired to area flooded.	Length of shore line, in miles.	Mean width of marginal strip, in feet.
New York, N. Y.:					
Ashokan	15 222	8 180	1.86	40	1 450
Kensico	4 500	2 218	2.03	30	630
Schoharie	2 372	1 145	2.07	16.5	610
Boston, Mass.:					
Wachusett	5 163	4 200	1.23	35	230
Providence, R. I.:					
Scituate	12 547	3 600	3.5	38	2 150
Akron, Ohio:					
Rockwell	2 065	769	2.7	20	535
Columbus, Ohio:					
Griggs	452	363	1.2	....	65
O'Shaughnessy	1 159	829	1.4	....	250
Youngstown, Ohio:					
Milton	3 344	1 657	2.0	23	606
The Mahoning Valley Sanitary District, Ohio:					
Meander Creek (proposed)	5 248	2 010	2.6	40	690

The writers have recently had occasion to consider the availability of Milton Reservoir for the purpose of providing a permanent source of public water supply for the Cities of Youngstown and Niles, Ohio, in connection with the project of the Mahoning Valley Sanitary District for providing a complete modern system of works for these cities. The existing conditions about the reservoir are entirely proper in connection with its present function of general regulation of the river, but radical procedure would be required in preparation for its use as a permanent source of potable supply. Careful estimate indicated that in order to gain the necessary sanitary control of the

waters and the marginal lands, the taking of an additional area of 3 000 acres of land over and above that now owned by the city would be required, at a cost exceeding $2 000 000. The original total cost of the reservoir, including land and construction, was $1 321 000.

The Mahoning Valley Sanitary District is now preparing to develop a relatively uncontaminated upland source of supply for Youngstown and Niles, so that these cities will shortly abandon the use of the badly polluted Mahoning River water. In connection with this project, the writers have examined precedent in the acquisition of land about various important reservoirs, and submit Table 7, as of possible interest.

It should be noted that, with the exception of the Columbus Reservoir, the "mean width of marginal strip", as stated in Table 7, is derived by dividing the area of land taken above reservoir level by the recorded length of flow line; irregularity of shore line, of course, has an important bearing on the practical fixing of taking lines which is not evaluated in the arrangement of Table 7. Thus, the proposed Meander Creek Reservoir has a very irregular shore line, whereas the two Columbus Reservoirs are of quite smooth outline, each being confined principally to the main valley of the stream.

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CROWN STRESSES IN A SKEW ARCH

Discussion*

BY MESSRS. B. F. JAKOBSEN, AND EDWARD GODFREY

B. F. JAKOBSEN,† M. AM. SOC. C. E. (by letter).‡—The writer is interested in arches as applied to dams and has studied Professor Rathbun's paper with great interest. Bearing in mind the complicated stress distribution in the structure and the relatively simple testing apparatus, the agreement between the experimental values and the calculated values based on Professor Rathbun's theory,§ seems very good; especially when account is taken of the fact that in order to simplify the mathematics some approximations must be made in the assumptions and that the measurements are not exact.

Assuming that the mathematical deductions are correct, the tests show that the fundamental assumptions made by Professor Rathbun lead to reliable results. The way is now open for the advocates of "horse sense"‖ to test their "theories" and submit the results to the profession. These theories, based on "horse sense," are likely to have the advantage of simplicity and the disadvantage of being incorrect, as may be inferred from the number of failures recorded.¶ An exact theory cannot be obtained for the reason that the modulus of elasticity is not the same for all parts of the structure and varies with time; the deformations continue to increase with time even when the load is constant,** temperatures vary, etc.

The development of a useful theory depends largely on the ability and skill of the scientist to distinguish the essential from the unimportant and to

* This discussion (of the paper by J. Charles Rathbun, M. Am. Soc. C. E., published in February, 1928, *Proceedings*, but not presented at any meeting of the Society), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr. (La Rue & Jakobsen), Los Angeles, Calif.

‡ Received by the Secretary, February 23, 1928.

§ *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

‖ *Loc. cit.*, pp. 673 and 679.

¶ *Loc. cit.*, p. 659.

** "Shrinkage and Time Effects in Reinforced Concrete," by F. R. McMillan, Univ. of Minnesota, Studies in Engineering, No. 3 (March, 1915).

formulate such assumptions as will lead to reliable results without undue mathematical complications.

The writer is especially interested in Fig. 8* because it shows the great and rapidly increasing deflection of the arch as the breaking load is approached. When the load was increased from zero to about 102 lb., the arch deflected about 0.01 in., while when the load was increased from 102 lb. to 142 lb. (the breaking load), the additional deflection was 0.023 in. This means that, in the sections subjected to bending stresses, the neutral axis was shifted toward the compression side and the section was able to carry a much larger bending moment than calculations based on the tensile strength of the material would indicate.

Professor Bach† has made tests on cast iron in bending and, at the same time, has tested the cast iron in tension and compression in order to obtain the stress-strain curve. By this means he found that the bending moment necessary to bring about failure was about 1.66 times greater than the theoretical bending moment based on the tensile strength of the cast iron. He was also able to calculate the distance which the neutral axis moved from its theoretical position through the center of gravity of the section.

Bending tests on plain concrete beams have shown that the bending moment necessary to bring about failure is about twice that computed from the tensile strength of the concrete. This must be due to a shifting of the neutral axis, as was found for cast iron. The same phenomenon must make itself felt also when tension is not present; that is, when the resultant lies within the middle third of the section. This would mean that the structures have considerably greater strength than calculations with a constant modulus of elasticity would indicate and that added strength has undoubtedly saved a number of poorly designed structures. Therefore, the fact that a structure which, according to theory, ought to fail and yet does not fail, does not mean that the theory is not to be relied on, but rather that the ordinary theory errs somewhat on the side of safety. This is a fortunate circumstance for engineers; it means that structures have, so to speak, a certain over-load capacity.

Professor Rathbun's tests should help to give confidence in theories based on the fundamental assumptions. He has performed a real service.

EDWARD GODFREY,‡ M. AM. SOC. C. E. (by letter).§—Structural engineering in its proper functioning consists not in working out a theoretical problem but in building a structure that cannot be assailed from the standpoint of other theory, even if that theory appears to be of a lower grade and to savor of practical knowledge.

The writer is in agreement with George F. Swain, Past-President, Am. Soc. C. E.,‖ both in questioning the correctness of applying the elastic theory to concrete or reinforced concrete arches and in considering a skew arch as

* *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, p. 497.

† "Elasticität und Festigkeit," Ninth Edition, p. 298; see, also, Figs. 18 and 19 of the writer's paper, "Stresses in Thick Arches of Dams," *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 508.

‡ Structural Engr., Pittsburgh, Pa.

§ Received by the Secretary, March 8, 1928.

‖ "Stresses, Graphical Statics, and Masonry."

anything but an unsymmetrical arch subject to twisting stresses. Professor Swain gives some very sound reasons why the elastic theory is inappropriate for the solution of reinforced concrete arch problems. The extremely complex theoretical treatment of this subject should be eliminated and the "row-of-blocks" method, which is frankly an approximation and not camouflaged with meaningless and misleading mathematics, should be substituted.

Professor Swain states very aptly, "Nobody can ever know which method is the more accurate, but he should realize that more mathematics does not mean more accuracy". If the question were merely one of arriving at two nearly identical results by different paths, there would be no reason to object to any engineer's selection of the hard mathematical path, but the objection which the writer urges against the elastic theory for a thing that can be proved not to be elastic is that a weaker structure undoubtedly is the result.

The one serious count against the elastic theory for reinforced concrete arches is that it assumes absolute rigidity in the abutments. This generally means that the design of the abutments is given no consideration whatever. The writer does not recall a single instance where a proponent of the elastic theory for arches considers the possibility of abutments not being absolutely, or even approximately, rigid. Furthermore, although replies to arguments against the elastic theory on the grounds of rotating, sinking, or shifting abutments state that this movement must be theoretically treated, no hint is given as to how the amount of rocking or shifting is to be determined.

Another basic assumption in the elastic theory is that the concrete is elastic. This is almost as serious an error as to assume fixed ends. To state that a thing is elastic from an engineering standpoint does not mean merely that it will return to its original shape after being stressed and the load removed. There are any number of heterogeneous combinations of springy materials, that could be made to fill this definition. To be elastic in the engineering, mathematical sense can only mean one thing, and that is that every part of the member so considered is not only elastic in the ordinary sense, but has the same modulus of elasticity. It is well known that concrete is far from fulfilling this condition. The modulus of elasticity of concrete is not constant. One mixture of given materials may have one value and another mixture of the same materials may have a value differing from the first by 50 to 100 per cent. The entire theoretical treatment of the arch is based on this value being constant, and no opinion can eliminate the bald fact that the whole treatment is so much waste effort, if the modulus of elasticity is not even approximately constant. The elastic theory further assumes an original unstressed condition of the arch. This means that if the arch could be submerged in a fluid of exactly its own specific gravity it would assume exactly the shape that would fit its abutments. It is doubtful if any one believes that shrinkage and expansion of concrete do not tend to distort any arch that can be built.

Professor Rathbun's tests proved the variability of the modulus of elasticity of plaster of Paris due to the use of varying quantities of water. Cement being, like plaster of Paris, a colloidal substance, will give a product varying in the same way. The argument that modern concrete is an exact product

does not require a man to be on a field job more than one day to answer in the negative.

There is still another sense in which concrete is not elastic. Experiments have shown that, when subject to long-time tests, it takes a permanent set. This is a fact that need give little concern in an arch designed on the non-elastic basis with its liberal depth of arch ring. In fact this will tend to equalize the stresses. However, in the thin, reinforced concrete arch ring, designed on the elastic theory with part of the cross-section possibly in tension a permanent set is apt to be serious.

Professor Rathbun's tests on skew arches seem to confirm his theory set forth in his earlier paper,* and agrees with that of Rankine for skew arches. Theoretically, such arches act as right arches; not, as many believe, showing a tendency for the load to take the short course to the abutments and overload the obtuse corners of such abutments. It is to be noted that in these tests great provision was necessary to maintain fixed abutments. It is also to be noted that when the abutments were allowed to shift only a little, the tendency to overload the obtuse corners was very manifest; that is, the reaction went to the obtuse corner. It is exactly this tendency that makes the skew arch, built as such, a menace.

In a laboratory on a small model it is possible to maintain rigid or fixed abutments. In a bridge, this is not a possibility. Tests on a full-sized arch bridge† (119 ft. span) have proved that rotation of the piers is perceptible although the piers are founded on rock. This recent light on the action of concrete arches and the instability of their abutments is of tremendous significance. If a pier founded on rock rotates, what would happen where the foundation is ordinary soil, and how is any complex theory, that makes for the absolute minimum thickness in an arch ring on the assumption of perfect fixedness of abutments, going to justify its existence?

Professor Rathbun's model test shows that a small endwise movement of a pier in the skew arch affects the distribution of the thrusts, throwing the main thrust into the obtuse corners of the abutments. In a bridge there will be a very heavy component of the thrust on the pier tending to move it in exactly that way. To take this thrust there would have to be constructed special thrust-bearings, and these would have to bear against vertical surfaces of earth, which offer but uncertain resistance. Professor Rathbun recognizes the need of this provision in his second conclusion,‡ but it is to be noted that in his tests it was taken care of by adjustment. No adjustment is possible in a concrete structure, and settlement is always one of the things to be expected. A settlement against a horizontal surface of earth in response to an active horizontal force is certain and unavoidable. Such settlement means that the reactions assumed in the design are not realized and that all the design calculations are in error.

* "Analysis of the Stresses in the Ring of a Concrete Skew Arch," *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

† *Engineering News-Record*, January 24, 1924, p 159.

‡ *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, p. 502.

The writer is in agreement with Professor Rathbun's* earlier ideas on the subject of skew arches, expressed in a letter in which, in commenting on the failure of a skew arch at Tacoma, Wash., he states:

"By means of models it has been repeatedly shown that the thrust is greater at the obtuse corner [in plan] than at the acute. * * * Waddell, in 'De Pontibus', says, 'A skew bridge is a structure the building of which should always be avoided when it is practicable'. After a great deal of study on the theory of the skew arch the writer has come to the conclusion that Waddell was right, even if the remark was written before the days of reinforced concrete".

Clyde T. Morris, M. Am. Soc. C. E., has stated† that tests on a skew arch in a laboratory show that the greater part of the reaction is close to the obtuse corner. In some cases the reaction at the acute corner is negative. Hool‡ states that,

"Skew arches may be treated exactly as right arches, the span being taken parallel to the center line of the roadway and not at right angles to the springing lines of the arch".

It is this idea which Professor Rathbun's tests and comments foster, but which practical considerations show to be dangerous. The writer is of the opinion that, if a skew arch must be used, it should be done by constructing separate parallel ribs not bonded in such manner as to transmit shear from one to another.

The Bendigo Arch, a skew structure that failed,§ had especially massive foundations built on solid rock. The materials were excellent. However, the arch failed by crushing the abutments at the obtuse corners.

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* *Engineering News-Record*, April 12, 1923, p. 682.
† *Loc. cit.*, April 20, 1922, p. 638.
‡ "Reinforced Concrete Construction," Vol. III, p. 43.
§ *Engineering Record*, April 16, 1910.

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PAPERS AND DISCUSSIONS

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INLETS ON SANDY COASTS

Discussion*

By W. M. Black, M. Am. Soc. C. E.

W. M. Black,† M. Am. Soc. C. E. (by letter.)‡—This excellent and closely reasoned paper appears opportunely as a continuation of the recent study of tides.§ The study of tidal action has received from the engineers of America far less attention than it merits. In designing works of engineering intended to guide, utilize, or modify the action of the currents of tidal waterways, the preliminary studies should be directed first to the source of the forces in action, to the tidal energy available, and to the reasons for the physical conditions by which the results of the exertion of this energy are manifested. It is evident that any operation which will demand the use of more energy than is available must prove unsuccessful unless at the same time provision is made for increasing the supply. Moreover, if at any point of a waterway, the free passage of the tide is choked, the energy available for useful work beyond is lessened. In a river with an alluvial bed, it is futile to form a channel with dimensions too great to be maintained by the current energy available. So, also, an undue narrowing of a lower reach of a navigable tidal waterway may prove destructive to the navigable channel above.

Colonel Brown's paper is a valuable contribution to the science of engineering. The description of the sand movement and of the tidal action at inlets and the mathematical formulas deducted therefrom cannot fail to be helpful to engineers dealing with problems of that nature. In commenting on the paper there is little to add.

* Discussion of the paper by Earl I. Brown, M. Am. Soc. C. E., continued from April, 1928, *Proceedings*.

† Maj. Gen., U. S. A. (*Retired*); Cons. Engr. (Black, McKenney & Stewart), Washington, D. C.

‡ Received by the Secretary, March 14, 1928.

§ "Tides and Their Engineering Aspects", by G. T. Rude, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., August, 1927, Papers and Discussions, p. 1069.

The writer's experience in general accords with Colonel Brown's observations. However, some further effects of wave action on sandy coasts can be stated. Under the influence of an onshore wind, perpendicular or oblique to the coast, the level of the ocean along the coast is raised. This creates a difference in head along the coast line, and the formation of a current close to the shore which flows toward the point of least resistance, the point of lower head. If the direction of the wind, and, consequently, the direction of wave movement, is oblique to the coast, the rush of water from the waves breaking on the shore advances up the shore obliquely to the shore line, and on reaching its highest point swings in the direction of the leeward side and returns again down the sloping beach obliquely to the leeward side, carrying with it some of the beach sand stirred up by the breaking wave. This action produces a continuous surface movement of the sandy beach to leeward. If a dike or groin be built to stop this movement, the first effect will be to produce a fill on the windward side with a recession of the shore line on the leeward side, as stated* by Colonel Brown, since no sand is moved in to replace that cut out and moved by the waves. After the fill to the windward of the dike has reached the limit set by the height and length of the groin, the sand will pass around the end and over the top of the groin, and the continuity of the shore line will be restored.

During heavy wave action, the sand of the sea bottom outside the beach low-water line is also stirred up by the breakers. This stirred up sand is carried to leeward by the coastal current formed as a result of the wind pressure. This movement is quite extensive, so that on a shallow offshore beach, immediately after a storm, there is found a series of troughs running parallel to the shore line, which are the channels formed by this coastal current. These furrows disappear a day or so after the storm has ceased.

During heavy winds the dry sand above high-water line is drifted in a manner similar to snow. This movement can be lessened by planting beach grasses and shrubs and by constructing sand fences.

Along the Atlantic Coast, sand is thus moved south by northeast storms and north by those from the southeast. The resultant movement is to the south, as is shown by the beach formations at inlets—long and pointed hooks on the north side of breaks in the cost line and blunted spits on the south side. A further proof is found in the size of the grains of beach sand which are much smaller along the Atlantic Coast of North Florida than on the New Jersey Coast.

The influence of the width and depth of inlets in determining the nature of the action of the tide, clearly set forth† by Colonel Brown, is illustrated by three consecutive inlets on the Atlantic Coast of North Florida, namely, the entrance to the St. Johns River, the entrance to St. Augustine Harbor, and Matanzas Inlet. The latter two are 35 and 49 miles, respectively, south of the mouth of the St. Johns River. At all these inlets, current and tidal observations were made under the writer's direction in 1888. The mouth of the St. Johns River was 2 100 ft. wide, and when the observations were

* *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, p. 507 *et seq.*
† *Loc. cit.*, p. 514.

made, was obstructed by an offshore bar, on the crest of which the depths ranged from 7 to 14 ft. The range of tide was 5 ft. Ebb slack occurred 3 hours before, and flood slack 2 hours after, high water. At St. Augustine, the inlet had a width of 2 000 ft. with an outer bar; and the depth on the bar crest ranged from 5 to 9 ft. The range of tide was 4.3 ft. Ebb slack occurred 1 hour and 10 min. after low water (5 hours and 5 min. before high water) and flood slack, 1 hour and 6 min. after high water. Matanzas Inlet was 1 000 ft. wide and the depth over the outer bar was 3.5 ft. Ebb slack occurred 11 min. before low water; and flood slack, 27 min. after high water.

It is seen that as the inlet narrowed and the depth on the bar decreased, the times of slack water inside the inlet approached more nearly the times of low and high water. At the mouth of the St. Johns River the tidal wave entered as a wave of oscillation. At the entrance to St. Augustine Harbor, the character of tidal wave action had been partly, and at Matanzas Inlet wholly, modified. Inside Matanzas Inlet the tide advanced as a wave of translation.

At the mouth of the Magdalena River, in Colombia, the tidal range is small, about 15 in., while the river discharge is great. There, the tides of the Carribean Sea act as a dam and cause a slackening of the outflowing river current, but there is no reversal of direction. During a prolonged dry season, the river water becomes brackish for some miles above the mouth, showing the existence of an under-current of salt water. This under-run of the salt water must be in a shallow sheet. As yet it has not been manifested in the current observations.

There is from time to time a sudden shift to the north of the location of the navigable pass across the outer bar of inlets after a slow movement to the south. The writer has always believed that this is due to the fact that the pass is the outlet for the major ebb tidal current movement from the inlet. When the southerly movement causes a lengthening of the channel between the sea and the inland waters, sufficient to decrease the slope appreciably, the difference of head between the inside and outside waters causes a break to be made in the bar crest at or near the shortest line between the heads. This frequently occurs during a heavy onshore gale. The littoral drift then again slowly pushes this break to the south. The profile of the crest of an outer drift and wave bar is more or less crescent in shape. The tidal wave of the sea arrives at all points of the bar crest nearly simultaneously, and the flood currents flow across this crest as over the rim of a saucer. The ebb currents reach the sea as a stream of water directed from a nozzle (the sides of the inlet) and spread in measure as the living force of the outflowing water becomes lessened and the pull of the lower head at the sides becomes effective.

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PAPERS AND DISCUSSIONS

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SILTING OF THE LAKE AT AUSTIN, TEXAS

Discussion*

By Messrs. Charles Schultz and John B. Hawley.

Charles Schultz,† M. Am. Soc. C. E. (by letter).‡—Because of the nature of civil engineering work, it is not often that engineers are afforded the opportunity to secure records, extending over the long periods of time, as given by the author. Certainly, this paper should make engineers consider this feature of design of reservoirs before proceeding with the expenditure of public moneys. Very little has been written along this line, and to one who has never had occasion to deal with this problem its possible seriousness seems almost unbelievable.

In 1914, the writer designed a dam in McKinney, Tex., to furnish water for a small country club. The dam was located along a deep ravine, having been constructed to hold back 18 ft. of water at the dam (100 ft. wide) and to form a lake ¼ mile long. As this was fed by small springs, no measurement was taken of the drainage area, which was flat, rolling, black land country, such as exists in a belt of land 100 miles wide, extending from the Red River to the Gulf of Mexico. This drainage area was approximately 1 000 acres, and in 10 years the run-off carried enough silt to fill the entire lake, which had to be abandoned. The average rainfall was approximately 40 in., and most of the territory was cultivated farm land.

On another occasion the writer constructed a levee in this same county, along Pilot Grove Creek, a tributary of the Trinity River, which has a width between hills of approximately 1 mile. The creek above this point was approximately 15 miles long, and carried so much silt that along the entire bank where the levee was constructed there was a fill of approximately 4 ft. that had been made since cultivation had begun, and stumps were encountered 4 ft. under the surface.

* Discussion of the paper by T. U. Taylor, M. Am. Soc. C. E., continued from April, 1928, *Proceedings*.

† City Engr., Tulsa, Okla.

‡ Received by the Secretary, March 19, 1928.

It is true that most rivers do not carry as much free sediment as some of the rivers in Texas, where the rate of rainfall is severe at times and the soil so light as to be easily washed away. However, this paper will remind engineers that these conditions should be studied and taken into account. The profession is certainly indebted to Professor Taylor for his very complete work.

JOHN B. HAWLEY,* M. AM. SOC. C. E. (by letter).†—This paper is the result of continuous observation and study by Dean Taylor during more than thirty-five years. He has repeatedly addressed Texas societies and associations, calling attention to the element of siltation and its menace to the success of water-storage projects. Data presented in the paper may be confidently accepted as accurate.

The author has not fully described the character of the drainage area of the Colorado River above Austin. Fig. 6 shows this region, with its areal geology, and the drainage area of the West Fork of the Trinity River above the Fort Worth water supply reservoir, "Lake Worth".

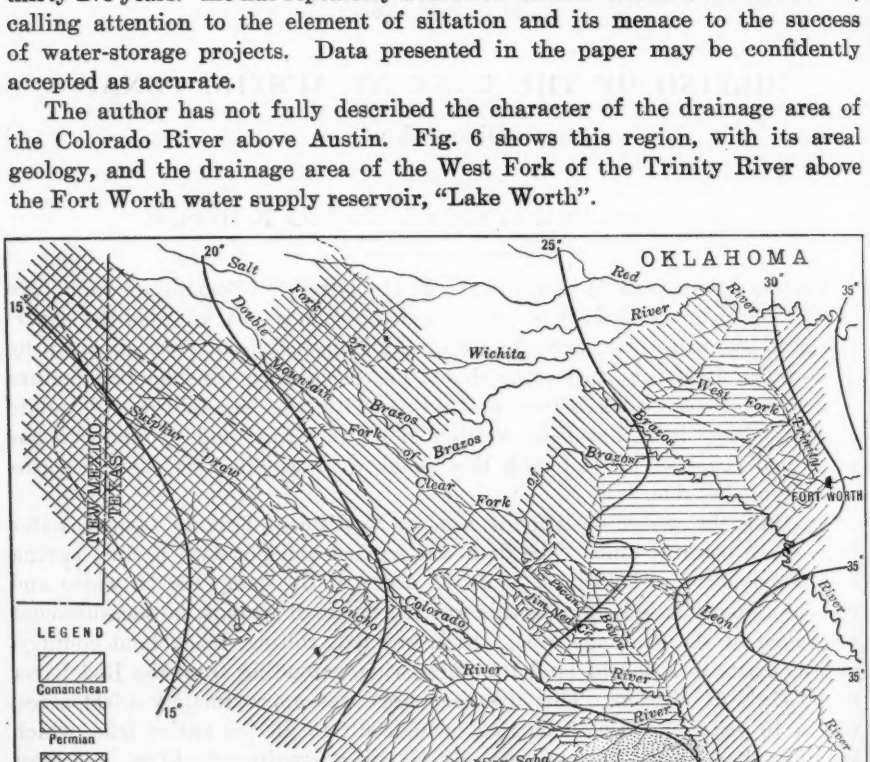

FIG. 6.—DRAINAGE AREA, COLORADO RIVER, ABOVE AUSTIN, TEX., AND OF TRINITY RIVER ABOVE LAKE WORTH.

The total drainage area of the Colorado above Lake Austin is approximately 38 000 sq. miles; that of the Trinity above Lake Worth is 1 800 sq. miles. The

* Cons. Engr., Fort Worth, Tex.

† Received by the Secretary, March 30, 1928.

areal geology, cultivated districts, and axial slopes of the two areas are quite similar; Austin being characterized by Comanchean (Lower Cretaceous), Permian, Pennsylvanian, Later Cenozoic, and the "Central Mineral Region" (Archæan), and Fort Worth by Comanchean, Permian, and Pennsylvanian.

On the Austin drainage area, the part designated "Central Mineral Region" (about 3 000 sq. miles) may be considered, for run-off and siltation purposes, as of the same general character as the Pennsylvanian. The upper 10 000 sq. miles (Later Cenozoic) has a mean annual rainfall of only 15 in.; many of the tributary creeks never reach the main stream at all, but lose themselves in sand flats and "sinks". It would seem improper as a siltation feature, therefore, to include this area. After deducting it, 28 000 sq. miles remain, for purposes of this discussion.

The average mean annual rainfall is approximately 24 in. The mean annual silt (for the two periods reported on by Dean Taylor) is about 2 800 acre-ft., or 0.1 acre-ft. of silt per sq. mile per annum. The Lake Worth drainage area is 1 800 sq. miles; the mean annual rainfall is (approximately) 28 in.; and the mean annual siltage since 1916, according to the U. S. Geological Survey and Texas Board of Water Engineers (approximately), 0.5 acre-ft. of silt per sq. mile per annum. This result was determined by "core soundings", because only a few original cross-sections of the lake basin were available.

The difference in character of the areal geology of the two areas is slight; the 4-in. difference in rainfall is scarcely sufficient to account for the vastly larger rate of siltation at Lake Worth. Medina Lake, 40 miles northwesterly from San Antonio, Tex., lies, with its entire drainage area (600 sq. miles), in the Comanchean rocks. Its axial slope is much steeper than that of the Colorado; its mean annual rainfall is about 28 in.; and its rate of siltation, about 0.3 acre-ft. per sq. mile per year. This drainage area is almost contiguous to the southern "divide" of the Colorado.

Personal inspection of substantially the entire Medina water-shed, in 1915, leads the writer to believe that the siltation rate at Medina Lake is naturally much less than at Lake Worth, due to the almost uniformly rocky nature of the Medina area.

Examination of Fig. 2(*a*)* indicates that even the old lake merely offered a rather large "channel section" for carrying storm waters, the velocity of which would probably be sufficient to carry a large proportion of fine silt over the dam. Fig. 2(*b*)* clearly shows that current velocities in times of flood were doubtless ample to transport all but the largest and heaviest silt particles over the spillway.

The Austin structure is substantially a "channel dam" and the lake created by it is merely a "channel lake" (Fig. 1†). On the Clear Fork of the Trinity, southwesterly from Fort Worth, are three "channel dams", built in 1895. The drainage area above the upper one of these dams is about 500 sq. miles. The geology is Comanchean, with an axial slope of 8 to 10 ft. per mile. The mean annual rainfall is 28 in. The river channel is 26 ft. deep and the dams are 14 ft. high.

* *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, p. 572.
† *Loc. cit.*, p. 570.

The first flood, after completion of these dams, left about 18 in. of silt deposit in each dam. The next flood carried out the silt left by its predecessor, and made a new deposit—and so on to the present time. In 1927—thirty-two years after construction—there is no more silt behind these dams than after the first flood.

In recent years, the writer has observed, at intervals, the discharge of the Colorado River over the crest of the Austin Dam. Its great turbidity and "soupy" nature, which is at times 15 000 to 20 000 parts per million, strongly supports the writer's idea.

If the floods from more than 30 000 sq. miles of catchment area have resulted in a siltation of only 2 800 acre-ft. per annum (instead of double or thrice this amount), it seems fairly clear that the bulk of the silt brought down by large river discharges must have passed over the dam. It would further seem that, had the impounded water body been of such contour as to constitute a "lake" in the usual concept of such a body, instead of a deepened river channel, its 50 000 acre-ft. capacity might have been 95% silted in 6 to 8 years.

Perhaps a 500 000 acre-ft. reservoir would have been the basis of an economically practicable project at Austin, but the one constructed never was, and never can be, an economic success because of an over-sized water-shed and an under-sized lake. A power company now contemplates constructing two 2 000 000 acre-ft. reservoirs farther up the river. Such reservoirs might help the situation at Austin Lake.

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PAPERS AND DISCUSSIONS

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UPWARD PRESSURES UNDER DAMS: EXPERIMENTS BY THE UNITED STATES BUREAU OF RECLAMATION

Discussion*

By B. F. Jakobsen, M. Am. Soc. C. E.

B. F. Jakobsen,† M. Am. Soc. C. E. (by letter).‡—This paper is an interesting account of some measurements of uplift pressures under dams. A study of Tables 3§ and 4‖ shows a progressive increase in uplift pressure intensities in nearly all the holes, except those nearest the down-stream face. This is very significant, and it appears that the water makes channels under the dam, progressing slowly until these channels finally reach the down-stream tail-water. Referring to Table 3 and comparing the values for May 2, 1923, with those of November 11, 1925, it will be noted that both the reservoir and tail-water elevations were lower on November 11, 1925, but most of the uplift pressures were higher, except in the holes, *D*, where these pressures have decreased and approached closer to the tail-water elevations.

It is also interesting to note in Fig. 19,¶ that in spite of grouting holes spaced 5 ft. apart under the cut-off wall and in spite of the drain pipe, *A*, the uplift pressure intensities in Pipe *B* are considerably higher than at the tail-water (see especially Station 13 + 00). Reading Column (*D*) under Station 13 + 00 in Table 4 and comparing these readings with the water elevation in the reservoir and the tail-water elevation, the gradual increase followed by a decrease, may be explained by the fact that the water slowly penetrates under the dam until it finally reaches the tail-water.

* Discussion of the paper by Julian Hinds, M. Am. Soc. C. E., continued from April, 1928, *Proceedings*.

† Cons. Engr. (La Rue & Jakobsen), Los Angeles, Calif.

‡ Received by the Secretary, March 31, 1928.

§ *Proceedings*, Am. Soc. C. E., March, 1928, Papers and Discussions, p. 698.

‖ *Loc. cit.*, p. 704.

¶ *Loc. cit.*, p. 707.

It would have been desirable if several pipes could have been located in fairly close proximity to each other; say, in groups, so that some information might have been gathered as to how regularly the uplift pressure occurs and how it is distributed under the dam. In the absence of any specific knowledge it would seem to the writer that the measured uplift should be assumed as acting under the entire base, although this would be an upper limit that cannot be reached, but can be approached.

It would be interesting to have these measurements continued because it is possible that silting might occur, the water might scour larger and larger channels as time goes on, or uplift might increase at one section and decrease at another.

All that experience can teach is that uplift does occur. It will undoubtedly vary for each particular site and may vary from section to section at any site. It is quite possible that some uplift may occur even in good rock and in spite of such precautions as grouting under the cut-off wall and by drainage wells close to the wall.

If it were not for the possibility of uplift, the design of a gravity dam, simple as it is, might be classed almost as an exact science. In reality, the first step toward the design of a so-called gravity dam, is to make an assumption as to how much uplift may be expected. If the base width of the gravity dam is less than about 85% of the height, some assumption as to uplift is involved. If the designer is too optimistic, an unsafe dam is likely to be the result.

Taken as a whole, the paper shows that uplift is a serious matter under gravity dams and that drainage and grouting cannot be relied on to eliminate it entirely.

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PAPERS AND DISCUSSIONS

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CONTINUOUS BEAMS OVER THREE SPANS

Discussion*

By Frank S. Bailey, Assoc. M. Am. Soc. C. E.

Frank S. Bailey,† Assoc. M. Am. Soc. C. E. (by letter).‡—Continuous beams have been studied so extensively in the past that the writer wondered when seeing the title of the author's paper what new treatment of the subject was possible. A reading of the paper, however, is sufficient to convince any designer of its value in eliminating the necessity for laborious computations in many cases.

Table 3,§ giving moment factors for maximum bending moments in three-span construction, should be a very real labor saver. A table giving the moment factors for three spans with fixed ends, would also be useful.

In a short paper entitled "Diagrams for Continuous Beams of Unequal Spans,"‖ the writer presented several curves of shears and moments for two and three-span beams, with supported and also with fixed ends, and generally carrying a uniform load over all spans. It is interesting to check the results in Table 3, with those found by the writer. The moment factors check with each other almost exactly for the case where all three spans are loaded. It should be noted that Table 3 gives factors for maximum bending moments but, while it gives a negative moment of $0.05\ w\ l^2$ as the maximum for the middle span in the case where the value of x is 0.50, this is really the minimum value of the moment, and occurs at the center of the span, as shown on Fig. 4 of the writer's paper previously mentioned, a negative moment extends entirely across the middle span and is a maximum adjacent to the supports.

* This discussion (of the paper by I. Oesterblom, M. Am. Soc. C. E., published in March, 1928, *Proceedings*, but not presented at any meeting of the Society) is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Asst. Engr., Metcalf & Eddy, Boston, Mass.

‡ Received by the Secretary, March 7, 1928.

§ *Proceedings*, Am. Soc. C. E., March, 1928, Papers and Discussions, p. 719.

‖ *Journal*, Boston Soc. of Civ. Engrs., Vol. 4, No. 8, October, 1917.

In comparing Fig. 3 of the writer's paper referred to, with the author's Table 3, it was found that the moments at the center of the middle span and at the two supports check very closely, but as a matter of fact, the maximum moment in the middle span does not occur at the center, but adjacent to the supports, where it is negative. This fact, of course, would be obvious by using Table 3 and plotting a diagram showing the moments at the supports as well as in the spans.

The maximum positive moment in the end span as shown on the writer's Fig. 3 is equal to $+ 0.024\ w\ l^2$, whereas Table 3 shows a maximum moment of $- 0.014\ w\ l^2$ in the end span, for all spans loaded. Here, again, the location of the maximum moment is really adjacent to the interior support.

The writer found a small positive moment in the end span while Table 3 gave a small negative moment as the maximum. On further reference to the paper the writer read* that, "it is necessary to introduce an approximation, by assuming that the secondary effects at the mid-spans will give results close enough to be added to the two principal moments." It is likely, therefore, that this approximation may account for the rather unimportant discrepancy.

It is to be hoped that the author's criticisms of existing building codes and other standard rules for moment factors, will have beneficial effects. Sponsors of the various codes undoubtedly wish them to be as trustworthy as possible, and defects when found will be corrected sooner or later.

Despite the best of building codes, accidents will happen if building inspectors and other officials and contractors are not extremely vigilant and perhaps sometimes even if they are. For several years the City of Boston has had a building code which will probably stand comparison with any other, and yet, because of some unforeseen weakness, a building in which a dance was being held a few years ago, suddenly collapsed, with shockingly tragic results.

* *Proceedings*, Am. Soc. C. E., March, 1928, Papers and Discussions, p. 715.

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PROGRESS REPORT OF SPECIAL COMMITTEE ON FLOOD-PROTECTION DATA

Discussion*

By Messrs. Gerard H. Matthes, F. G. Switzer, Harrison P. Eddy, C. E. Grunsky, Arthur E. Morgan, Arthur N. Talbot, A. J. Hammond, William Bowie, and John R. Freeman.

Gerard H. Matthes,† M. Am. Soc. C. E.—The progress report of this Committee is a plea for making available facts pertaining to floods, to serve as basis for the intelligent planning of flood control works and for educating the public toward a better understanding of the flood peril. The Committee proposes that this work be done comprehensively for all the rivers of the United States, thus creating a repository of flood data, complete and reliable to a degree that could not possibly be attained by individual, or even by cooperative effort of practising engineers.

It has been argued that research work of this kind properly belongs within the scope of activities of the Water Resources Branch of the U. S. Geological Survey. That Bureau is well qualified to undertake such work and in the past has made special flood studies for a number of rivers, the results of which have been published in the Water Supply Paper series. Inadequate appropriations have operated to stop these studies and Congress, although appealed to by the Board of Direction of the Society, has shown no inclination to vote funds for special compilations of flood data by that Bureau.

Good work has been done by a few State bureaus in making detailed studies of the floods that have occurred within their commonwealths, the States of California and Pennsylvania being notable examples. Although such State work is deserving of encouragement, it does not fill the requirements expressed by the Committee, for the reason that no two States are likely to follow identical methods, nor are the results of State work likely to be uniform as regards degree of completeness. Obviously, arbitrary limits would necessarily be imposed by the extent of funds available in each case. The net result would be

* This discussion (of the Progress Report of the Special Committee on Flood-Protection Data, presented at the Annual Meeting, New York, N. Y., January 18, 1928 and published in March, 1928, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr., New York, N. Y.

a heterogeneous collection of flood data that would lack that degree of practical usefulness to the Engineering Profession which the Committee primarily is seeking to bring about. A glance at the long list of interstate rivers cannot fail to impress one with the need of comprehensive and systematic work by a centralized agency.

The Committee's plan includes the collecting of data pertaining to floods that have occurred prior to the establishment of gauge records. The work done along this line by the Pennsylvania State Water Supply Commission has demonstrated the practicability of referencing early flood stages to present-day gauges with sufficient accuracy to make the results of practical value in estimating the probable frequencies at which floods of various magnitudes occur. There is more material of this kind available than is commonly supposed. Diligent research work is needed to bring it to light and to correlate it with present-day flood stages.

The attitude of the Committee toward flood formulas, as expressed in the report* to the effect that a general flood formula or series of formulas would not serve the needs of the profession, is not made in disparagement of existing formulas and their use, but in the firm belief that the Committee's duty primarily is the furnishing of the basic facts for whatever use may be made of them by the profession, including the derivation of formulas. Indeed, owing to the present increased interest in flood control, there is likely to be an increased desire for quick though approximate methods of estimating maximum flood discharges by means of formulas, and it is probable that new formulas will be devised. It is, therefore, all the more important that basic data for the derivation of such formulas be provided and be made more complete and more accurate than has been the case hitherto. Many of the early formulas were based on scant and often unreliable data, the only kind then available, and are doubtless subject to revision. Many elaborate studies of flood frequencies by probability and other methods have been based on flood data containing inconsistencies and discrepancies.

As an example may be mentioned the long record of observations on Tohickon Creek at Point Pleasant, Pa., which has tempted many an engineer to include it in his studies, not realizing that because of the gauge being near the mouth of the creek it has always been subject to back-water from Delaware River floods, thus effectually making the record valueless for flood studies. The speaker recalls high-water stages on the Allegheny River, appearing in early U. S. Weather Bureau publications, that were caused by back-water from ice jams. In the absence of proper notation, these high readings may easily be mistaken for flood crests and as such have doubtless operated to distort frequency studies in which they were utilized.

There is urgent need of a careful revision of flood data in order to eliminate this kind of error and also to distinguish between flood crest stages and 24-hour average flood stages. Some of the long lists of maximum floods that have been published in the past did not differentiate between these two stages and may have been misleading rather than useful in the preparation of flood formulas and diagrams.

* *Proceedings*, Am. Soc. C. E., March, 1928, Society Affairs, p. 164.

The Committee's plan of procedure is framed with special reference to taking care of such matters as these and should fill a long felt want among hydraulic engineers. In addition to listing flood stages and the corresponding discharges in second-feet and in second-feet per square mile of drainage area, the plan includes giving (for the more important floods) the discharge from day to day, or at lesser intervals if available, for the entire flood period, thus affording material for the preparation of flood hydrographs and for computing total flood volumes discharged. Standard forms for listing the various kinds of data were adopted by the Committee some years ago and have been published.* These forms will be found worthy of study in this connection.

It is a sad commentary on American progressiveness that so little interest has been evinced of late years in so important a matter as the protection of life and property against floods. This is broadly speaking and not in indictment of Federal and State authorities. Their hands have been largely tied by lack of public interest.

There is no denying the fact that the present generation, taking people by and large, knows less and cares less about the behavior of the streams in its respective neighborhoods than did its forefathers more than 100 years ago. River stages, especially the records of extreme stages, were necessarily matters of vital interest and common knowledge in the days when the population was concerned with natural waterways in the daily routine of life. No small part of this knowledge was handed down to the early settlers by the American Indian. He was the river man *par excellence*, and this is true more especially of the Indian tribes east of the Rocky Mountains. Much of their early knowledge of streams, so badly needed to-day, has vanished with them.

The apparent present-day inability to prevent the recurrence of great flood disasters is not chargeable to lack of engineering skill, but to the supine indifference toward the flood problem generally that is exhibited by thinking citizens during times when rivers are normal. It is this indifference that has made flood control provisions unthinkable for many localities in the United States. The inclination of many people to look on great floods as visitations from Heaven against which man is powerless is ingrained, and this is traceable in no small measure to the two accounts of the great deluge contained in the Scriptures. These accounts describe the Deluge, which so far as the best available information is concerned, consisted of nothing more than extraordinary floods in the Euphrates River,† as an act of God, indeed, as the deliberate act of a wrathful god. Sunday schools and even the common schools have consistently perpetuated this belief. If, instead, they were to teach that floods, both great and small, have their appointed places and proper functions in Nature's household, and that for civilized man to be caught in their path is an act of ignorance on his part, a saner attitude toward floods on the part of the general public would result.

* *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1065.

† Sumerian and Babylonian legends deciphered from cuneiform writings on tablets are in substantial accord to the effect that it was an intense rainfall lasting not more than six or seven days that produced the widespread inundation of the very flat valley lands along the Euphrates and Tigris Rivers in Mesopotamia. The Hebrew versions of the Deluge in the Book of Genesis agree in so many respects with these early legends as to leave no doubt that they refer to the same flood.

About $500 000 is now being spent annually by the Egyptian Ministry of Public Works* in collecting information, largely of an hydrologic nature, pertaining to the flow of the Nile and its major tributaries, the object being to bring about more effective control and beneficial use of the flood waters of that stream. This is not a new study. Rather, it was started decades ago and some of the continuous records now available extend back 60 years. There is probably no river in the world for which flood records are so complete as they are for the Nile, extending as they do over a 1 300-year period. This period contains a gap of 200 years with no observations, leaving a net total of 1 100 years of flood records. Nevertheless, an annual expenditure of $500 000 for further research appears economically justified. So far as the speaker is aware no annual appropriations to compare with this have ever been made for hydrological studies of any American river or river system.

An opportunity exists for some agency, be it the Society or some commission or foundation, or possibly a Government bureau, to perform a much needed service by educating the American public to understand the behavior of its natural streams and the utilization and control of the waters thereof. The Society, at least, can do much toward starting such a movement by setting about to secure the fundamental data and by publishing these in condensed but convenient form for use as proposed in the several progress reports of the Committee.

The undertaking is one of no mean proportions and may require years of painstaking research work and the expenditure of large sums. The time for initiating such an undertaking is now, while the widespread interest in flood control that has resulted from the flood disasters of 1927 favors doing research work of this kind.

Vast sums are being spent in this country for research work of many kinds, but little has been spent in investigating the causes of floods and flood damage with a view toward preventing their recurrence. When it is considered that one American astronomical observatory has at considerable expense and labor completed the cataloguing of nearly 250 000 stars, including in each instance the right ascension, declination, magnitude, spectroscopical analysis, and spectroscopical classification by groups, the wonder grows that similar expenditures of time and money have not been directed toward cataloguing the 250 000 floods that have ravaged American streams. If astronomical studies of academic interest are worth the effort stated, then how much more should this be true of flood studies having to do with the practical problem of protecting life and property.

F. G. SWITZER,† Assoc. M. AM. SOC. C. E. (by letter).‡—After Mr. Matthes' able discussion and the further emphasis given by the other speakers there can

* From "Progress in the Study of the Hydrology of the Nile in the Last Twenty Years," an Address by Dr. H. E. Hurst, Director General, Physical Dept., Ministry of Public Works, Egypt, delivered before the Royal Geographical Society, London, at its June 13, 1927 meeting, and published in the *Royal Geographical Journal*, November, 1927, pp. 440–460, which includes a discussion by Sir Murdock MacDonald containing additional data.

† Cons. Engr. and Prof. of Hydr. Eng., Sibley School of Mech. Eng., Coll. of Eng., Cornell Univ., Ithaca, N. Y.

‡ Received by the Secretary, January 26, 1928.

be no doubt as to the value of the work of this Committee. Surely funds can be obtained for working up data already collected.

In order to give concrete evidence to Mr. Morgan's statement regarding the unreliability of some of the published data on floods the writer wishes to quote a case in question. In studying the floods of a certain Southern river, it was desired to verify the magnitude of the maximum recorded flood. Values ranging from 250 000 to 400 000 sec-ft. were found, that were reported by presumably reliable engineers. The difficulty of estimating these quantities is well known; but the engineer using such information has a hard task to decide between such extremes. Much work is needed in checking information already available.

A new angle of attack has been presented* by C. R. Pettis, M. Am. Soc. C. E., which appears to have considerable merit. Is it too much to ask of the practicing engineer that he report to this Committee the results of his flood studies and a résumé of his methods? Such a collection of data and methods would serve as an extremely valuable source of information for research.

In 1924 and 1925 the writer had flood problems in connection with certain hydro-electric power developments. Part of the studies then made, served as a basis for his paper entitled, "Probability of Flood Flows", published by the Society.† A grant from the Heckscher Research Foundation, at Cornell University, has made it possible to engage an assistant to continue these studies on a broader basis.

Thus far, the flood problem has been presented from the viewpoint of the designer. Does this work meet the needs of the constructor? Cannot further information be obtained that will be of more value to him? Why not find the magnitudes of floods likely to occur in each month of the year?

HARRISON P. EDDY,‡ M. AM. SOC. C. E.—Probably no one would question the need of compiling, into a usable form, the existing data relating to floods. Records are available, extending over considerable periods of time. Others are so fragmentary as to be of comparatively little value except as they are supplemented by other information. The problem is to gather the material and to re-arrange it into such form as to be of greatest significance.

Any one who has tried to solve a flood problem in any part of the country must have found the acquisition of the desired data to be an undertaking of considerable magnitude. Analysis and interpretation is quite as large a problem. More often than not the results of such a study are buried in office files, or are presented in reports which are not printed and, therefore, are not available to other engineers.

This Committee has accumulated a large volume of data, although the work cannot be called complete, and more information is becoming available with the passing of time. The greater part of the work of compiling the information has been done. Some search may be required to obtain records relating to floods of earlier years which, although not providing data of accuracy

* "A New Theory of River Flood Flow."

† *Proceedings*, Am. Soc. C. E., April, 1927, Papers and Discussions, p. 563.

‡ Cons. Engr. (Metcalf & Eddy), Boston, Mass.

comparable to those obtained by actual measurement, nevertheless, may be invaluable in their application to particular streams, and in effect may increase materially the period covered by some records. This is an age of research. In almost every direction some form of research is being undertaken; and the speaker conceives that one of the first steps necessary is to bring together the available information and make it serviceable. The Committee has not yet been able to work over the accumulated data and put them in form for use.

That is briefly how the matter stands to-day. There is no doubt of the need; there is no doubt that a great deal of material is available—most of it has already been accumulated; but there is grave doubt whether the work required to complete the study and to make the data convenient for use can be financed. The problem to-day is one of ways and means to carry the work to completion.

C. E. GRUNSKY,* PAST-PRESIDENT, AM. SOC. C. E.—The speaker desires to express his interest in the report of the Committee and hopes that the Committee, despite the pessimistic attitude expressed in the report, will be continued, whether or not it gets the allotment of money that it asks for from the Society. Mr. Matthes, who has presented this question in a very lucid manner states that means should be found for making the necessary basic studies throughout the entire country in a comprehensive way.

The magnitude of a flood is not dependent alone on the quantity of water that falls, nor on the rate at which it falls; it is dependent as well on the retardation to which it is subjected as it flows from the water-shed area to the point of outfall. This varies for every water-shed; no two are alike.

The work that the Water Resources Branch of the Federal Government is doing in measuring stream flow should not be curtailed. It should be extended. There should be no hesitation by the Government in appropriating large sums for this study. It is only a few years ago that even so obvious a matter as the printing of the collected material was under discussion. There should be no hesitancy whatever in giving all the basic information to the public in printed form. Printing is about the cheapest thing which the Federal Government can do.

It is timely that a special effort be made to have the stream gauging work of the Water Resources Branch of the U. S. Geological Survey extended and to make available more complete information relating to stream flow (particularly to the flow at the time of flood) than has been done in the past.

The speaker was Assistant State Engineer in California about forty years ago when studies were being made of how to protect about 1 000 000 acres of land in the Sacramento Valley which were annually inundated by the overflow waters of the San Joaquin and Sacramento Rivers. It was comparatively easy to estimate the output of water from tributary streams. It was a difficult problem to combine the individual stream discharges for a determination of the flow of the main stream after the overflow lands were withdrawn

* Cons. Engr. (C. E. Grunsky Co.), San Francisco, Calif.

from inundation and could no longer serve as retarding basins. The method of making the combination has been published.*

Flood control is a subject of general interest. Each State has its problems. In the collection of data, there should be co-operation between the States and the United States Government, but the direction thereof should be by some Federal department. Special effort might well be made by the Committee and by individual engineers throughout the country to secure, from Congress, liberal appropriations for this purpose. It might well be said that no money can be expended by any Government to greater advantage than by faclitating research work. Activity along lines of research will contribute its share toward increasing the number of consumers, who, theretofore active as producers contributed to periodic over-production. Their employment on research improves market conditions. It is wise for the Nation to spend money liberally. Efficiency, and not over-stressed economy, will bring greatest benefit to the Nation.

Because of its bearing on the possibility of a measure being passed in Congress that will make a practically continuing appropriation for studies of the country's water resources, the speaker desires to relate a personal experience of about twenty-five years ago. A writer of prominence stated that he had just written a strong editorial against an appropriation of $40 000 by the State of California to be used for co-operating with the U. S. Geological Survey in the matter of stream gauging. He stated that he knew that the State of California had already investigated its water problems ten or twelve years previously; that all the needed information had already been collected and that the proposed appropriation would be a pure waste of money.

It was explained to him over the telephone that the engineers had been and still were in sore need of such information as was then being collected by the Water Resources Branch of the Federal Government for a period of fifty years back; that a river did not flow either at its maximum, or at its minimum, at the behest of the engineer; that the records must be continuous; and that engineers fifty years in the future would be extremely thankful for any data relating to flood flow, etc., which could be now assembled. He replied, "I had not thought of that; I think I will write an editorial in favor of the appropriation." The appropriation was made.

ARTHUR E. MORGAN,† M. AM. SOC. C. E.—It would be unfortunate for the work of this Committee to be interrupted. Some good work has been done, but a great deal more remains. With a comparatively small expenditure of time and money, engineers might possibly get the co-operation of the U. S. Weather Bureau, so that rainfall records, which are necessary in flood-control studies, can be made more pertinent to their needs.

The Weather Bureau is not an organization of engineers. Its personnel has probably made very few flood studies for engineering purposes and the manner in which the data are recorded and assembled, is not always satisfactory to the engineer. For instance, at different stations in the same region,

* *Transactions*, Am. Soc. C. E., Vol. LXI (December, 1908), p. 332.

† Pres., Antioch Coll.; Pres., Dayton Morgan Eng. Co., Dayton, Ohio.

records are made at different times of the day. The Weather Bureau has had the requirement, theoretically, that the time of beginning and ending of rainfall should be recorded. At least until the last few years, the speaker can state that that requirement was largely ignored; and when a study of rainfall is made through the U. S. Weather Bureau records, it is found that such a general failure to record beginnings and endings of the time of rainfall is a serious defect in the data.

Possibly the Weather Bureau would be very glad to co-operate with the engineers of the country if the engineering needs of weather records were made clear. One of the chief disappointments to hydrologists has been that Weather Bureau records are not more in accordance with engineering needs.

Another point is the need of examining very critically the origin and accuracy of the flood records in common use among engineers.

From the pioneer records of the late Emil Kuichling, M. Am. Soc. C. E., down to the present time there is found a great accumulation of records of great storms. As these records pass from one textbook to another, and from one publication to another, they come to bear the authority of inspired material. They are used as authoritative records. When followed back, especially as records of extreme flood flows, probably not one out of five is so dependable that it can be used with any assurance.

Studies of flood data should go back of those records to the data from which they were secured; and where that data cannot be found, where the engineer cannot get back to the basis of those judgments, he had better use the records with a great deal of caution.

This raises the question of the kind of records that will be kept of floods yet to come. Quite probably, the next great flood will have its maximum discharges in localities where gauging stations are too few, and estimates of flood flow will have to be made from cross-sections of the overflowed area, just as in the past.

It would be a great help to the collection of flood data if the exact manner in which those observations should be made, is determined and presented with clearness, precision, and conciseness, so that it would be possible for engineers of limited experience, by following such directions, to make more dependable estimates of flood flows.

These three factors—the perfection of weather records; the clearing up of a mass of flood data so that engineers can better appraise their value; and the co-operation of engineers in measuring floods as they pass—would greatly aid in the assembling of dependable data.

ARTHUR N. TALBOT,* PAST-PRESIDENT, AM. SOC. C. E.—Ever since the formation of the Special Committee on Flood Protection Data there has been a general expression of opinion that its field of work comprises a highly important engineering problem; and, further, that its plans for collecting data and making analyses and interpretations and presenting methods of considering the problems, involved work of a kind that should be carried out by the Society. The question has always been, however, where and how the money

* Prof. of Municipal and San. Eng., Emeritus, Univ. of Illinois, Urbana, Ill.

required for the project may be obtained; and that question the Committee has brought up again and again. The amount asked for—$10 000 a year—is a considerable sum for the Society to provide for the work of a single committee. The amount is not large in view of the needs of the work, but, doubtless, sources of funds other than the Society must be found.

Concerning the question of appropriations being made by the Government for carrying out parts of the project, the speaker has been informed that, within the Department interested, efforts have been made more than once to insert items in the annual budget for work to be done by the Government. The funds now available seriously limit the collection of data. Even the matter of analyses and comparisons, to bring the information to the point where it would be useful in a large way, has not been found practicable, considering the other duties of the personnel of the Department. The recommendations for the appropriation of such items have been stricken out "somewhere up the line". Somehow there did not seem to be the insistent demand that was necessary for their retention in the budget. It would appear that, if ever there is to be a feeling in Congress or in the Executive Departments of the Government, if ever there is to be a chance of appropriation for the work, it is now, because of the need that has been brought before the people in the past year by the floods both in the South and in the Northeast. It would also seem that the present is a most opportune time for obtaining an appropriation. If this is not successful, a strong effort should be made to raise the money from some other source.

Recently, an engineer connected with one of the Departments in Washington, stated that if several influential engineers, familiar with the situation, who know of the need for the appropriation and who have the ability to talk about it with authority, would go directly to the budget maker, General Lord, and to President Coolidge to make their request, he believed the item would be inserted. He thought that this method was the only way to get action, for Congress, except in great emergencies, is loath to insert an item in budgets that have been prepared in the regular way. In response to an expression of doubt that the President would interfere with the recommendations that had been made by the Bureaus and the Departments, this engineer, who is quite familiar with the methods used in making up the budget, said that he felt confident a successful presentation of the needs could be made in this way. This statement is given as the remark of a man familiar with Government procedure. Doubtless, there are many who would be quite competent to judge of the steps which should be taken to secure money for these activities, either from the Government or from other sources. However, it seems that the question at the present time is not so much what to do in the way of Committee work as how to get the money to carry on that work.

A. J. HAMMOND,* M. AM. SOC. C. E.—The U. S. Department of the Interior has taken up the matter of collecting stream-flow data and is engaged at present in the development of a program. It seems that, for a number of years, appropriations have been entirely inadequate for the U. S. Geological

* Chf. Engr., Mellon-Stuart Co., Chicago, Ill.

Survey to carry on such work as is needed. Every one knows that the information is valuable so far as it has gone, but that it has not been sufficient.

There is a bill before Congress that will increase the appropriation for this work from $147 000 to $420 000 in 1929, and for a period thereafter, $500 000 per year. It is believed that, in a period of twenty years, this amount will make it possible to get the desired information of the water resources of the country.

The engineers of the country should get behind this measure. The Geological Survey, it seems, is the proper organization to function in that respect, and the expenditure of money involved—$500 000 per year—represents a tremendous amount of work.

It is difficult to predict how much can be done with an expenditure of $10 000, even if that amount became available from the Board of Direction of the Society. Therefore, at the proper time, each engineer should get behind this movement to get the additional funds proposed by the Interior Department.

In the proposed survey of the waterways of the country, the subject of flood prevention and flood control will be one of the primary objects; and with that money available it seems as if it would be fully feasible for the members of that survey to make an efficient study of the problem.

William Bowie,* M. Am. Soc. C. E.—It would seem that the time has come to take up seriously the problem of the country's water resources. This subject should not be opposed, because it has a world of merit. However, it would seem that the Society should consider the question of whether or not it would be well to have a Bureau of the Federal Government devoted entirely to questions involving the development of water resources in their relation to electric power, river development, and flood control, and for other uses to which the water supply of the country can be adapted.

As all know, the primary purpose of the U. S. Geologcial Survey is to work in theoretical and economic geology. There are many engineers in that Bureau, but they are subject to the control of the directing officials, who are geologists. It would seem that the development of the country's water resources is a real and most important engineering work. Whether or not there has previously been any suggestion that there should be a special engineering bureau devoted solely to water resources is not known but, judging from expressions in daily papers and technical journals, it would seem that the time is about ripe for water resources to be given a special status. The U. S. Geological Survey has done splendid work in starting engineering operations along a number of lines but, as most of its engineering lines of activity developed, special bureaus were created. This occurred with the Reclamation Service, the National Park Service, the Bureau of Mines, and probably others.

If a new bureau were created to care for water problems, the best engineers available should be placed in charge of it. There is no intention to intimate that those in charge of the Water Resources Branch of the U. S. Geological Survey are not now eminent and competent engineers, but they do not have that

* Chf., Div. of Geodesy, U. S. Coast and Geodetic Survey, Washington, D. C.

freedom in a geological organization that they would have if they were in a separate bureau. There should be no adverse criticism of the geologists who are doing the very best that they can, but it seems to be certain that engineers would have just as great difficulty in guiding scientific research in geological problems as geologists are probably now having in handling the engineering phases of the water problems of the country.

It is not intended to make a motion or recommendation of a definite nature in regard to the matter under discussion; but it is well to have expression of thought on a very important engineering work of the Government with a view to obtaining the correct solution after giving the matter careful consideration. Of course, the Society can not create a Government Bureau, but its advice, no doubt, would be highly valued by the officials of the Government who deal with engineering needs.

JOHN R. FREEMAN,* PAST-PRESIDENT, AM. SOC. C. E. (by letter).†—The extremely important work of safeguarding against damage by future floods does not need to wait for much further progress in compilations of maximum flood discharge.

If one will study the various records,‡ he can find abundant grounds for believing that almost any drainage area of from 2 000 to 4 000 sq. miles, located anywhere from Maine to Illinois, inclusive, and as far south as the Carolinas, is liable to receive, sooner or later, a rainfall averaging 8 or 10 in. in depth within 2 days over this entire area.

In the case of any particular stream no one can tell whether it will come next year or four hundred years hence.

Study of the topography in New England and New York and elsewhere in the northern belt of States, convinced the writer long ago that evidences of these rare rainfalls and floods of extreme violence can be found sculptured in the topography in many localities. The record is in the form of gulleys, or in steep slopes of sand or gravel banks, which banks are evidently the result of post-glacial action and at a date which, while prehistoric, is so recent that the slopes have not yet had time to become greatly flattened by weathering.

Thaddeus Merriman, M. Am. Soc. C. E., Chief Engineer of the New York Catskills Supply, has observed such evidences of great floods within the Schoharie Basin. Professor Charles P. Berkey has stated that this region presents evidence of floods greater than any recorded on that stream within the historic period, but plainly not extremely ancient.

The late James B. Francis, Past-President and Hon. M. Am. Soc. C. E., when consulted about fifty years ago as to the proper dimensions for spillways for the reservoir dams of the Boston Water-Works in the Central River water-shed, recommended that they provide for a rate of flow equivalent to a run-off at a uniform rate of 6 in. per 24 hours. It is of interest to note that there were few, if any, localities that suffered from the recent floods in Vermont and Western Massachusetts where the rate of run-off was greater than this.

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* Cons. Hydr. Engr., Providence, R. I.

† Received by the Secretary, April 4, 1928.

‡ See, for instance, Technical Repts., The Miami Conservancy Dist., Vol. V.

Mr. Francis had made careful studies of the great storm of 1869 which swept up through New England from Central Connecticut, with a total precipitation of about 10 in., over an area of about 5 000 sq. miles, and an average of 8 in. over an area of about 10 000 sq. miles. He studied other great floods and came to the conclusion that anywhere in New England a run-off of 6 in. within 24 hours might be expected. This is equivalent to 162 cu. ft. per sec. per sq. mile.

The further observations of the past fifty years have justified his prudence, but have shown that, for smaller drainage areas of 2 to 10 sq. miles, this figure should perhaps be doubled for conditions favoring rapid delivery.

Nearly fifty years ago, the writer assisted, in a minor capacity, the late Hiram F. Mills, Hon. M. Am. Soc. C. E., in studies made for the City of Westfield, Mass., which had just previously been visited by a severe flood and also was in the course of the recent flood along the Westfield River. Mr. Mills recommended that an island in the river be cleared of brush and kept clear; that the dam near the city be lengthened; and that a dike a few feet high be built alongside the river. This dike is still in service and immediately after the recent flood, the writer learned that it had served to prevent all damage by the floods of the past fifty years so far as it extended, but that the town had grown down stream beyond its limits.

These facts show that engineers of broad horizon, skilled in such matters, could work out several solutions for problems of flood prevention fifty years ago before they had the far more abundant data since collected. It is not necessary to wait for more data before recognizing all the places where flood disasters may occur. The writer urges that surveys for flood protection should not be made to wait a year, or even a day, for more data to be collected.

Immediately after the recent Vermont flood the writer addressed a letter to the Governor of the State of Rhode Island, urging that a flood protection survey of the State be begun forthwith; that some competent engineer should be set at work sketching the drainage area of every stream along which it was conceivable that flood damage might occur. He urged that at each locality where bridges, dwellings, factories, or other structures liable to damage were located, the engineers simply compute the volume that would pass under Mr. Francis' old rule of a 6-in. depth of run-off from this water-shed within 24 hours (162 cu. ft. per sec. per sq. mile) and then estimate, from a survey of the adjacent channel, the height to which such a flood would rise. By this simple means one could very quickly locate all the danger points within the United States.

This preliminary rough and ready method should be followed up by a study in which the length, breadth, and declivity of the water-shed should be taken into account; also the modification of the natural flow by the existing detention reservoirs. The problem can also be studied in the light of the new theory presented recently for private circulation by Major Charles R. Pettis, Corps of Engineers, U. S. Army, M. Am. Soc. C. E., which carries a page of approval by Sherman M. Woodward, M. Am. Soc. C. E., which of itself is commendation of the highest order.

Major Pettis has derived an extremely simple formula in which the probable maximum quantity, in cubic feet per second, is the product of a simple equation containing only three factors: (1) A constant; (2) the probable maximum average depth of rainfall over the entire water-shed in a 100-year period; and (3) the width of drainage basin, in miles.

It will surprise most engineers to find how largely the maximum flow is dependent simply on the width of drainage area without any particular attention to its length or the declivity of the stream. Major Pettis presents more than fifty illustrations of the application of his formula to water-sheds, from Massachusetts to Virginia and Ohio, which show an amazing agreement between the results of the formula and the highest flood flow ever yet recorded.

There is altogether too great a tendency for people to be satisfied with provisions that will withstand the greatest flood within the memory of the oldest inhabitant or the greatest flood recorded within the last two hundred years. There has been far too great a tendency to provide a margin of safety of from only 1 to 3 ft. greater flood head.

For many years past, after having studied the flood factors in many places, the writer has never felt satisfied when designing a spillway on an important reservoir dam, to provide for less than double the maximum rate of flood flow ever known on that stream, and to check up by Mr. Francis' rule.

A factor of safety in the form of a multiplier not smaller than 2, and sometimes larger, is needed in the design of flood protection works just as much as the ordinary factor of safety is needed in designing the truss of a bridge where the elastic limit of stress in a part must be at least double that produced by the greatest loading, or that the computed breaking strength of a part must be four times the greatest load assumed.

MEMOIRS OF DECEASED MEMBERS

NOTE.—Memoirs will be reproduced in the volumes of *Transactions*. Any information which will amplify the records as here printed, or correct any errors, should be forwarded to the Secretary prior to the final publication.

WARREN MARTIN ARCHIBALD, M. Am. Soc. C. E.*

DIED JANUARY 16, 1928.

Warren Martin Archibald, the son of Charles D. and Martha R. Archibald, was born in Medford, Mass., on August 25, 1876. After receiving his preliminary education, Mr. Archibald was registered from 1895 to 1899 as a Special Student in the Massachusetts Institute of Technology, at Cambridge, Mass. During the summers he was employed as Rodman for the New York and New England Railroad Company, and also for the City of Medford. After leaving college, he was engaged as Rodman and Instrumentman for the Metropolitan Water-Works, in Boston, and Clinton, Mass.

In 1900, he entered the employ of Ford, Bacon and Davis, Consulting Engineers of New York, N. Y., remaining with this firm until 1908. During this time he acted as Draftsman and Designer on street railway construction, in Atlanta, Ga., later taking full charge of lines, grades, and construction for one division of track work in Birmingham, Ala. Subsequently, he went to Kansas City, Mo., where he was engaged as Chief Designer of special track construction and Resident Engineer on tunnel reconstruction, in charge of the building of three bridges across the Kaw River. From 1904 to 1908, he was in charge of track and building construction for the Nashville Railway and Lighting Company, in Nashville, Tenn. This work included the design and construction, with Company forces, of steel and concrete structures for a fireproof car barn and terminal building, power-houses, bridges, etc., valued at $2 000 000.

In May, 1908, Mr. Archibald returned to New England, where he served as Resident Engineer on the construction of a 12-mile, high-pressure pipe line and temporary dam and intake, in New Britain, Conn. In 1909, he acted as Superintendent for the contractor on dam construction, in Russellville, Ark.

During the following two years he was again with the firm of Ford, Bacon, and Davis, as Resident Engineer in Houston, Tex., on the design and construction, with Company forces, of a new underground conduit system, complete, with the necessary sewers, manholes, paving, wires, transformers, poles, etc., for operating the entire system.

In 1911, Mr. Archibald was appointed Engineer of Maintenance of Way with the Houston Electric Company, at Houston, in full charge of all track and overhead line construction. This work included paving, drainage, and public relations as well as the usual track and overhead problems. He was also engaged in a consulting capacity with the Galveston-Houston-Interurban Railway Company.

* Memoir prepared from information on file at Society Headquarters.

From June to September, 1917, he served as Superintendent of steam railroad track construction at Camp Travis, San Antonio, Tex., after which he again entered the employ of Ford, Bacon, and Davis, as Senior Engineer in charge of field and office work in the Report and Valuation Department. His work consisted of reports on public utilities, inclusive of all the States east of the Rocky Mountains and foreign countries. These reports related to financing, rate cases, etc., and demanded a complete inspection of the properties as to their condition and maintenance as well as to the general business conditions in the immediate vicinity. During 1920, he was Assistant to the Chief Engineer on the report and design for a water supply system, sewerage system, and sewage disposal for the Cities of Athens and Pireaus, Greece. Twice, during this period, he made trips into Honduras on valuation reports and the inspection of railroads and tram tracks, together with the shops and their equipment.

In 1924, Mr. Archibald resigned his position because of ill health caused by an attack of influenza some years before. He took a year's needed rest, and in June, 1925, entered the employ of Henry A. Symonds, M. Am. Soc. C. E., as Resident Engineer at Lynnfield, Mass. In 1926, he was appointed Assistant Engineer in the City Engineer's Office at Medford, and was made Acting City Engineer in December of that year. Just previous to his death, he had been told of his appointment as City Engineer.

He was a member of the Houston, Tex., Lodge of the Benevolent and Protective Order of Elks and of the Kiwanis Club and Mount Hermon Lodge, A. F. and A. M., of Medford. His church affiliations were Unitarian.

Mr. Archibald was quiet and unassuming, of a stern integrity, and always loyal to friend or employer. A former Chief Engineer, with whom he had been associated for several years, says of him,

"We have been through many a hard campaign together and no one knew better than I his sterling qualities as a man and his remarkable ability as an engineer".

He was never married, and is survived by his sister, Jennie S. Archibald, of Medford, Mass.

Mr. Archibald was elected a Member of the American Society of Civil Engineers on January 3, 1911.

GEORGE ANSEL CARPENTER, M. Am. Soc. C. E.*

DIED DECEMBER 15, 1927.

George Ansel Carpenter, the son of George Albert and Lydia Clark (Gage) Carpenter, was born on March 4, 1864, in what later became the City of Pawtucket, R. I. He was educated in the public schools, having been graduated from the High School in 1883.

On leaving High School, Mr. Carpenter entered the employ of the late D. Lawrence Wilkinson, a Civil Engineer of Pawtucket, and remained with him until August, 1888. Mr. Wilkinson became City Engineer when

* Memoir prepared by Francis B. Marsh and Frank E. Winsor, Members, Am. Soc. C. E.

Pawtucket was incorporated as a City in 1886 and Mr. Carpenter became his First Assistant. From August, 1888, to February, 1891, he was Chief Engineer for the Board of Sewer Commissioners of that portion of the Town of Lincoln, R. I., which shortly afterward became the City of Central Falls.

In February, 1891, Mr. Carpenter succeeded Mr. Wilkinson as City Engineer of Pawtucket and held this position continuously until his death, during a period of nearly thirty-seven years. The City of Pawtucket increased in population several fold during this time and his re-appointment year after year, regardless of the changes in political administration, is a striking tribute to his ability and to the universal confidence which he inspired, both among the politicians of all parties and among his fellow citizens. As City Engineer, he designed and supervised the construction of highways, sewers, water-works, bridges, and other municipal structures, and was called on, and expected to advise regarding a variety of problems covering almost the entire field of municipal activities, both engineering and non-engineering.

From April, 1899, to May, 1911, Mr. Carpenter was also Chief Engineer on investigations for grade-crossing elimination in the Cities of Pawtucket and Central Falls, in which work the late Edwin P. Dawley, M. Am Soc. C. E., acted as Consulting Engineer. His plans were finally adopted, and the work was completed in 1915.

The sewerage system of Pawtucket was designed and built by Mr. Carpenter, as were also the major additions to the original water-works. In recent years, routing of street traffic, street widenings and improvements better to accommodate motor vehicles, systems of street signals, etc., occupied his attention, with a resultant marked improvement in a difficult traffic problem.

Although Mr. Carpenter devoted his entire professional life to his native city, refusing to be attracted by offers of more alluring financial rewards elsewhere, his interests were broad and his acquaintanceship large.

He was a member of the Boston Society of Civil Engineers for 33 years and of the New England Water Works Association for 17 years, and was President of the latter at the time of his death. He was also a member of the American Society for Municipal Improvements for 22 years and was active in its work. He was one of a small group who, in 1915, organized the Providence Engineering Society. This Society, which had grown to a membership of 1 200 before he died, owes much of its marked success to his wise counsel and sustained interest. He served as its President in 1918-19.

Mr. Carpenter was a Past Master of Union Lodge No. 10, A. F. and A. M., and a member of the Holy Sepulchre Commandery, Knights Templar, of Pawtucket, and of Palestine Temple, Nobles of the Mystic Shrine, of Providence. He was a member of the High Street Universalist Society, of which he was President for about 20 years, a member of the Board of Trustees of the Pawtucket Memorial Hospital, and a member of the Pawtucket Lions Club.

He was a keen analyst, with an unusual facility of expression, carrying his points clearly and forcibly, but so kindly as to disarm opposition and to leave no sting among those at variance with him. He was extremely modest,

with friendly qualities which endeared him to a multitude of lifelong friends, as well as to the younger men of the profession who were so fortunate as to be associated with him. His native city owes him a debt of gratitude which is inestimable, and his influence in the community will continue to live after him and be an inspiration to those who follow.

On January 5, 1888, Mr. Carpenter was married to Jennie Smith Shepardson, of Pawtucket, who, with two daughters, Gladys Randall (Mrs. Norman R. Earle) and Margaret Allyn (Mrs. Raymond C. Newton), both of Pawtucket, survives him.

Mr. Carpenter was elected a Member of the American Society of Civil Engineers on May 7, 1902.

ROBERT L'HOMMEDIEU TATE, M. Am. Soc. C. E.*

DIED DECEMBER 20, 1927

Robert L'Hommedieu Tate, the son of Robert W. and Susan (L'Hommedieu) Tate, was born at Union Springs, N. Y., on September 3, 1888.

He attended the public schools of New York City, and was graduated from Morris High School in 1907. He then entered Cornell University, Ithaca, N. Y., from which he was graduated in 1912 with the degree of Civil Engineer.

Mr. Tate's preliminary professional apprenticeship was as a Surveyor and Topographer on the Costa Rica-Panama Boundary Survey in 1912. Subsequent to his return to the United States, he was engaged for a short period in the office of F. A. Molitor, M. Am. Soc. C. E., Consulting Engineer, New York City. Later, he became a Draftsman on a hydro-electric project in West Virginia.

In 1914, he was an Assistant Engineer for the Delaware, Lackawanna and Western Railway Company on Maintenance of Way and Construction, which position he retained until 1917.

During the participation by the United States in the World War, between May, 1917, and July, 1918, Mr. Tate was engaged in the military service of his country. He attended the first Officers' Training Camp, at Madison Barracks, N. Y., and on April 16, 1917, was commissioned a First Lieutenant in the Engineer Officers Reserve Corps. On May 26, 1917, he was ordered to active duty, having been assigned to the 303d Engineers, the Divisional Engineer Combat Regiment of the 78th Division. He went overseas with his regiment in May, 1918, was promoted to the rank of Captain in July, 1918, and served in the St. Mihiel and Meuse-Argonne offensives. During a portion of this period he acted as Regimental Personnel Adjutant. At the close of the war he returned to the United States, and was honorably discharged in June, 1919.

As a Junior Officer, Mr. Tate's military service was founded on the same patriotism as that which prompted so many of the younger engineers to "learn the game" at Officers' Training Camps. With many others of the same age he

* Memoir prepared by Frederic Molitor, M. Am. Soc. C. E.

"did his bit" with fortitude and success, both from a military and an engineering standpoint.

Following his discharge from the Army, Mr. Tate again entered foreign service as an Assistant Engineer on the design and construction of a town and plant for the mining of bauxite in Dutch Guiana. Here, he designed and constructed a complete town site, with business and residential buildings for 4 000 people, including the installation of electric lights, as well as a sewerage system and a surface mining plant. This work entailed a stadia survey, under difficult topographic conditions, through dense tropical jungles along the boundary line.

He returned from Dutch Guiana in April, 1921, and from the early part of 1922 until June, 1924, was engaged with A. B. Cohen, M. Am. Soc. C. E., Consulting Engineer, whom he assisted in the design, plans, and estimates for a reinforced concrete bridge that was to carry the Hudson Boulevard at Journal Square, Jersey City, N. J. From 1924 to the time of his death, he was engaged as a Resident Engineer on the Hudson Boulevard project, in charge of the field work incident to the construction of this bridge.

Mr. Tate's professional career was marked by a close attention to detail, thorough research studies, and a complete application to the problems assigned to him. He gave promise of a broad and distinguished future, and his many friends in the Engineering Profession expressed keen regret that such a career should have been cut short by an untimely and early death. Mr. Tate was noted for the warm friendships that he made in the profession and in the Army.

At the time of his death, he held a commission as Major in the Engineer Officers Reserve Corps. He is survived by his mother and his widow, Leonora T. (Van Derhoef) Tate.

Mr. Tate was elected a Junior of the American Society of Civil Engineers on March 4, 1913; an Associate Member on April 17, 1917; and a Member on December 14, 1925.

PHILIP AYLETT, Assoc. M. Am. Soc. C. E.*

DIED NOVEMBER 28, 1927.

Philip Aylett was born on April 30, 1867, at "Montville", the Aylett Plantation on the Mattaponi River, in King William County, Virginia. Although of unassuming mien, he could have boasted of an ancestry scarcely equaled by any of his peers. He could have claimed, among others, Alexander Spottswood, one of the early Governors of Virginia, and Patrick Henry, one of the founders of this Nation. His father was William Roane Aylett, who as Colonel commanded the 53d Virginia Regiment of the Confederate Army, and his mother was Alice Brockenbrough Aylett, of Richmond County, Virginia.

After Mr. Aylett had received a thorough preparatory training, he was entered, in 1884, as a Cadet in The Virginia Military Institute, at Lexington,

* Memoir prepared by Richard T. Goodwyn, Esq., Athens, Ga.

Va., from which he was graduated as a Civil Engineer in 1888. Afterward, he accepted a position as Principal and Instructor of Mathematics at Sunnyside Academy in Virginia. He held this position for one year and then commenced his life work as a Civil Engineer.

It would take more space than is permitted, to tell of the various surveys, engineering projects, etc., undertaken and accomplished by Mr. Aylett. His work carried him south as far as New Orleans, La.; west, as far as St. Louis, Mo.; and north, as far as Albany, N. Y. Early in his career he was engaged as Assistant Engineer in charge of tunnel work for the Baltimore and Ohio Railroad Company, at Harper's Ferry, W. Va. At another time, he was Construction Engineer on the Albermarle Sound Bridge for the Norfolk and Southern Railroad Company, and during 1894 and 1895 he was Engineer of Bridges on the Seaboard Air Line Railroad, at Portsmouth, Va. In 1906, he was appointed Principal Assistant Engineer on the Mississippi River Bridge at St. Louis. Traces of his engineering handiwork may be found in widely distributed places, among which are McKeesport, and Johnstown, Pa., Radford and Manchester, Va., and Winston, N. C., and Algiers, La.

The last years of his life were spent in and around New York, N. Y., where he was connected with the Valuation Department of the New York Central Railroad Company as well as with other similar important enterprises.

During the years between 1889 and 1927, Mr. Aylett gave unstintedly of his time in the furtherance of his chosen profession. He not only accomplished the work before him, but also found time to invent and patent a lock-joint pipe of very definite value, as well as a suspension system for constructing arch bridges without falsework. He also contributed illustrated articles on bridges, foundations, etc., to various publications, including *The Engineering News, The Cement Age,* and *Engineering and Contracting.*

He was the author of Part VIII of "European Bridges", in Volume III of Hool's "Concrete Bridges and Culverts", and relative to this, made a trip abroad in 1914 to inspect concrete bridges.

Mr. Aylett was a man of strong personality and of unblemished character, who took particular delight in trying to uplift the down trodden and oppressed. Although small in stature, he was of great mentality. A propensity for speaking, possibly inherited from his paternal great-grandfather, Patrick Henry, led him as a college student, and, later, as a mature man, to discard the usual reticence of the average engineer, thus evincing his ability as a forceful and fluent speaker. He was a member of the National Security League and during the World War, and thereafter, proved a valuable speaker on "Americanization and What It Means".

Mr. Aylett was married in 1900 to Hilma C. Fernquist, of Hartford, Conn., who, with one daughter, Elsie, survives him. While sitting at his desk with the day's work before him, he suffered a stroke of apoplexy and was called to the Great Beyond.

Mr. Aylett was elected an Associate Member of the American Society of Civil Engineers on June 3, 1896.

HUNTER IMBODEN SNYDER, Assoc. M. Am. Soc. C. E.*

DIED JANUARY 28, 1928.

Hunter Imboden Snyder was born on March 29, 1884, at Elizabethton, Tenn. He attended the public schools at Covington, Va., and at Knoxville, Tenn. His technical training was received at the University of Tennessee, at Knoxville, where he was a student in Civil Engineering from October, 1901, to June, 1902, and from October, 1903, to June, 1904.

Mr. Snyder's early experience in engineering work included the following assignments: From June, 1902 to October, 1903, various positions preparatory to that of Assistant Resident Engineer with the Knoxville, La Follette, and Jellico Railway Company; from June, 1904, to July, 1906, Assistant to the Engineer in Charge, the Knoxville Terminal Buildings and Clinch Avenue Viaduct for the Louisville and Nashville Railroad Company; from July, 1906, to January, 1907, Assistant to the General Superintendent and Field Engineer for the Oliver Company on the construction of a reinforced concrete warehouse at Nashville, Tenn.; from January to November, 1907, Superintendent for the L. B. Davidson Company, of Knoxville, in charge of concrete work for the South and Western Railroad Company; from November, 1907, to May, 1908, Inspector of concrete work for the Knoxville Division of the Louisville and Nashville Railroad Company; and from May, 1908, to March, 1909, Designing Engineer in charge, the Memphis Branch of the Contractors Engineering Supply Company, of Knoxville.

In March, 1909, Mr. Snyder entered private practice as Consulting Engineer on structural work. He was continuously employed in this field until July, 1915, and his work in this connection consisted of the design and construction of many important reinforced concrete warehouses, among which were the following: A five-story cold storage warehouse at Tampa, Fla.; a five-story warehouse for the Cheek Neal Coffee Company and a ten-story warehouse for Delcher Brothers, at Jacksonville, Fla.; also a warehouse for the Dalton Flour Mills, at Dalton, Ga. Other similar work included a mill construction warehouse for the De Sota Oil Company at Memphis, Tenn. He was Consulting Engineer for the F. W. Long Company on the construction of the sub-structure for the Municipal Lighting Plant and the remodeling of a power plant into a sub-station for the City of Jacksonville. He also acted as Consulting Engineer for the Architects on a hospital building for the St. Luke's Hospital Association at Jacksonville.

From 1913 to 1915, Mr. Snyder was associated with H. G. Perring, M. Am. Soc. C. E., as Consulting Engineer on the design and construction of various projects in Florida, as follows: A double-tube tunnel under St. Johns River at Jacksonville; a paving program for the City of St. Augustine; structural work on the Duvall County Court House and the Duvall County Armory; the Barstow Jail; the Palm Beach County Court House; the Tampa City Hall; the Hillsboro Hotel, in Tampa, and private road work in Baker County. Mr. Snyder was also associated with Mr. Perring on the following bridge

* Memoir prepared by J. L. Savage, M. Am. Soc. C. E.

work: A reinforced concrete bridge over the Halifax River, at Daytona; a bridge over the San Sebastian River for the City of St. Augustine; bridges over McGirts Creek and Trout Creek and a number of other smaller bridges in Duvall County, Florida.

Mr. Snyder's engineering work during 1915 brought him in contact with various bond houses in connection with the purchase of county and district bonds and his success in this field attracted much attention. In July, 1915, he accepted an attractive offer from Dewitt, Trimball, and Company and thereafter was employed by different bond houses as follows: From July, 1915, to June, 1916, with Dewitt, Trimball, and Company, Chicago, Ill.; from July, 1916, to March, 1918, with Seasongood and Mayer Company, Cincinnati, Ohio; from March, 1918 to May, 1919, witth C. W. McNear and Company, Chicago, and from May, 1919, to February, 1920, with R. M. Grant and Company, Chicago and New York, N. Y.

In 1920 he was compelled to give up his work that he might seek a more healthful climate and from then until his death, on January 28, 1928, he spent most of his time in Colorado, in 1922, definitely establishing his home in Denver.

Mr. Snyder was a splendid type of man, having a high order of integrity, a brilliant mind, and a certain natural friendliness which gave him an unusually magnetic personality. Although practically an invalid for the last few years of his life, he never lost his cheerful disposition nor his hopeful outlook. His continued interest in the activities of the Society and in engineering activities in general, was a source of surprise and satisfaction to those of his engineering friends who knew of his long enforced separation from his work.

His life was marked by devotion to his parents and to his wife, Nancy Abell Snyder, all of whom survive him.

Mr. Snyder was elected an Associate Member of the American Society of Civil Engineers on January 7, 1913.

FRANK WAGNER, Assoc. M. Am. Soc. C. E.*

DIED DECEMBER 24, 1927.

Frank Wagner was born in Philadelphia, Pa., on December 16, 1887. He received his preparatory school training in the public schools of Philadelphia, having been graduated from the Northeast Manual Training School in 1907. The same year he entered the Civil Engineering Department of the University of Pennsylvania from which institution he was graduated in 1911 with the degree of Bachelor of Science in Civil Engineering.

While at college he worked on structural and railroad work during his summer vacations and after his graduation he was engaged with the American Bridge Company, at the Pencoyd Plant, as Structural Draftsman. With the advent of the Blankenburg Administration, he entered the employ of the

* Memoir prepared by William G. Grove, M. Am. Soc. C. E.

City of Philadelphia, remaining from April, 1912, until June, 1918, first as Inspector and, later, as Assistant Engineer, specializing in roadway construction and the general improvement of highways.

In June, 1918, he became Inspector for the Merchant Shipbuilding Corporation, of Bristol, Pa., but only remained in that position a few months, transferring early the following year to the United States Railroad Administration, then to Cox and Stevens, and, finally, to the United States Army, where he remained until the spring of 1921. During this three-year period, he was engaged in inspecting machinery and other war materials, and also barges for service on the Mississippi River and the New York Barge Canal. About the latter part of this period, he was transferred to Cordova, Ala., to take charge of the design and construction of a large coal tipple. While at Cordova, in the spring of 1921, he suffered a nervous breakdown and was obliged to give up engineering and spend more than a year recuperating at Wernersville, Pa.

Restored to health in 1923, Mr. Wagner was for a time Town Engineer of Branford, Conn., and later, was with the Johns-Manville Company, as Estimator on building materials. In October, 1925, attracted by the Florida boom, he went South. After engaging in several projects he became, in 1926, Resident Engineer on the Matanzas Bascule Bridge, at St. Augustine, Fla., for the P. T. Cox Company, and remained on that structure until its completion in the middle of 1927.

During the spring of 1927, Mr. Wagner had an attack of influenza from which he never fully recovered. Soon after his return North in July, he suffered another nervous breakdown and went a second time to Wernersville for a complete rest. He seemed to be making progress toward recovery until about a week before Christmas when he had a relapse and was obliged to go to the hospital, where he passed away on December 24, 1927.

Mr. Wagner was never married. He was a member of the Lutheran Church and of the Masonic Order, and was a Licensed Engineer in the States of Florida and Pennsylvania. Perhaps his outstanding characteristic was his loyalty to his friends, particularly those with whom he grew up from childhood. His passing means a great personal loss to those who knew him best.

Mr. Wagner was elected an Associate Member of the American Society of Civil Engineers on May 19, 1924.